8 ARITHMETIC PROGRESSION

THEORY

8.1 INTRODUCTION :

In practical life, we must have observed many things follow a certain pattern, such as the petals of a sunflower, the holes of a honeycomb, the grains on a maize cob, the spirals on a pineapple on a pipe cone. In our day-to-day life we see patterns of geometric figures on clothes, picture, posters. They make the learners motivated to form such new pattern.

For example,

[→] [←] [→] [←] [?]

Solution: [→]

Likewise number patterns are also faced by learners. In their study, number pattern play an important role in the field of mathematics.

Ex. (i) 2, 4, 6, 8, 10 then next number is 12.

(ii) $4, \frac{1}{2}, \frac{1}{16}, \frac{1}{128}$ next number is $\frac{1}{1024}$.

Idea on A.P. was given by mathematician Carl Friedrich Gauss, who was a young boy, stunned his teacher by adding up

$1 + 2 + 3 + \ldots\ldots.. + 99 + 100$ within a few minutes. Here's how he did it:

He realised that adding the first and last numbers, 1 and 100, gives, 101; and adding the second and second last numbers, 2 and 99, gives 101, as well as $3 + 98 = 101$ so on .

Thus he concluded that there are 50 sets of 101. So the series is :

$$50(1 + 100) = 5050.$$

In this chapter, you will study only Arithmetic Progression (A.P.) and Geometric Progression (G.P.).

8.2 SEQUENCE :

The number patterns or arrangement of numbers according to definite rule or a set of rules is called a Sequence.

The various numbers occurring in a sequence are called its terms. The n^{th} term of the sequence is denoted by x_n. The n^{th} term is also called the general term of the sequence.

For example,

(i) The number $\langle 1, 4, 9, 16, \ldots\ldots.. \rangle$ represent a sequence written according to the rule $x_n = n^2$, $n \in N$.

(ii) The number $\langle 1, 3, 5, 7, \ldots\ldots... \rangle$ represent a sequence written according to the rule $x_n = 2n - 1$, $n \in N$.

(iii) The numbers $\langle 2, 3, 5, 7, 11, 13, ... \rangle$ represent a sequence of prime numbers.

In every sequence it is not always possible to write a specific formula.

Sequence is a set of terms which may be real, complex and an algebric expression arranged in a define order according to certain rule.

Ex. 1, 2, 3.............

$(x + 1), (2x + 2), (3x + 3), ...$

8.3 SERIES :

If $\langle x_1, x_2, x_3, ... \rangle$ is a sequence, then the expression $x_1 + x_2 + x_3 + ...$ is called the series associated with the given sequence.

8.4 PROGRESSION :

Those sequence whose terms follow certain patterns are called Progressions.

In this chapter, you will study two types of progressions (i) Arithmetic Progression (A.P.) and (ii) Geometric Progression (G.P.)

8.5 ARITHMETIC PROGRESSION (A.P.) :

A sequence is called an arithmetic progression, if the difference of a term and previous term is always same.

The difference is called the common difference of arithmetic progression.

The sequence $\langle x_1, x_2, x_3, ..., x_n, ... \rangle$ is called an arithmetic progression (A.P.), if $d = x_2 - x_1 = x_3 - x_2 = x_n - x_{n-1} = ...$

It is a sequence whose terms decrease or increase by a fix/constant number. This constant number is called common difference of A.P. and it generally denoted by 'd'.

$[d = a_{n+1} - a_n]$

Illustrations

Illustration 1

Find the A.P. whose 1st term is 2 & common difference is 3.

Solution

. Given : First term (a) = 2 & Common difference (d) = 3.

A.P. is 2, 5, 8, 11, 14,.......

Illustration 2

Show that the sequence on defined by $a_n = 4n + 5$ is an A.P. Also find its common difference.

Solution

$a_1 = 9, a_2 = 13, a_3 = 17, ...$ in A.P.

We have $a_n = 4n + 5$ .....(i)

Replacing n by (n + 1) we get

$a_{n+1} = 4(n + 1) + 5 = 4n + 4 + 5$

$a_{n+1} = 4n + 9$ .....(ii)

$d = a_{n+1} - a_n \Rightarrow d = (4n + 9) - (4n + 5) \Rightarrow d = 4$

Remarks:

(i) The common difference 'd' should be independent of n.

8.5.1 General Term :

a, a + d, a + 2d, a + 3d.........represents an arithmetic progression where a is the first term and d the common difference. This is called the general form of an A.P.

Let 'a' be the first term and 'd' be the common difference of an A.P. Then its

n^{th} term is $a_n = a + (n - 1)d$

n^{th} term of an A.P. from the end:

If 'a' be the first term and 'd' be the common difference of an A.P. having 'm' terms. Then the n^{th} term from the end is $(m - n + 1)^{th}$ term from the beginning.

[n^{th} term from the end: $a_{m-n+1} = [a + (m - n + 1 - 1)d] = [a + (m - n)d]$

8.5.2 Sum of First n terms of an A.P.:

The sum S_n of n terms of an A.P. with first term 'a' and common difference 'd' is given by

(i) $S_n = \frac{n}{2}[2a + (n-1)d]$

(ii) $S_n = \frac{n}{2}[a + l]$ where $l = a + (n - 1)d$, l = last term

8.5.3 Selection of Terms in an A.P.:

No. of terms	Terms	Common difference
3	a – d, a, a + d	d
4	a – 3d, a – d, a + d, a + 3d	2d
5	a – 2d, a – d, a, a + d, a + 2d	d
6	a – 5d, a – 3d, a – d, a + d, a + 3d, a + 5d	2d

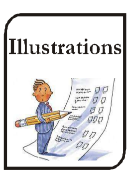

Illustration 2

The sum of three numbers in A.P. is 27, and their product is 504, find them

Solution

Let the three terms be (a – d), a and (a + d) where d is common different. It is given that

$(a - d) + a + (a + d) = 27$

or $3a = 27 \Rightarrow a = 9$

and $(a - d)(a)(a + d) = 504$

or $a(a^2 - d^2) = 504 \Rightarrow 9(81 - d^2) = 504$

$\Rightarrow 81 - d^2 = \frac{504}{9} \Rightarrow d^2 = 81 - 56 = 25 \Rightarrow d^2 = 25$

or $d = +5$ or $d = -5$

Therefore, the terms are: 9 – 5, 9, 9 + 5 = 4, 9, 14 **Ans.**

8.5.4 Arithmetic Means :

Three quantities are in arithmetical progression, the middle one is said to be the arithmetic mean of the other two.

Thus a is the arithmetic mean between $a - d$ and $a + d$

(i) Insertion of a single arithmetic mean between a and b

Let A be the arithmetic mean of a and b. Then

a, A, b are in A.P.

$A - a = b - A$

$$2A = a + b \quad \Rightarrow \quad A = \frac{a+b}{2}$$

(ii) Insertion of n arithmetic means between a and b

Let $A_1, A_2, A_3 ....A_n$ be n arithmetic means between two quantities a and b. Then

$a, A_1, A_2 ....A_n, b$ is an A.P.

$$\text{Clearly } b = a_{n+2} = a + [(n+2) - 1]\,d \quad \Rightarrow \quad d = \frac{b-a}{n+1}$$

Thus, the n arithmetic means between a and b are as follow:

$$A_1 = a + d = a + \frac{b-a}{n+1};\ A_2 = a + 2d = a + \frac{2(b-a)}{n+1};\ ....\ A_n = a + nd = a + \frac{n(b-a)}{n+1}.$$

$$A_n = \left(a + \frac{n(b-a)}{n+1}\right)$$

These are required arithmetic means between a and b.

Note: **Sum of 'n' arithmetic mean inserted between two numbers a and b is**

$$S = \frac{n}{2}(a+b) \text{ where } n = \text{number of arithmetic mean.}$$

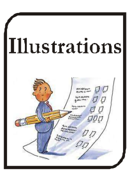

Illustration 3

Insert three arithmetic means between 3 and 19.

Solution

Let A_1, A_2, A_3 be three arithmetic means between 3 and 19. Then

$$A_1 = 3 + \frac{1(19-3)}{3+1} = 3 + \frac{16}{4} = 7$$

$$A_2 = 3 + \frac{2(19-3)}{3+1} = 3 + \frac{32}{4} = 11$$

$$A_3 = 3 + \frac{3(19-3)}{3+1} = 3 + \frac{48}{4} = 15$$

8.5.5 Properties of Arithmetic Progression :

Property–1: If a constant is added to or subtracted from each term of an AP, then the resulting sequence is also an AP with the same common difference.

Property–2: If each term of a given AP is multiplied or divided by a non-zero constant k, then the resulting sequence is also an AP with common difference kd or d/k respectively. Where d is the common difference of the given AP.

Property–3: In a finite AP the sum of the terms equidistant from the beginning and end is always same and is equal to the sum of first and last term i.e. $a_k + a_{n-(k-1)} = a_1 + a_n$
For all k = 1, 2, 3........(n – 1)

Property–4: Three numbers a, b, c are in AP if 2b = a + c

Property–5: A sequence is an AP if its nth term is a linear expression in n i.e. $a_n = An + B$ where A, B are constants. In such a case the coefficient of n in a_n is the common difference of the AP.

Property–6: A sequence is an AP if the sum of its first n terms is of the form $An^2 + Bn$ where A, B are constants independent of n. In such a cases the common difference is 2A. i.e. 2 times the coefficient of n^2.

Property–7: If the terms of an AP are chosen at regular intervals then they form an AP.

8.6 SOME IMPORTANT RESULTS

(i) The sum of first *n* positive integers

$S_n = 1 + 2 + 3 + 4 + ... + n$

$$\left[S_n = \frac{n(n+1)}{2}\right]$$

Note : The sum of first 100 natural numbers is 5050.

(ii) The sum of square of first n positive integers

$S_n = 1^2 + 2^2 + 3^2 + ... + n^2$

$$\left[S_n = \frac{n(n+1)(2n+1)}{6}\right]$$

(iii) The sum of the cubes of first 'n' positive integers

$S_n = 1^3 + 2^3 + 3^3 + ... + n^3$

$$\left[S_n = \left\{\frac{n(n+1)}{2}\right\}^2\right]$$

(iv) $[a_n = S_n - S_{n-1}]$

where a_n = nth term, S_n = sum of n terms

8.7 GEOMETRIC PROGRESSION (G.P.)

The sequence $\langle x_1, x_2, x_3 \ldots\ldots\ldots x_n \ldots\ldots\ldots\rangle$ is called a geometric progression (G.P.) if $\frac{x_2}{x_1} = \frac{x_3}{x_2} = \ldots\ldots$

$\frac{x_n}{x_{n-1}} = \ldots\ldots$, where none of $x_1, x_2, \ldots\ldots\ldots\ldots, x_n, \ldots\ldots$ is zero.

In general $\frac{x_{n+1}}{x_n}$ = constant (say r), $n \in N$

The constant ratio r is called the common ratio of the G.P. If the first term x_1 of the G.P. be taken as a, then the standard form of G.P. is $\langle a, ar, ar^2, \ldots\ldots\ldots\rangle$

8.7.1 Formula for General Term of a G.P.:

The n^{th} term of the G.P. written in standard form is given by $a_n = ar^{n-1}$, $n \in N$

8.7.2 Formula for Sum of First n terms of a G.P.:

The sum of first n terms of the geometric series

$$S_n = \frac{a(r^n - 1)}{r - 1}, \text{ if } |r| > 1 \text{ and } S_n = \frac{a(1 - r^n)}{1 - r} \text{ if } |r| < 1$$

Note : When $r = 1$, then

$S_n = a + a + a + \ldots\ldots$ upto n terms = na.

8.7.3 Formula for the Sum of Infinite terms of a G.P.:

If $|r| < 1$, the sum of infinite terms (S) of the G.P.

$$S = \frac{a}{1 - r}$$

8.7.4 Selection of terms in G.P.:

No. of terms	Terms	Common Ratio
3	$\frac{a}{r}$, a, ar	r
4	$\frac{a}{r^3}, \frac{a}{r}$, ar, ar^3	r^2
5	$\frac{a}{r^2}, \frac{a}{r}$, a, ar, ar^2	r
6	$\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r}$, ar, ar^3, ar^5	r^2

Note : Three numbers a, b, c are in G.P. if and only if $\frac{b}{a} = \frac{c}{b}$ or if and only if $b^2 = a.c.$

8.7.5 Geometric Mean (G.M.) of two terms a and b :

If a, G, b are in G.P. (a and b are positive), then G is the GEOMETRIC MEAN of numbers a and b.

We get $G = \sqrt{ab}$.

Inserting n Geometric means between two terms a and b:

Let a and b be positive numbers. Let $G_1, G_2, \ldots\ldots, G_n$ be such that $a, G_1, G_2, \ldots\ldots G_n, b$ is a G.P.

Then $b = x_{n+2} = ar^{n+1} \quad \Rightarrow \quad r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

Thus the n geometric means between a and b are :

$$G_1 = ar = a\left(\frac{b}{a}\right)^{\frac{1}{n+1}}; \quad G_2 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}; \quad \ldots\ldots, G_n = ar^n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$$

8.8 RELATIONSHIP BETWEEN A.M. AND G.M.:

Let A and G be A.M. and G.M. of two given positive real numbers a and b respectively, then

$$A = \frac{a+b}{2} \text{ and } G = \sqrt{ab}.$$

Thus, we have, $A - G = \frac{a+b}{2} - \sqrt{ab} = \frac{a+b-2\sqrt{ab}}{2} = \frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{2} \geq 0$

$\Rightarrow \quad A - G \geq 0. \quad$ Hence $\quad A \geq G.$

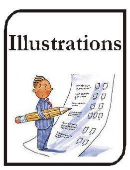

Illustration 4:

Find the sum of : $1, (1 + 2), (1 + 2 + 2^2), (1 + 2 + 2^2 + 2^3) \ldots\ldots(1 + 2 + \ldots\ldots 2^{2000})$

Solution

$T_1 = 2^1 - 1, T_2 = 2^2 - 1, T_3 = 2^3 - 1, \ldots\ldots\ldots, T_{2001} = 2^{2001} - 1$

[Since there are 2001 terms]

Adding, $T_1 + T_2 + \ldots\ldots, T_{2001}$

$$= 2^1 + 2^2 + 2^{2001} - 2001 = 2\left(\frac{2^{2001}-1}{2-1}\right) - 2001 = 2^{2002} - 2 - 2001$$

$\Rightarrow \quad 2^{2002} - 2003$

Illustration 5 :

2^{nd} term of a G.P. is 30 and 4^{th} terms is 750. Find the 3^{rd} term.

Solution

Let 'a' be the 1^{st} term and 'r' the comon ratio.

Now, $ar = 30$ .....(i)

$ar^3 = 750$ .....(ii)

Dividing (ii) by (i),

$r^2 = 25, r = 5$

$\therefore \quad a = \left(\frac{30}{5}\right) = 6$

$\therefore \quad 3^{rd}$ term $= ar^2 = 6(5)^2 = 150$

SOLVED EXAMPLES

Example 1

If a, b, c are in A.P., Prove that b + c, c + a, a + b are in A.P.

Solution

$b + c, c + a, a + b$ will be in A.P.

$(c + a) - (b + c) = (a + b) - (c + a)$

$a - b = b - c$

$2b = a + c$

Thus a, b, c are in A.P.

$\Rightarrow$ $b + c, c + a, a + b$ will be in A.P.

Example 2

If a^2, b^2, c^2 are in A.P. then prove that $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in A.P.

Solution

$$\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$$

or $\frac{a}{b+c}+1, \frac{b}{c+a}+1, \frac{c}{a+b}+1$ are in A.P.

or $\frac{a+b+c}{b+c}, \frac{b+c+a}{c+a}, \frac{c+a+b}{a+b}$ are in A.P.

or $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

or $$\frac{1}{c+a}-\frac{1}{b+c}=\frac{1}{a+b}-\frac{1}{c+a}$$

or $$\frac{(b-a)}{(c+a)(b+c)}=\frac{(c-b)}{(a+b)(c+a)}$$

$$\frac{b-a}{b+c}=\frac{c-b}{a+b}$$

or $b^2-a^2=c^2-b^2$ $\Rightarrow$ $2b^2 = a^2 + c^2$

Thus, a^2, b^2, c^2 are in A.P. $\Rightarrow$ $\frac{a}{b+c}, \frac{b}{a+c}, \frac{c}{a+b}$ are in A.P.

Example 3

If the sum of m terms of an AP is the same as the sum of its n terms, show that the sum of its (m + n) terms is zero.

Solution

Let 'a' be the first term and 'd' be the common difference of the given AP then

$S_m = S_n$

$\frac{m}{2}[2a + (m-1)d] = \frac{n}{2}[2a + (n-1)d]$

$m[2a + (m-1)d] = n[2a + (n-1)d]$

$2am + m(m-1)d = 2an + n(n-1)d$

$2a(m-n) + \{m(m-1) - n(n-1)\}d = 0$

$2a(m-n) + \{(m^2 - n^2) - (m-n)\}d = 0$

$(m-n)[2a + (m+n-1)d] = 0$

$2a + (m+n-1)d = 0$ .....(i) $[\because m - n \neq 0]$

Now, $S_{m+n} = \frac{m+n}{2}[2a + (m+n-1)d] = \frac{m+n}{2} \times 0 = 0$

Example 4

The interior angles of a polygon are in AP. The smallest angle is 120° and the common difference is 5°. Find the number of sides of polygon.

Solution

Let there be n sides of the polygon. Then the sum of its interior angles is given by

$S_n = (2n-4)$ right angles

or $S_n = (n-2) \times 180°$ .....(i)

Hence the interior angles term an AP with first term a = 120° and common difference d = 5°

$S_n = \frac{n}{2}[2a + (n-1)d]$

$(n-2) \times 180 = \frac{n}{2}[2 \times 120° + (n-1) \times 5°]$

$(n-2) \times 360 = n[5n + 235] \Rightarrow n^2 - 25n + 144 = 0$

$\Rightarrow (n-16)(n-9) = 0 \Rightarrow n = 16$ or $n = 9$

But when n = 16 the last angle

$a_n = a + (n-1)d$

$= 120 + (16-1)5°$

$= 120 + 75 = 195°$ which is not possible

Hence, n = 9

Example 5

If the p^{th} term of an AP is 1/q and q^{th} term 1/p. Prove that the sum of the first pq terms is $\frac{1}{2}(pq+1)$.

Solution

p^{th} term of an AP = $a_p = a + (p-1)d$

or $\frac{1}{q} = a + (p-1)d$ .....(i)

q^{th} term of an AP = $a_q = a + (q-1)d$

or $\frac{1}{p} = a + (q-1)d$ .....(ii)

Solving equation (i) and (ii), we get

$$d[(p-1)-(q-1)] = \frac{1}{q} - \frac{1}{p}$$

$$d\,[p-q] = \frac{p-q}{pq} \quad \Rightarrow \quad \left[d = \frac{1}{pq}\right]$$

Putting the value of 'd' in equation (i), we get

$$a + (p-1) \times \frac{1}{pq} = \frac{1}{q}$$

$$a + \frac{1}{q} - \frac{1}{pq} = \frac{1}{q}$$

$$\left[a = \frac{1}{pq}\right]$$

Now the sum of first pq terms we have

$$a = \frac{1}{pq},\ d = \frac{1}{pq},\ n = pq$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{pq} = \frac{pq}{2}\left[\frac{2}{pq} + (pq-1) \times \frac{1}{pq}\right] = \frac{pq}{2}\left[\frac{2}{pq} + 1 - \frac{1}{pq}\right] = \frac{pq}{2}\left[\frac{pq+1}{pq}\right]$$

or $\left[S_{pq} = \frac{1}{2}(pq+1)\right]$

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CONCEPT APPLICATION LEVEL - I [NCERT Questions]

Q.1 Write first four terms of the A.P. when the first term a and the common difference d are given as follows:

(i) $a = 10, d = 10$ (ii) $a = -2, d = 0$ (iii) $a = 4, d = -3$ (iv) $a = -1, d = \frac{1}{2}$

(v) $a = -1.25, d = -0.25$

Sol. (i) First term = a = 10

Second term = 10 + d = 10 + 10 = 20

Third term = 20 + d = 20 + 10 = 30

Fourth term = 30 + d = 30 + 10 = 40

Hence , first four terms of the given AP are 10, 20, 30, 40.

(ii) First term = a = –2

Second term = –2 + d = –2 + 0 = –2

Third term = –2 + d = –2 + 0 = –2

Fourth term = –2 + d = –2 + 0 = –2

Hence , first four terms of the given AP are –2, –2, –2, –2.

(iii) First term = a = 4

Second term = 4 + d = 4 + (–3) = 1

Third term = 1 + d = 1 + (–3) = –2

Fourth term = –2 + d = –2 + (–3) = –5

Hence , first four terms of the given AP are 4, 1, –2, –5.

(iv) First term = a = –1

Second term $= -1 + d = -1 + \frac{1}{2} = \frac{-1}{2}$

Third term $= \frac{-1}{2} + d = \frac{-1}{2} + \frac{1}{2} = 0$

Fourth term $= 0 + d = 0 + \frac{1}{2} = \frac{1}{2}$

Hence , first four terms of the given AP are $-1, \frac{-1}{2}, 0, \frac{1}{2}$.

(v) First term = a = –1.25

Second term = –1.25 + d = –1.25 + (–0.25) = –1.50

Third term = –1.50 + d = –1.50 + (–0.25) = –1.75

Fourth term = –1.75 + d = –1.75 + (–0.25) = –2.00

Hence , first four terms of the given AP are –1.25, –1.50, –1.75, –2.00

Q.2 Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the n^{th} term of the A.P.

	a	d	n	a_n
(i)	7	3	8	…
(ii)	–18	…	10	0
(iii)	…	–3	18	–5
(iv)	–18.9	2.5	…	3.6
(v)	3.5	0	105	…

Sol. (i)
$a_n = a + (n-1)d$
$\Rightarrow a_n = 7 + (8-1)3 \quad \Rightarrow a_n = 7 + (7)3$
$\Rightarrow a_n = 7 + 21 \quad \Rightarrow a_n = 28$

(ii)
$a_n = a + (n-1)d$
$\Rightarrow 0 = -18 + (10-1)d \quad \Rightarrow 18 = 9d$
$\Rightarrow d = \frac{18}{9} \quad \Rightarrow d = 2$

(iii)
$a_n = a + (n-1)d$
$\Rightarrow -5 = a + (18-1)(-3) \quad \Rightarrow -5 = a - 51$
$\Rightarrow a = 51 - 5 \quad \Rightarrow a = 46$

(iv)
$a_n = a + (n-1)d$
$\Rightarrow 3.6 = -18.9 + (n-1)(2.5) \quad \Rightarrow 3.6 + 18.9 = (n-1)(2.5)$
$\Rightarrow 22.5 = (n-1)(2.5) \quad \Rightarrow n - 1 = \frac{22.5}{2.5}$
$\Rightarrow n - 1 = 9 \quad \Rightarrow n = 9 + 1$
$\Rightarrow n = 10$

(v)
$a_n = a + (n-1)d$
$\Rightarrow a_n = 3.5 + (105-1)0 \quad \Rightarrow a_n = 3.5$

Q.3 In the following APs, find the missing terms in the boxes:

(i) 2, □, 26 **(ii) □, 13, □, 3** **(iii) 5, □, □, $9\frac{1}{2}$**

(iv) –4, □, □, □, □, 6 **(v) □, 38, □, □, □, –22**

Sol. (i) Let the common difference of the given AP be d.
Then, Third term = 2 + d + d = 2 + 2d
According to the question,
$2 + 2d = 26 \quad \Rightarrow 2d = 26 - 2$
$\Rightarrow 2d = 24 \quad \Rightarrow d = \frac{24}{2} = 12$
So, second term = 2 + d = 2 + 21 = 14
Hence, the missing term in the box is **14**.

(ii) Let the first term and the common difference of the given AP be a and b respectively.

Second term = 13.

$\Rightarrow \quad a + (2 - 1)d = 13 \quad \Rightarrow \quad a + d = 13$ .....(1)

Fourth term = 3

$\Rightarrow \quad a + (4 - 1)d = 3 \quad \Rightarrow \quad a + 3d = 3$ .....(2)

Solving (1) and (2), we get

$a = 18,\ d = -5$

Therefore,

Third term = $a + (3 - 1)d = a + 2d = 18 + 2(-5) = 18 - 10 = 8$

Hence, the missing terms in the boxes are **18** and **8**.

(iii) Let the common difference of the given A.P. be d.

$a = 5$

Fourth term = $9\frac{1}{2} \quad \Rightarrow \quad 5 + (4 - 1)d = \frac{19}{2} \quad [\because\ a_n = a + (n - 1)d]$

$\Rightarrow \quad 3d = \frac{19}{2} - 5 \quad \Rightarrow \quad 3d = \frac{9}{2} \quad \Rightarrow \quad d = \frac{3}{2}$

Therefore,

Second term = $5 + \frac{3}{2} = \frac{13}{2} = 6\frac{1}{2}$

and Third term $= \frac{13}{2} + \frac{3}{2} = 8$

Hence, the missing terms in the boxes are $\mathbf{6\frac{1}{2}}$ and **8**.

(iv) Let the common difference of the given AP be d.

$a = -4$

Sixth term = 6

$\Rightarrow \quad -4 + (6 - 1)d = 6 \quad [\because\ a_n = a + (n - 1)d]$

$\Rightarrow \quad -4 + 5d = 6 \quad \Rightarrow \quad 5d = 6 + 4$

$\Rightarrow \quad 5d = 10 \quad \Rightarrow \quad d = \frac{10}{5} \quad \Rightarrow \quad d = 2$

Therefore,

Second term $= -4 + 2 = -2$

Third term $= -2 + 2 = 0$

Fourth term $= 0 + 2 = 2$

Fifth term $= 2 + 2 = 4$

Hence, the missing terms in the boxes are **–2, 0, 2, 4**.

(v) Let the first term and the common difference of the given A.P. be a and d respectively.

Second term = 38

$\Rightarrow \quad a + (2 - 1)d = 38 \quad [\because\ a_n = a + (n - 1)d]$

$\Rightarrow \quad a + d = 38$ .....(1)

Sixth term = – 22

$\Rightarrow \quad a + (6 - 1)d = -22 \quad \Rightarrow \quad a + 5d = -22$

Solving (1) and (2), we get .....(2)

$a = 53,\ d = -15$

Therefore

Third term $= 53 + (3 - 1)(-15)$ $[\because\ a_n = a + (n-1)d]$

$= 53 - 30 = 23$

Fourth term $= 53 + (4 - 1)(-15)$ $[\because\ a_n = a + (n-1)d]$

$= 53 - 45 = 8$

Fifth term $= 53 + (5 - 1)(-15)$ $[\because\ a_n = a + (n-1)d]$

$= 53 - 60 = -7$

Hence, the missing terms in the boxes are **53, 23, 8, –7**.

Q.4 Which term of the AP : 3, 8, 13, 18, ... is 78.

Sol. The given AP is 3, 8, 13, 18, ...

Here, $a = 3,\ d = 8 - 3 = 5$

Let the n^{th} term of the A.P. be 78.

Then $a_n = a + (n-1)d$

$\Rightarrow 78 = 3 + (n-1)(5)$ $\Rightarrow 5(n-1) = 78 - 3$

$\Rightarrow 5(n-1) = 75$ $\Rightarrow n - 1 = \frac{75}{5}$

$\Rightarrow n - 1 = 15$ $\Rightarrow n = 15 + 1$ $\Rightarrow n = 16$

Hence, 16th term of the A.P. is 78.

Q.5 Check whether –150 is a term of the A.P. : 11, 8, 5, 2, ...

Sol. The given list of numbers is 11, 8, 5, 2.........

$a_2 - a_1 = 8 - 11 = -3$

$a_3 - a_2 = 5 - 8 = -3$

$a_4 - a_3 = 2 - 5 = -3$

i.e., $a_{k+1} - a_k$ is the same every time. So the given list of numbers forms an A.P. with first term $a = 11$ and the common difference $d = -3$.

Let –150 be the n^{th} term of the given A.P.

Then, $a_n = -150$

$\Rightarrow a + (n-1)d = -150$ $\Rightarrow 11 + (n-1)(-3) = -150$ $\Rightarrow -3(n-1) = -150 - 11$

$\Rightarrow -3(n-1) = -161$ $\Rightarrow 3(n-1) = 161$ $\Rightarrow n - 1 = \frac{161}{3}$

$\Rightarrow n = \frac{161}{3} + 1$ $\Rightarrow n = \frac{164}{3}$

But n should be a positive integer. So –150 is not a term of 11, 8, 5, 2, ...

Q.6 Find the 31^{st} term of an A.P. whose 11^{th} term is 38 and the 16^{th} term is 73.

Sol. Let the first term and the common difference of the AP be a and d respectively.

Then, 11^{th} term = 38 (Given)

$\Rightarrow a + (11 - 1)d = 38$ $[\because a_n = a + (n-1)d]$

$\Rightarrow a + 10d = 38$ ......(1)

and 16^{th} term = 73

$\Rightarrow a + (16 - 1)d = 73$ $[\because a_n = a + (n-1)d]$

$\Rightarrow a + 15d = 73$ .....(2)

Solving (1) and (2), we get

$a = -32,\ d = 7$

Therefore, 31^{st} term

$= a + (31 - 1)d = a + 30d = -32 + (30)(7) = -32 + 210 = 178$

Hence, the 31st term of the AP is 178.

Q.7 The 17^{th} term of an A.P. exceeds its 10^{th} term by 7. Find the common difference.

Sol. Let the first term and the common difference of the A.P. be a and d respectively.

According to the question,

$a_{17} = a_{10} + 7$

$\Rightarrow\ a + (17 - 1)d = a + (10 - 1)d + 7 \qquad [\because a_n = a + (n - 1)d]$

$\Rightarrow\ a + 16d = a + 9d + 7 \qquad \Rightarrow\ 16d - 9d = 7$

$\Rightarrow\ 7d = 7 \qquad \Rightarrow\ d = \frac{7}{7} = 1$

Hence, the common difference is 1.

Q.8 How many three-digit numbers are divisible by 7 ?

Sol. The three-digit numbers divisible by 7 are :

105, 112, 119, 126, ..., 994

$a_2 - a_1 = 112 - 105 = 7$

$a_3 - a_2 = 119 - 112 = 7$

$a_4 - a_3 = 126 - 119 = 7$

i.e. $a_{k+1} - a_k$ is the same every time. So the above list of numbers form an A.P. with the first term a = 105 and the common difference d = 7.

Last term (l) = 994

Let their be n terms in this A.P.

Then, n^{th} term = l

$\Rightarrow\ a + (n - 1)d = 994$

$\Rightarrow\ 105 + (n - 1)7 = 994 \qquad \Rightarrow\ (n - 1)7 = 994 - 105 \qquad \Rightarrow\ (n - 1)7 = 889$

$\Rightarrow\ n - 1 = \frac{889}{7} \qquad \Rightarrow\ n - 1 = 127 \qquad \Rightarrow\ n = 127 + 1$

$\Rightarrow\ n = 128$

Hence, there are 128 three-digit numbers divisible by 7.

Q.9 For what value of n, are the n^{th} terms of two APs : 63, 65, 67,....... and 3, 10, 17,...... equal?

Sol. **First AP :** 63, 65, 67, ...

Here, $a = 63; \quad d = 65 - 63 = 2$

$\therefore\ n^{th}$ term $= 63 + (n - 1)2 \qquad [\because a_n = a + (n - 1)d]$

Second AP : 3, 10, 17,........

Here, $a = 3; \quad d = 10 - 3 = 7$

$\therefore\ n^{th}$ term $= 3 + (n - 1)7 \qquad [\because a_n = a + (n - 1)d]$

If the n^{th} terms of the two APs are equal, then

$63 + (n - 1)2 = 3 + (n - 1)7 \qquad \Rightarrow\ (n - 1)2 - (n - 1)7 = 3 - 63$

$\Rightarrow\ (n - 1) - (2 - 7) = -60 \qquad \Rightarrow\ (n - 1) - (-5) = -60$

$\Rightarrow\ n - 1 = \frac{-60}{-5} \qquad \Rightarrow\ n - 1 = 12$

$\Rightarrow\ n = 12 + 1 \qquad \Rightarrow\ n = 13$

Hence, for n = 13, then nth terms of the two APs are equal.

Q.10 Find the 20^{th} term from the end of the AP : 3, 8, 13,......., 253.

Sol. The given AP is 3, 8, 13, ..., 253

Here, $a = 3$; $d = 8 - 3 = 5$; $l = 253$

Let the number of terms of the AP be n.

Term, n^{th} term $= l$

$\Rightarrow \quad 3 + (n - 1)5 = 253 \qquad [\because\ a_n = a + (n - 1)d]$

$\Rightarrow \quad (n - 1)5 = 253 - 3 \qquad \Rightarrow \quad (n - 1)5 = 250 \qquad \Rightarrow \quad n - 1 = \dfrac{250}{5}$

$\Rightarrow \quad n - 1 = 50 \qquad \Rightarrow \quad n = 50 + 1 \qquad \Rightarrow \quad n = 51$

So, there are 51 terms in the given AP.

Now, 20^{th} term from the last term

$= (51 - 20 + 1)^{th}$ term from the beginning

$= 32^{th}$ term from the beginning

$= 3 + (32 - 1)5 \qquad [\because\ a_n = a + (n - 1)d]$

$= 3 + 155 = 158$

Hence, the 20^{th} term from the last term of the given AP is 158.

Aliter: Let us write the given AP in the reverse order. Then the AP becomes

253, 248, 243, ..., 3

Here, $a = 253$; $d = 243 - 253 = -5$

Therefore, Required term

$= 20^{th}$ term of the AP $= 253 + (20 - 1)(-5) \qquad [\because\ a_n = a + (n - 1)d]$

$= 253 - 95 = 158$

Hence, the 20^{th} term from the last term of the given AP is 158.

Q.11 Subba Rao started work in 1995 at an annual salary of M5000 and received an increment of M200 each year. In which year did his income reach M7000?

Sol. Here, a = M5000; d = M200; l = M7000

Suppose that his income reached M7000 after n years.

Then, $l = a + (n - 1)d$

$\Rightarrow \quad 7000 = 5000 + (n - 1)200 \qquad \Rightarrow \quad (n - 1)200 = 7000 - 5000$

$\Rightarrow \quad (n - 1)200 = 2000 \qquad \Rightarrow \quad n - 1 = \dfrac{2000}{200}$

$\Rightarrow \quad n - 1 = 10 \qquad \Rightarrow \quad n = 10 + 1 = 11$

Hence, his income reached M7000 in 11^{th} year.

Q.12 Ramkali saved M5 in the first weak of a year and then increased her weekly saving by M1.75. If in the n^{th} week, her weekly savings become M20.75, find n.

Sol. Here, a = M5; d = M1.75; a_n = M20.75

We know that

$a_n = a + (n - 1)d$

$\Rightarrow \quad 20.75 = 5 + (n - 1)(1.75) \qquad \Rightarrow \quad (n - 1)(1.75) = 20.75 - 5$

$\Rightarrow \quad (n - 1)(1.75) = 15.75 \qquad \Rightarrow \quad n - 1 = \dfrac{15.75}{1.75}$

$\Rightarrow \quad n - 1 = 9 \qquad \Rightarrow \quad n = 9 + 1 = 10$

Here, the required value of n is 10.

Q.13 In an A.P. :

(i) given $a = 5, d = 3, a_n = 50$, find n and S_n.

(ii) given $a = 7, a_{13} = 35$, find d and S_{13}.

(iii) given $a = 2, d = 8, S_n = 90$, find n and a_n.

(iv) given $l = 28, S = 144$, and there are total 9 terms. Find a.

Sol. (i) Here, $a = 5, d = 3, a_n = 50$

We know that

$$a_n = a + (n-1)d$$

$\Rightarrow 50 = 5 + (n-1)3$ $\Rightarrow (n-1)3 = 50 - 5$

$\Rightarrow (n-1)3 = 45$ $\Rightarrow n - 1 = \frac{45}{3}$

$\Rightarrow n - 1 = 15$ $\Rightarrow n = 15 + 1$

$\Rightarrow n = 16$

Again, we know that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\Rightarrow S_n = \frac{16}{2}[2(5) + (16-1)3]$

$\Rightarrow S_n = 8[10 + 45]$

$\Rightarrow S_n = 8(55)$

$\Rightarrow S_n = 440$

(ii) Here, $a = 7, a_{13} = 35$

We know that

$$a_n = a + (n-1)d$$

$\Rightarrow a_{13} = a + (13-1)d$ $\Rightarrow a_{13} = a + 12d$

$\Rightarrow 35 = 7 + 12d$ $\Rightarrow 12d = 35 - 7$

$\Rightarrow 12d = 28$ $\Rightarrow d = \frac{28}{12}$

$\Rightarrow d = \frac{7}{3}$

Again, we know that

$S_n = \frac{n}{2}[2a + (n-1)d]$ $\Rightarrow S_{13} = \frac{13}{2}[2a + (13-1)d]$

$\Rightarrow S_{13} = \frac{13}{2}[2a + 12d]$ $\Rightarrow S_{13} = \frac{13}{2}\left[2(7) + 12\left(\frac{7}{3}\right)\right]$

$\Rightarrow S_{13} = \frac{13}{2}(14 + 28)$ $\Rightarrow S_{13} = \frac{13}{2}(42)$

$\Rightarrow S_{13} = (13)(21)$ $\Rightarrow S_{13} = 273$

(iii) Here, $a = 2$, $d = 8$, $S_n = 90$

We know that

$$S_n = \frac{n}{2}[2a + (n-1)d] \Rightarrow 90 = \frac{n}{2}[2(2) + (n-1)8]$$

$\Rightarrow 90 = n[2 + (n-1)4]$ $\Rightarrow 90 = n[4n - 2]$

$\Rightarrow 90 = 2n[2n - 1]$ $\Rightarrow 45 = n[2n - 1]$

$\Rightarrow 45 = 2n^2 - n$ $\Rightarrow 2n^2 - n - 45 = 0$

$\Rightarrow 2n^2 - 10n + 9n - 45 = 0$ $\Rightarrow 2n(n-5) + 9(n-5) = 0$

$\Rightarrow n - 5 = 0$ or $2n + 9 = 0$ $\Rightarrow n = 5$ or $n = \frac{-9}{2}$

$\Rightarrow n = \frac{-9}{2}$ is inadmissible as n, being the number of terms, is a natural number.

Again, we know that

$a_n = a + (n-1)d$

$\Rightarrow a_n = 2 + (5-1)8$

$\Rightarrow a_n = 2 + (4)8$

$\Rightarrow a_n = 34$

(iv) Here, $l = 28$, $S = 144$, $n = 9$

We know that

$$S_n = \frac{n}{2}(a + l) \Rightarrow S = \frac{9}{2}(a + l)$$

$\Rightarrow 144 = \frac{9}{2}(a + 28)$ $\Rightarrow \frac{(144)(2)}{9} = a + 28$

$\Rightarrow 32 = a + 28$ $\Rightarrow a + 28 = 32$

$\Rightarrow a = 32 - 28$ $\Rightarrow a = 4$

Q.14 How many terms of the AP: 9, 17, 25,........ must be taken to give a sum of 636?

Sol. The given AP is 9, 17, 25,.......

Here $a = 9$, $d = 17 - 9 = 8$

Let n terms of the AP must be taken.

Then, $S_n = 636$

$\Rightarrow \frac{n}{2}[2a + (n-1)d] = 636$ $\Rightarrow \frac{n}{2}[2(9) + (n-1)8] = 636$

$\Rightarrow n[9 + (n-1)4] = 636$ $\Rightarrow n[9 + 4n - 4] = 636$

$\Rightarrow n[(4n + 5)] = 636$ $\Rightarrow 4n^2 + 5n - 636 = 0$

$\Rightarrow 4n^2 + 53n - 48n - 636 = 0$ $\Rightarrow n(4n + 53) - 12(4n + 53) = 0$

$\Rightarrow (4n + 53)(n - 12) = 0$ $\Rightarrow 4n + 53 = 0$ or $n - 12 = 0$

$\Rightarrow n = \frac{-53}{4}$ or $n = 12$

$\Rightarrow n = \frac{-53}{4}$ is inadmissible as n, being the number of terms is natural number .

$\therefore n = 12$

Hence, 12 terms of the AP must be taken.

Q.15 Find the sum of first 22 terms of an AP in which d = 7 and 22[nd] term is 149.

Sol. Here, $d = 7$, $a_{22} = 149$

Let the first term of the AP be a.

We know that

$a_n = a + (n - 1)d$

$\Rightarrow$ $a_{22} = a + (22 - 1)d$ $\Rightarrow$ $a_{22} = a + 21d$

$\Rightarrow$ $149 = a + (21)(7)$ $\Rightarrow$ $149 = a + 147$

$\Rightarrow$ $a + 147 = 149$ $\Rightarrow$ $a = 149 - 147$

$\Rightarrow$ $a = 2$

Again, we know that

$S_n = \frac{n}{2}[2a + (n - 1)d]$ $\Rightarrow$ $S_{22} = \frac{22}{2}[2(2) + (22 - 1)7]$

$\Rightarrow$ $S_{22} = (11)[4 + 147]$ $\Rightarrow$ $S_{22} = (11)(151)$

$\Rightarrow$ $S_{22} = 1661$

Hence, the sum of first 22 terms of the AP is 1661.

Q.16 Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Sol. Let the first term and the common difference of the AP be a and d respectively.

Second term = 14 (Given)

$\Rightarrow$ $a + (2 - 1)d = 14$ $(\because a_n = a + (n - 1)d)$

$\Rightarrow$ $a + d = 14$ .....(1)

Third term = 18 (Given)

$\Rightarrow$ $a + (3 - 1)d = 18$ $(\because a_n = a + (n - 1)d)$

$\Rightarrow$ $a + 2d = 18$ .....(2)

Solving equation (1) and equation (2), we get

$a = 10, d = 4$

Now, sum of first 51 terms of the AP

$= S_{51}$

$= \frac{51}{2}[2a + (51 - 1)d]$ $\left[\because S_n = \frac{n}{2}[2a + (n - 1)d]\right]$

$= \frac{51}{2}[2a + 50d] = 51[a + 25d] = (51)(10 + 25 \times 4)$

$= (51)(10 + 100) = (51)(110) = 5610$

Q.17 Show that a_1, a_2,...., a_n,.... form an AP where a_n is defined as $a_n = 3 + 4n$. Also find the sum of the first 5 terms.

Sol. We have $a_n = 3 + 4n$

Put $n = 1, 2, 3, 4,$......... in succession, we get

$a_1 = 3 + 4(1) = 3 + 4 = 7$

$a_2 = 3 + 4(2) = 3 + 8 = 11$

$a_3 = 3 + 4(3) = 3 + 12 = 15$

$a_4 = 3 + 4(4) = 3 + 16 = 19$

$\vdots \quad \vdots \quad \vdots \quad \vdots$

$\therefore$ $a_2 - a_1 = 11 - 7 = 4$

$a_3 - a_2 = 15 - 11 = 4$

$a_4 - a_3 = 19 - 15 = 4$

i.e. $a_{k+1} - a_k$ is the same every time.

So, $a_1, a_2, \ldots, a_n, \ldots$ form an AP.

Here, $a = a_1 = 7$

$d = a_2 - a_1 = 4$

$\therefore$ Sum of the first 15 terms

$= S_{15}$

$$= \frac{15}{2}[2a + (15 - 1)d] \qquad [\because S_n = \frac{n}{2}[2a + (n-1)d]\ ]$$

$$= \frac{15}{2}[2a + 14d] = 15[a + 7d] = (15)(7 + 7 \times 4) = (15)(7 + 28) = (15)(35) = 525$$

Q.18 If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3^{rd}, the 10^{th} and the n^{th} terms.

Sol. We have

Sum of the first n terms $= 4n - n^2$ $\Rightarrow$ $S_n = 4n - n^2$

Put, $n = 1$

$S_1 = 4(1) - (1)^2 = 4 - 1 = 3$ $\Rightarrow$ $a_1 = 3$

Hence, the first term is 3.

Put $n = 2$

$S_2 = 4(2) - (2)^2 = 8 - 4 = 4$

Hence, the sum of two terms is 4.

Second term $= S_2 - S_1 = 4 - 3 = 1$

Put, $n = 3$

$S_3 = 4(3) - (3)^2 = 12 - 9 = 3$

$\therefore$ 3^{rd} term $= a_3 = S_3 - S_2 = 3 - 4 = -1$

Put, $n = 9$ and 10

$S_9 = 4(9) - (9)^2 = 36 - 81 = -45$

and $S_{10} = 4(10) - (10)^2 = 40 - 100 = -60$

$\therefore$ 10^{th} term $= a_{10} = S_{10} - S_9 = -60 - (-45) = -15$

$S_{n-1} = 4(n-1) - (n-1)^2 = 4n - 4 - (n^2 - 2n + 1)$

$= 4n - 4 - n^2 + 2n - 1 = 6n - n^2 - 5$

$\therefore$ n^{th} term $= a_n$

$= S_n - S_{n-1} = (4n - n^2) - (6n - n^2 - 5) = 5 - 2n$

Q.19 Find the sum of the first 40 positive integers divisible by 6.

Sol. The first 40 positive integers divisible by 6 are : 6, 12, 18, 24, ...

Here, $a_2 - a_1 = 12 - 6 = 6$

$a_3 - a_2 = 18 - 12 = 6$

$a_4 - a_3 = 24 - 18 = 6$

i.e. $a_{k+1} - a_k$ is the same every time.

So, the above list of numbers form an A.P.

Here, $a = 6, d = 6, n = 40$

$\therefore$ Sum of the first 40 positive integers

$= S_{40}$

$= \frac{40}{2}[2a + (40 - 1)d]$ $\quad [\because S_n = \frac{n}{2}[2a + (n-1)d]]$

$= 20[2a + 39d] = 20[2 \times 6 + 39 \times 6]$

$= (20)(12 + 234) = (20)(246) = 4920$

Q.20 A sum of M700 is to be used to give seven cash prizes to students of a school for their overall academic performance . If each prize is M20 less than its preceding prize, find the value of each of the prizes.

Sol. Since each prize is M20 less than its preceding prize, therefore, the values of the seven successive cash prizes will form an AP.

Let the first prize be Ma.

Then the winner prizes, in succession, will be M(a – 20), M(a – 40), M(a – 60), etc.

Here, $A = a; d = (a - 20) - a = -20; S_n = 700$

We know that

$$S_n = \frac{n}{2}[2A + (n-1)d]$$

$\Rightarrow \quad 700 = \frac{7}{2}[2a + (7-1)(-20)] \quad \Rightarrow \quad 700 = \frac{7}{2}[2a - 120]$

$\Rightarrow \quad 700 = 7(a - 60) \quad \Rightarrow \quad a - 60 = \frac{700}{7}$

$\Rightarrow \quad a - 60 = 100 \quad \Rightarrow \quad a = 100 + 60$

$\Rightarrow \quad a = 160$

$\Rightarrow$ Value of first prize = M160

$\therefore$ Value of second prize = M160 – M20 = M140

$\therefore$ Value of third prize = M140 – M20 = M120

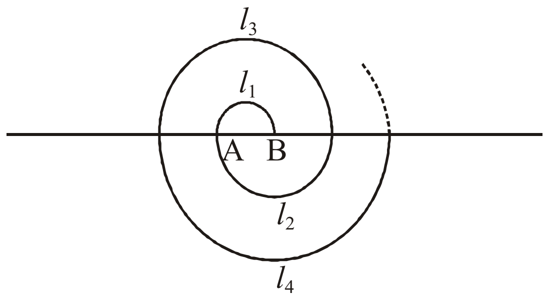
$\therefore$ Value of fourth prize = M120 – M20 = M100

$\therefore$ Value of fifth prize = M100 – M20 = M80

$\therefore$ Value of sixth prize = M80 – M20 = M60

$\therefore$ Value of seventh prize = M60 – M20 = M40

Q.21 A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ... as shown in figure. What is the total length of such a spiral made up of thirteen consecutive semicircles? (Take $\pi = \frac{22}{7}$)



[Hint : Length of successive semicircles is l_1, l_2, l_3, l_4,...... with centres at A, B, A, B,....., respectively.]

Sol. Lengths (in cm) of successive semicircles with centres at A, B, A, B, ... are

$\pi(0.5)$, $\pi(0.1)$, $\pi(1.5)$, $\pi(2.0)$, ... respectively

$\therefore$ $a_2 - a_1 = \pi(1.0) - \pi(0.5) = \pi(0.5)$

$a_3 - a_2 = \pi(1.5) - \pi(1.5) = \pi(0.5)$

$a_4 - a_3 = \pi(2.0) - \pi(1.5) = \pi(0.5)$

i.e. $a_{k+1} - a_k$ is the same every time.

So, the above list of numbers form an A.P.

Here, $a = \pi(0.5)$, $d = \pi(0.5)$, $n = 13$

$\therefore$ Total length of the spiral

$= S_{13}$

$= \frac{13}{2}[2a + (13-1)d]$ $\quad [\because S_n = \frac{n}{2}[2a + (n-1)d]\ ]$

$= \frac{13}{2}[2a + 12d] = 13(a + 6d)$

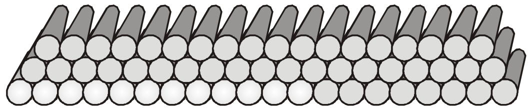
$= (13)\ [\pi(0.5) + 6\pi(0.5)]$ cm

$= (13)\ [7\pi(0.5)]$ cm

$= (13)\ [7 \times \frac{22}{7} \times 0.5]$ cm

$= (13)\ (11)$cm $= 143$ cm

Q.22 200 logs are stacked in the following manner : 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see figure). In how many rows are the 200 logs placed and how many logs are in the top row?



Sol. The numbers of logs in the bottom row, next row, row next to it and so on form the sequence 20, 19, 18, 17......

$\therefore$ $a_2 - a_1 = 19 - 20 = -1$

$a_3 - a_2 = 18 - 19 = -1$

$a_4 - a_3 = 17 - 18 = -1$

i.e. $a_{k+1} - a_k$ is the same every time.

So, the above sequence form an A.P.

Here, $a = 20,\ d = -1,\ S_n = 200$

We know that

$$S_n = \frac{n}{2}[2a + (n-1)d] \quad \Rightarrow \quad 200 = \frac{n}{2}[2(20) + (n-1)(-1)]$$

$$\Rightarrow \quad 200 = \frac{n}{2}[40 - n + 1] \quad \Rightarrow \quad 200 = \frac{n}{2}(41 - n)$$

$\Rightarrow$ $400 = n(41 - n)$ $\Rightarrow$ $n(41 - n) = 400$

$\Rightarrow$ $41n - n^2 = 400$ $\Rightarrow$ $n^2 - 41n + 400 = 0$

$\Rightarrow$ $n^2 - 25n - 16n + 400 = 0$ $\Rightarrow$ $n(n - 25) - 16(n - 25) = 0$

$\Rightarrow$ $(n - 25) - (n - 16) = 0$ $\Rightarrow$ $n - 25 = 0$ or $n - 16 = 0$

$\Rightarrow$ $n = 25$ or $n = 16$ $\Rightarrow$ $n = 25, 16$

Hence, the number of rows is either 25 or 16.

Now, Number of logs in top row

= Number of logs in 25^{th} row

$= a_{25}$

$= a + (25 - 1)d$ $\quad [\because\ a_n = a + (n-1)d]$

$= a + 24d = 20 + 24(-1)$

$= 20 - 24 = -4$

which is not possible.

Therefore, $n = 16$

and Number of log in to row

= Number of logs in 16th row

$= a_{16}$

$= a_a + (16 - 1)d$ $\quad [\because\ a_n = a + (n-1)d]$

$= a + 15d = 20 + 15(-1)$

$= 20 - 15 = 5$

Q.23 In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato and the other potatoes are placed 3 m apart in a straight line. Their are ten potatoes in the line (see figure). Each competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

[Hint: To pick up the first potato and the second potato, the total distance (in metres) run by competitor is $2 \times 5 + 2 \times (5 + 3)$].

Sol. To drop the first potato in the bucket, the distance run = 2×5 m.

To drop the second potato in the bucket the distance run = $2 \times (5 + 3)$m.

To drop the third potato in the bucket the distance run = $2 \times (5 + 3 + 3)$m. and so on.

$\therefore$ $a_2 - a_1 = 2 \times (5 + 3)m - 2 \times 5m = 2 \times 3m = 6$ m.

$a_3 - a_2 = 2 \times (5 + 3 + 3)m - 2 \times (5 + 3)m = 2 \times 3m = 6$ m.

i.e. $a_{k+1} - a_k$ is the same every time.

So, the above distance (in m) form an A.P.

Here, $a = 2 \times 5 = 10$m, $d = 6$ m, $n = 10$

$\therefore$ The total distance competitor has to run.

$= S_{10}$

$= \frac{10}{2}[2a + (10 - 1)d]$ $\left[\because S_n = \frac{n}{2}[2a + (n - 1)d]\right]$

$= 5[2a + 9d]$m $= 5[2 \times 10 + 9 \times 6]$m

$= 5[20 + 54]$m $= 370$ m

CONCEPT APPLICATION LEVEL - II [Previous Year Questions]

Q.1 Which term of A.P. is 21, 42, 63, 84, ... is 210?
(A) 9^{th} (B) 10^{th} (C) 11^{th} (D) 12^{th}

Q.2 If the n^{th} term of sequence is $3 + 2n$, then the sum of its first 20 terms is :
(A) 480 (B) 520 (C) 500 (D) 460

Q.3 What is the n^{th} term in the arithmetic series given below?
$3 + 7 + 11 + 15 + 19 + ...$
(A) 4n (B) 3 + 4n (C) 2n + 1 (D) 4n – 1

Q.4 Everybody shakes hands with everybody else. If total number of hands shaken were 66, then how many persons were present? **[NIMO]**
(A) 11 (B) 12 (C) 13 (D) 14

Q.5 Two AP's have the same common difference. The first term of one of these is 3 and that of the other is 8. What is the difference between their 10^{th} terms? **[NIMO]**
(A) 5 (B) 10 (C) 15 (D) 19

Q.6 For an A.P., the sum of n terms of the sequence is $\frac{n}{2}(A63 - 7n)$, find the 10^{th} term of the sequence:
(A) 12 (B) 11 (C) 15 (D) 24 **[IOM-11]**

Q.7 If the 15^{th} term of an A.P. is 121 and 25^{th} term is 201, then the 35^{th} term of the A.P. is **[IOM-11]**
(A) 292 (B) 281 (C) 264 (D) 275

Q.8 The sum of three numbers in A.P. is 21 and their product is 231. Find the numbers. **[IOM-12]**
(A) 3, 7 and 11 (B) 4, 8 and 12 (C) 5, 11 and 13 (D) 2, 3 and 5

Q.9 Find the sum of the first 25 terms of the A.P. whose second term is 9 and 4^{th} term is 21. **[IMO-12]**
(A) 1740 (B) 1470 (C) 1720 (D) 1875

Q.10 A body falls 16 metres in the first second of its motion, 48 metres in the second, 80 metres in the third and so on. How long will it take to fall 4096 metres? **[IOM-12]**
(A) 16 seconds (B) 18 seconds (C) 8 seconds (D) 6 seconds

Q.11 Four numbers have been inserted between 4 and 24. Find the common difference. **[IOM-12]**
(A) 4 (B) 6 (C) 10 (D) 8

Q.12 Which of these terms of the sequence given is the first negative term?

$15, 13\frac{3}{4}, 12\frac{1}{2}, 11\frac{1}{4}, ...$ **[NSTSE-2013]**

(A) 12^{th} (B) 13^{th} (C) 14^{th} (D) 18^{th}

Q.13 The sum of the third and seventh terms of an A.P. is 6 and their product is 8, then common difference is

(A) ±1 (B) ±2 (C) $\pm\frac{1}{2}$ (D) $\pm\frac{1}{4}$ **[NTSE-2013]**

Q.14 The sum of all two digit numbers each of which leaves remainder 3 when divided by 5 is **[Delhi NTSE Stage-1 2013]**

(A) 952 (B) 999 (C) 1064 (D) 1120

Q.15 If $a_1, a_2, ..., a_{19}$ are the first 19 term of an AP and $a_1 + a_8 + a_{12} + a_{19} = 224$. Then $\sum_{i=1}^{19} a_i$ is equal to **[Harayana NTSE Stage-1 2013]**

(A) 896 (B) 1064 (C) 1120 (D) 1164

Q.16 The sum of 18 consecutive natural numbers is a perfect square. The smallest possible value of this sum is **[Harayana NTSE Stage-1 2014]**

(A) 144 (B) 169 (C) 225 (D) 289

Q.17 The sum $\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+.......+\frac{99}{1+99^2+99^4}$ **[Harayana NTSE Stage-1 2014]**

(A) 0.46 and 0.47 (B) 0.47 and 0.48 (C) 0.48 and 0.49 (D) 0.49 and 0.50

Q.18 If x and y are two positive real numbers such that their sum is one, then the maximum value of $x^4y + xy^4$ is **[Harayana NTSE Stage-1 2014]**

(A) $\frac{1}{6}$ (B) $\frac{1}{8}$ (C) $\frac{1}{12}$ (D) $\frac{1}{16}$

Q.19 If $a_1, a_2, a_3, ...$ is an arithmetic progression with common difference 1 and $\sum_{i=1}^{98} a_i = 137$, then the value of $a_2 + a_4 + a_6 + ... a_{98}$ is **[Harayana NTSE Stage-1 2014]**

(A) 67 (B) 83 (C) 93 (D) 98

Q.20 A club consists of members whose ages are in A.P. the common difference being 3 months. If the youngest members of the club is just 7 years old and the sum of the ages of all the members is 250 years, then the number of members in the club are **[Karnataka NTSE Stage-1 2014]**

(A) 15 (B) 20 (C) 25 (D) 30

Q.21 The first term of an A.P. is 5, the last term is 45 and the sum is 400. Then the fourth term of A.P. is **[Rajasthan NTSE Stage-1 2016]**

(A) 13 (B) 11 (C) 15 (D) 14

Q.22 If S_1 , S_2, S_3, ... , S_r are the sums of first n terms of r arithmetic progressions whose first terms are 1, 2, 3, ... and whose common differences are 1, 3, 5, ... respectively, then the value of $S_1 + S_2 + S_3 + ...+ S_r$ is **[NTSE Stage-2 2016]**

(A) $\frac{(nr-1)(nr+1)}{2}$ (B) $\frac{(nr+1)nr}{2}$ (C) $\frac{(nr-1)nr}{2}$ (D) $\frac{n(nr+1)}{2}$

Q.23 Three positive integers a_1, a_2 and a_3 are in A.P. such that $a_1 + a_2 + a_3 = 33$ and $a_1 \times a_2 \times a_3 = 1155$. Find the values of a_1, a_2, a_3 **[IMO-2016]**

(A) 15, 20, 17 (B) 10, 11, 12 (C) 7, 11, 15 (D) 7, 15, 20

Q.24 Amit gets pocket money from his father every day. Out of the pocket money, he saves M2.75 on first day and on each succeeding day he increases his saving by 25 paise. Find the amount saved by Amit on 14^{th} day. **[IMO-2016]**

(A) M6 (B) M12 (C) M8 (D) M10

Q.25 If there are (2n – 1) terms in an A.P., then the ratio of the sum of its odd terms to its even terms is ___________ **[IOM-2016]**

(A) $\frac{n+1}{n}$ (B) $\frac{n-1}{n}$ (C) $\frac{n}{n-1}$ (D) $\frac{n}{n+1}$

Q.26 Four numbers are in arithmetic progression. The sum of first and the last term is 8 and the product of both the middle terms is 15. The least number of the series is **[IOM-2016]**

(A) 4 (B) 3 (C) 2 (D) 1

Q.27 If the first, second and last the terms of an A.P., be a, b, 2a respectively, then its sum will be **[IOM-2016]**

(A) $\frac{ab}{b-a}$ (B) $\frac{ab}{2(b-a)}$ (C) $\frac{3ab}{2(b-a)}$ (D) $\frac{3ab}{4(b-a)}$

CONCEPT APPLICATION LEVEL - III

SECTION-A

- **Fill in the blanks**

Q.1 If the sum of n terms of a series is $5n^2 + 2n$ then the second term is____________.

Q.2 The solution of the equation $(x + 1) + (x + 4) + (x + 7) + ... + (x + 28) = 155$ is given by $x =$_________.

Q.3 The sum of n terms of an AP is n^2. Then the common difference is __________.

SECTION-B

- **Multiple choice questions with one correct answer**

Q.1 If arithmetic mean of a and b is $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ then the value of n is

(A) –1 (B) 0 (C) 1 (D) None

Q.2 $\frac{S_n}{S_m} = \frac{n^4}{m^4}$ (where S_k is the sum of first k terms of an AP $a_1\ a_2\ a_3......\infty$) then the value of $\frac{a_{m+1}}{a_{n+1}}$ in terms of m and n will be

(A) $\left(\frac{2m+1}{2n+1}\right)^3$ (B) $\left(\frac{2n+1}{2m+1}\right)^3$ (C) $\left(\frac{2m-1}{2n+1}\right)^3$ (D) $\left(\frac{2m+1}{2n-1}\right)^3$

Q.3 If $a_1, a_2, a_3.........a_n$ are in AP where $a_i > 0$ for all i then the value of

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + ... + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}$$

(A) $\frac{1}{\sqrt{a_1}+\sqrt{a_n}}$ (B) $\frac{1}{\sqrt{a_1}-\sqrt{a_n}}$ (C) $\frac{n}{\sqrt{a_1}-\sqrt{a_n}}$ (D) $\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$

Q.4 8th term of the series $2\sqrt{2}, \sqrt{2}, 0...........$ will be

(A) $-5\sqrt{2}$ (B) $5\sqrt{2}$ (C) $10\sqrt{2}$ (D) $-10\sqrt{2}$

Q.5 If the ratio of the sum of n terms of two APs is (3n–13) : (5n+21), then the ratio of 24th terms of the two progression is

(A) 2 : 3 (B) 2 : 1 (C) 1 : 2 (D) None of these

Q.6 Find the sum of all integers between 50 and 500 which are divisible by 7.

(A) 17966 (B) 1177996 (C) 17766 (D) 17696

Q.7 The sum of all 2 digit odd numbers is

(A) 2475 (B) 2530 (C) 4905 (D) 5049

Q.8 If the p^{th}, q^{th} and r^{th} terms of an A.P. are a,b,c respectively, find the value of $a(q-r)+b(r-p)+c(p-q)$

(A) 2 (B) 1 (C) 0 (D) 3

Q.9 If $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in AP then

(A) a,b,c are in AP

(B) a^2,b^2,c^2 are in AP

(C) $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ are in AP

(D) None of these

Q.10 If the first term of an AP is 17, the last term is $-12\frac{3}{8}$ and the sum is $25\frac{7}{16}$, then find the common difference.

(A) $-\frac{43}{18}$ (B) $-\frac{45}{17}$ (C) $-\frac{47}{16}$ (D) $\frac{47}{16}$

Q.11 If the sum of the first 2n terms of the AP 2,5,8...... is equal to the sum of the first n terms of the A.P. 57, 59, 61.......then n is equal to

(A) 10 (B) 12 (C) 11 (D) 13

Q.12 Let T_r be the rth term of an AP, for r = 1,2,3.........for some positive integers m, n. We have $T_m = 1/n$ and $T_n = 1/m$ then T_{mn} equals

(A) $\frac{1}{mn}$ (B) $\frac{1}{m}+\frac{1}{n}$ (C) 1 (D) 0

Q.13 A body falls 16 metres in the first second of its motion, 48 m in the second, 80 m in the third, 112 m in the fourth and so on. How far does it fall during the 11th second of its motion?

(A) 338 m (B) 340 m (C) 334 m (D) 336

Q.14 The numbers a,b,c,d,e form an AP then the value of a–4b+6c–4d+e is

(A) 1 (B) 2 (C) 0 (D) None of these

Q.15 The sequence $a_1, a_2, a_3, ..., a_n$ form an AP. Then $a_1^2 - a_2^2 + a_3^2 - a_4^2 + ... + a_{2n-1}^2 - a_{2n}^2$ is equal to

(A) $\frac{n}{2n-1}\left(a_1^2 - a_{2n}^2\right)$ (B) $\frac{2n}{(n-1)}\left(a_{2n}^2 - a_1^2\right)$ (C) $\frac{n}{n+1}\left(a_1^2 - a_{2n}^2\right)$ (D) None of these

Q.16 If a,b,c,d,e,f are arithmetic mean between 2 and 12, then a + b + c + d + e + f is equal to

(A) 14 (B) 42 (C) 84 (D) None of these

Q.17 If S_1, S_2 and S_3 denotes the sum of first n_1,n_2 and n_3 terms respectively of an AP then

$$\frac{S_1}{n_1}(n_2 - n_3) + \frac{S_2}{n_2}(n_3 - n_1) + \frac{S_3}{n_3}(n_1 - n_2) =$$

(A) 0 (B) 1 (C) $S_1S_2S_3$ (D) $n_1n_2n_3$

Q.18 If a,b,c,d,e,f are in AP then e–c is equal to

(A) 2(c–a) (B) 2(d–c) (C) 2(f–d) (D) (d–c)

Q.19 If the roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$ are in AP then their common difference will be

(A) ±1 (B) ±2 (C) ±3 (D) ±4

Q.20 If $<a_n>$ and $<b_n>$ be two sequence given by $a_n = (x)^{1/2n} + (y)^{1/2n}$ and $b_n = x^{1/2n} - y^{1/2n}$ for all $n \in N$.

Then $a_1a_2a_3.......a_n$ is equal to

(A) $x - y$ (B) $\frac{x+y}{b_n}$ (C) $\frac{x-y}{b_n}$ (D) $\frac{xy}{b_n}$

SECTION-C

- **More Than One Correct :**

Q.1 Which of the following represents an A.P.?

(A) 0.2, 0.4, 0.6,..... (B) 29, 58, 87, 116 ....

(C) 15, 45, 135, 405..... (D) 3, 3.5, 4.5, 8.5,......

Q.2 If $t_n = 6n + 5$, then $t_{n+1} =$

(A) 6(n + 1) + 17 (B) 6(n – 1) + 17 (C) 6n + 11 (D) 6n – 11

Q.3 $S_n = 54 + 51 + 48 + ...$ n terms = 513. Value of n is

(A) 18 (B) 19 (C) 15 (D) None of these

Q.4 If $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A.P. then which of the following is in A.P. ?

(A) a, b, c (B) a^2, b^2, c^2 (C) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ (D) bc, ac, ab

SECTION-D

- **Assertion & Reason**

Instructions: In the following questions as Assertion (A) is given followed by a Reason (R). Mark your responses from the following options.

(A) Both Assertion and Reason are true and Reason is the correct explanation of 'Assertion'

(B) Both Assertion and Reason are true and Reason is not the correct explanation of 'Assertion'

(C) Assertion is true but Reason is false

(D) Assertion is false but Reason is true

Q.1 **Assertion :** 1, 2, 4, 8,......... is a G.P., 4, 8, 16, 32 is a G.P. and 1 + 4, 2 + 8, 4 + 16, 8 + 32, ... is also a G.P.

Reason : Let general term of a G.P. with common ratio r be T_{k+1} and general term of another G.P. with common ratio r be T_{k+1} then the series whose general term $T''_{k+1} = T'_{k+1} + T'_{k+1}$ is also a G.P. with common ratio r.

Q.2 **Assertion :** The sum of the series with the n^{th} term, $t_n = (9 - 5n)$ is (465), when number of terms n = 15.

Reason : Given series is in A.P. and sum of n terms of an A.P. is $S_n = \frac{n}{2}[2a + (n-1)d]$.

SECTION-E

- **Match the following (one to one)**

Column-I and **column-II** contains **four** entries each. Entries of column-I are to be matched with some entries of column-II. Only One entry of column-I may have the matching with the one entry of column-II and one entry of column-II may have matching with only one entry of column-I.

Q.1

Column I	**Column II**
(i) Sum of the first 20 terms of the AP –6, 0, 6, 12...........	(P) 7500
(ii) $a_n = 4n + 5$ is an AP then the sum of the 100 terms in the series	(Q) 1020
(iii) Sum of all odd numbers between 100 and 200	(R) 3050
(iv) The sum of integers from 1 to 100 that are divisible by 2 or 5 is	(S) 20700

Q..2

Column I (A.P.)	**Column II (n^{th} term)**
(i) 119, 136, 153, 170, ...	(P) 13 – 3n
(ii) 7, 11, 15, 19, ...	(Q) 9 – 5n
(iii) 4, –1, –6, –11, ...	(R) 3 + 4n
(iv) 10, 7, 4, 3,	(S) 17n + 102

ANSWER KEY

CONCEPT APPLICATION LEVEL - II

Q.1	B	Q.2	A	Q.3	D	Q.4	B	Q.5	A	Q.6	C	Q.7	B
Q.8	A	Q.9	D	Q.10	A	Q.11	A	Q.12	C	Q.13	D	Q.14	B
Q.15	B	Q.16	C	Q.17	D	Q.18	C	Q.19	B	Q.20	C	Q.21	A
Q.22	B	Q.23	C	Q.24	A	Q.25	A	Q.26	D	Q.27	C		

CONCEPT APPLICATION LEVEL - III

SECTION-A

Q.1	17	Q.2	x = 1	Q.3	d = 2

SECTION-B

Q.1	C	Q.2	A	Q.3	D	Q.4	A	Q.5	C	Q.6	D	Q.7	A
Q.8	C	Q.9	B	Q.10	C	Q.11	C	Q.12	C	Q.13	D	Q.14	C
Q.15	A	Q.16	B	Q.17	A	Q.18	B	Q.19	C	Q.20	C		

SECTION-C

Q.1	AB	Q.2	BC	Q.3	AB	Q.4	CD

SECTION-D

Q.1	A	Q.2	D

SECTION-E

Q.1	(i)-(Q), (ii)-(S), (iii)-(P), (iv)-(R)	Q..2	(i)-(S), (ii)-(R), (iii)-(Q), (iv)-(P)