7 QUADRATIC EQUATION

7.1 INTRODUCTION

A polynomial of degree one is called linear polynomial and that of degree two is called quadratic polynomial. For example $x^2 + 4$, $2x^2 + 3x + 4$, are quadratic polynomials.

A quadratic polynomial can have at most three terms namely the terms containing x^2, x and the constant term. The general form of a quadratic polynomial in x is $ax^2 + bx + c$, where a, b, c are real numbers and $a \neq 0$. When we equate this polynomial to zero we get a quadratic equation.

7.2 QUADRATIC EQUATION

(i) Let $p(x) = ax^2 + bx + c$, where $a, b, c \in R$ be the quadratic polynomial, then $p(x) = 0$, i.e. $ax^2 + bx + c = 0$ is called a **quadratic equation** where a, b, c are real numbers and $a \neq 0$. Since $a \neq 0$, quadratic equations in general are of the following types

(a) $b = 0, c \neq 0$ i.e. $ax^2 + c = 0$

(b) $b \neq 0, c = 0$ i.e. $ax^2 + bx = 0$

(c) $b = 0, c = 0$ i.e. $ax^2 = 0$

(d) $b \neq 0, c \neq 0$ i.e. $ax^2 + bx + c = 0$

(ii) An equation involving the square of unknown quantity (variable) and no other higher power, is called a quadratic equation or a second degree equation.

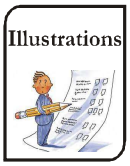


Illustration 1

Which of the following are quadratic equations.

(i) $3x^2 - 8x = 0$ ***(ii)*** $x^2 + \frac{1}{x^2} = 8$ ***(iii)*** $x^2 - 6x + 5\sqrt{x} - 7 = 0$

Solution

(i) $3x^2 - 8x = 0$ is a quadratic equation.

7.3 METHODS OF SOLVING QUADRATIC EQUATIONS

METHOD: 1

7.3.1 Solution of quadratic equations By factorisation method:

Solve: $x^2 - 7x = -10$;

$\Rightarrow$ $x^2 - 7x + 10 = 0$

$\Rightarrow$ $(x - 2)(x - 5) = 0$

$\Rightarrow$ $x = 2$ and $x = 5$

Hence the roots of equation are $x = 2$ and $x = 5$

METHOD: 2

7.3.2 Solution of quadratic equations by completion of square method

Illustration showing solution of a quadratic equation using completion of square:

Find the roots of $2x^2 + 6x + 1 = 0$

Solution

Here the coefficient of x^2 is not equal to 1 so first we make the coefficient of $\mathbf{x^2}$ equal to 1.

Hence the given equation can be written as : $\mathbf{2(x^2 + 3x + 1/2) = 0}$

(Dividing both sides of the equation by 2)

Now, we have $x^2 + 3x + \frac{1}{2} = 0$

Here coefficient of x is 3, so adding and subtracting $\left(\frac{3}{2}\right)^2$ to the given equation:

$$x^2 + 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + \frac{1}{2} = 0 \quad \Rightarrow \quad \left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + \frac{1}{2} = 0$$

$$\Rightarrow \quad \left(x + \frac{3}{2}\right)^2 - \frac{7}{4} = 0 \quad \Rightarrow \quad \left(x + \frac{3}{2}\right)^2 = \frac{7}{4}$$

$$\Rightarrow \quad x + \frac{3}{2} = \pm\frac{\sqrt{7}}{2} \quad \Rightarrow \quad x = -\frac{3}{2} \pm \frac{\sqrt{7}}{2}$$

Hence, $x = \frac{-3+\sqrt{7}}{2}$, $x = \frac{-3-\sqrt{7}}{2}$

Steps used to solve quadratic equation by the completion of square method :

Step-1 : Make the coefficient of x^2 to 1 (if any)

Step-2 : Add and subtract the square of half of the coefficient of x and simplify

Step-3 : In this way we obtain an expression of the form $A^2 - B^2 = 0$ which gives $A^2 - B^2 = (A+B)(A-B) = 0$.

Step-4 : Equate each factor is equal to zero. The values of x so obtained are required zeros of the given quadratic equation.

METHOD: 3

7.3.3 Solution of quadratic equations by the formula method (Shridharacharya method)

The solution of quadratic equation $ax^2 + bx + c = 0$, where $a, b, c \in R$ and $a \neq 0$ is given by the formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

this formula is known as "Shridharchaya formula".

So roots are

$$\mathbf{x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}} \quad \text{and} \quad \mathbf{\frac{-b - \sqrt{b^2 - 4ac}}{2a}}$$

7.4 DISCRIMINANT AND THE NATURE OF ROOTS

(i) Discriminant

For the quadratic equation $ax^2 + bx + c = 0$, the expression $b^2 - 4ac$ denoted by '**D**' is called **DISCRIMINANT** of the equation.

(ii) Nature of the roots:

Nature of quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$, depends upon the value of the expression $b^2 - 4ac$ i.e. discriminant.

Let the two roots α and β obtained using the Discriminant method, are as follows:

$$\alpha = \frac{-b + \sqrt{D}}{2a}, \qquad \beta = \frac{-b - \sqrt{D}}{2a}$$

The nature of the roots and their value has been summarised in the table below:

S. No.	Find D	Nature of Roots	Value of the real roots
1.	$D > 0$	Real and distinct	$\frac{-b \pm \sqrt{D}}{2a}$
2.	$D = 0$	Real and equal (or repeated roots)	Each $= \frac{-b}{2a}$
3.	$D < 0$	No real roots	None
4.	$b = 0, \frac{c}{a} < 0$	Equal in magnitude but opposite in sign real and distinct.	$\sqrt{\frac{-c}{a}}, -\sqrt{\frac{-c}{a}}$
5.	$c = 0$	Real and distinct one being zero	0, -b/a
6.	$c = a$, $b^2 - 4ac > 0$	Real distinct and reciprocal to each other	-

IMPORTANT RESULTS:

Let a and b are two real numbers then

(1) If $a \geq b \Leftrightarrow -a \leq -b$ e.g. $2 < 3$ but $-2 > -3$

(2) If $ax + b \leq 0 \Rightarrow x \leq -\frac{b}{a}$ (3) If $ax + b \geq 0 \Rightarrow x \geq \frac{-b}{a}$

(4) If $x^2 - a^2 \geq 0 \Rightarrow (x + a)(x - a) \geq 0$

Either $x + a \geq 0 \,\&\, x - a \geq 0$

$\Rightarrow x \geq -a \,\&\, x \geq a$

$\Rightarrow x \geq a$ (because if $x \geq a$ then it is understood that $x \geq -a$)

or $x - a \leq 0 \,\&\, x + a \leq 0$

$\Rightarrow x \leq a \,\&\, x \leq -a$

$\Rightarrow x \leq -a$ (because if $x \leq -a$ it is understood that $x \leq a$

For Example, $x \geq 5 \Rightarrow x \geq -5$

Hence $x^2 - a^2 \geq 0 \Rightarrow x \geq a$ *or* $x \leq -a$

(5) If $x^2 - a^2 \le 0 \Rightarrow (x-a)(x+a) \le 0$

Either $x+a \ge 0$ & $x-a \le 0$	or $x+a \le 0$ & $x-a \ge 0$
$\Rightarrow x \ge -a$ & $x \le a$	$\Rightarrow x \le -a$ & $x \ge a$
$\Rightarrow -a \le x \le a$	(There are no such values of x
Hence $x^2 - a^2 \le 0 \Leftrightarrow -a \le x \le a$	which satisfies the above relation)

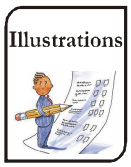


Illustration 2

Determine the nature of the roots of $3x^2 + 11x + 10 = 0$

Solution

$3x^2 + 11x + 10 = 0$

given a = 3, b = 11, c = 10

$D = b^2 - 4ac \Rightarrow D = (11)^2 - 4 \times 3 \times 10 = 121 - 120 = 1 \Rightarrow D = 1$

Since $D > 0$

$\Rightarrow$ Roots are real and unequal.

Illustration 3

Find the value of k for the equation $3x^2 + kx + 3 = 0$ ***have real and equal roots***

Solution

Here, a = 3, b = k, c = 3

for real and equal roots, $b^2 - 4ac = 0$

$(k)^2 - 4 \times 3 \times 3 = 0 \Rightarrow k^2 = 36$, $k = \pm 6$

7.5 RELATION BETWEEN ROOTS AND COEFFICIENTS OF $ax^2 + bx + c = 0$

If α, β are roots of $ax^2 + bx + c = 0$

then, sum of roots = $\alpha + \beta = -b/a$

product of roots $\alpha\beta = c/a$

(i) Symmetric functions of roots of a quadratic equation: Let α and β be the roots of a quadratic equation. An expression in α and β are interchanged, is known as a symmetric function in α and β. To evaluate a symmetric function of the roots of a quadratic equation in terms of its coefficients, we always express it in terms of $\alpha + \beta$ and $\alpha\beta$.

The following results are very useful for the same.

(i) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$.

(ii) $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

(iii) $\alpha^2 - \beta^2 = (\alpha - \beta)(\alpha + \beta) = (\alpha + \beta)\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

(iv) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$

(v) $\alpha^3 - \beta^3 = (\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2)$

(vi) $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$

(vii) $\alpha^4 - \beta^4 = (\alpha + \beta)(\alpha - \beta)(\alpha^2 + \beta^2) = (\alpha + \beta)(\alpha - \beta)[(\alpha + \beta)^2 - 2\alpha\beta]$

(ii) If α, β are the roots of $ax^2 + bx + c = 0$ then

(a) $\alpha^2 + \beta^2 = \dfrac{b^2 - 2ac}{a^2}$ (b) $\alpha - \beta = \dfrac{\sqrt{b^2 - 4ac}}{a}$

(c) $\alpha^2 - \beta^2 = \dfrac{-b}{a^2}\sqrt{b^2 - 4ac}$ (d) $\alpha^3 - \beta^3 = \dfrac{3abc - b^3}{a^3}$

(e) $\alpha^4 - \beta^4 = \dfrac{b^4 - 4ab^2c + 2a^2c^2}{a^4}$ (f) $\alpha^6 + \beta^6 = \dfrac{9a^2b^2c^2 + b^6 - 6ab^4c + 2a^3c^3}{a^6}$

(g) $\dfrac{\alpha + \beta}{\alpha^{-1} + \beta^{-1}} = \dfrac{c}{a}$ (h) $\sqrt{\dfrac{\alpha}{\beta}} + \sqrt{\dfrac{\beta}{\alpha}} = \dfrac{-b}{\sqrt{ac}}$

(iii) If one root of $ax^2 + bx + c = 0$ is twice the other then $2b^2 = 9ac$.

(iv) If one root of the equation $x^2 + px + q = 0$ is three times the other then $16q = 3p^2$.

(v) If the roots of the equation $x^2 - px + q = 0$ are consecutive numbers then $p^2 = 4q + 1$

(vi) (a) If the ratio of the roots of the equation $ax^2 + bx + c = 0$ is m : n then $(m + n)^2ac = mnb^2$
(b) If one root of the quadratic equation is k times the other then $(k + 1)^2ac = kb^2$.

(vii) If the roots of the equation $x^2 - px + q = 0$ differ by unity then $p^2 = 4q + 1$

(viii) If unity is a root of $ax^2 + bx + c = 0$ then the other root is $\dfrac{c}{a}$.

(ix) Roots are equal in magnitude but opposite in sign if $b = 0$

(x) Both roots are zero if $b = c = 0$.

7.6 EQUATION REDUCIBLE TO QUADRATIC FORM

Let us understand this topic by taking illustrations

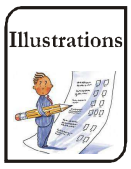


Illustration 4

Solve the equation for x $\left(\dfrac{x}{x+1}\right)^2 + 6 - 5\left(\dfrac{x}{x+1}\right) = 0$***; x ≠ 0***

Solution :

[Hint: Let $\dfrac{x}{x+1} = y$]

$y^2 + 6 - 5y = 0 \Rightarrow y^2 - 5y + 6 = 0$
$y^2 - 2y - 3y + 6 = 0 \Rightarrow y(y - 2) - 3(y - 2) = 0$
$y - 2 = 0$ or $y - 3 = 0 \Rightarrow y = 2$ or 3

Case-I

if $y = 2$

$\Rightarrow \quad \frac{x}{x+1} = 2 \quad \Rightarrow \quad x = 2x + 2$

$\Rightarrow \quad 2x - x = -2 \quad \Rightarrow \quad x = -2$

Case-II

If $y = 3$

$\Rightarrow \quad \frac{x}{x+1} = 3 \quad \Rightarrow \quad x = 3x + 3 \quad \Rightarrow \quad 3x - x = -3$

$\Rightarrow \quad 2x = -3 \quad \Rightarrow \quad x = -3/2$

Ans. $-3/2, -2$

Illustration 5

Solve the equation for x: $\left(x^2 + \frac{1}{x^2}\right) - 3\left(x - \frac{1}{x}\right) - 2 = 0 \;\; x \neq 0$

Solution :

[Hint: Put $x - \frac{1}{x} = y$ **]**

Put, $x - \frac{1}{x} = y$ .....(1)

squaring both sides

$\left(x - \frac{1}{x}\right)^2 = y^2$

$\Rightarrow \quad x^2 + \frac{1}{x^2} - 2 = y^2 \quad \Rightarrow \quad x^2 + \frac{1}{x^2} = y^2 + 2$ .....(2)

Now,

$\left(x^2 + \frac{1}{x^2}\right) - 3\left(x - \frac{1}{x}\right) - 2 = 0$

$(y^2 + 2) - 3(y) - 2 = 0$ (from (1) & (2))

$y^2 + 2 - 3y - 2 = 0$

$\Rightarrow \quad y^2 - 3y = 0 \quad \Rightarrow \quad y(y - 3) = 0$

$\Rightarrow \quad y = 0 \text{ or } y - 3 = 0 \quad \Rightarrow \quad y = 0 \text{ or } 3$

Case I : If $y = 0$

$\Rightarrow \quad x - \frac{1}{x} = 0 \quad \Rightarrow \quad \frac{x^2 - 1}{x} = 0$

$\Rightarrow \quad x^2 - 1 = 0 \quad \Rightarrow \quad (x + 1)(x - 1) = 0$

$\Rightarrow \quad x = \pm 1$ .....(3)

Case II : if $y = 3 \Rightarrow x - \frac{1}{x} = 3$

$\Rightarrow \quad x^2 - 3x - 1 = 0 \quad \Rightarrow \quad x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2 \times 1}$

$\Rightarrow \quad x = \frac{3 \pm \sqrt{9+4}}{2} \quad \Rightarrow \quad x = \frac{3 \pm \sqrt{13}}{2}$ .....(4)

from (3) & (4)

$x = \pm 1, \frac{3 \pm \sqrt{13}}{2}$

Ans. $\left\{\pm 1, \frac{3 \pm \sqrt{13}}{2}\right\}$

Illustration 6

Solve the equation for x: $x^4 + 2x^3 - 13x^2 + 2x + 1 = 0$

Solution :

[Hint: Divide both sides by x^2]

$x^4 + 2x^3 - 13x^2 + 2x + 1 = 0$

dividing both the sides by x^2,

we get $x^2 + 2x - 13 + \frac{2}{x} + \frac{1}{x^2} = 0$ .....(1)

Let $x + \frac{1}{x} = y$ .....(2)

squaring both sides,

$x^2 + \frac{1}{x^2} = y^2 - 2$ .....(3)

substituting the values of $\left(x + \frac{1}{x}\right)$ and $\left(x^2 + \frac{1}{x^2}\right)$ in eq. (1) from equations (2) and (3),

we get $(y^2 - 2) + 2(y) - 13 = 0$

$y^2 - 2 + 2y - 13 = 0$

$y^2 + 2y - 15 = 0 \quad \Rightarrow \quad y^2 + 5y - 3y - 15 = 0$

$\Rightarrow \quad (y + 5)(y - 3) = 0 \quad \Rightarrow \quad y + 5 = 0$ or $y - 3 = 0$

$\Rightarrow \quad y = -5$, or 3

Case-I :

If $y = -5 \quad \Rightarrow \quad x + \frac{1}{x} = -5$

$\Rightarrow \quad \frac{x^2 + 1}{x} = -5 \quad \Rightarrow \quad x^2 + 5x + 1 = 0$

$\Rightarrow \quad x = \frac{-5 \pm \sqrt{25 - 4}}{2} = \frac{-5 \pm \sqrt{21}}{2}$

Case II :

if $y = 3$

$\Rightarrow \quad x + \frac{1}{x} = 3 \quad \Rightarrow \quad x^2 - 3x + 1 = 0$

$\Rightarrow \quad x = \frac{3 \pm \sqrt{9-4}}{2} \quad \Rightarrow \quad x = \frac{3 \pm \sqrt{5}}{2}$

Ans. $\left\{\frac{-5 \pm \sqrt{21}}{2}\right\}, \frac{3 \pm \sqrt{5}}{2}$

7.7 TO FORM A QUADRATIC EQUATION WITH ROOTS α AND β

A quadratic equation with roots α & β is given by

$x^2 - (\text{sum of roots})\, x + \text{product of roots} = 0$

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$x^2 - Sx + P = 0$

where, S denotes the sum of roots and P denotes the product of roots.

7.8 CONDITION FOR COMMON ROOTS

Consider two quadratic equations

$a_1x^2 + b_1x^2 + c_1 = 0$ .....(i) $(a_1 \neq 0)$

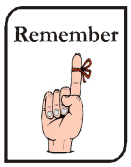
and $a_2x^2 + b_2x + c_2 = 0$ .....(ii) $(a_2 \neq 0)$

(1) If one root is common then,

$(a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1) = (c_1a_2 - c_2a_1)^2$

(2) If both roots are common then,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$



1. The graph of a quadratic function (expression) is called a parabola . The point at which its direction changes is called its turning point, commonly called the vertex of the parabola.
2. The graph of the function is concave upwards when $a > 0$ and concave downwards when $a < 0$.
3. If the graph has no points in common with the x – axis, the roots of the equation are imaginary and cannot be determined from the graph.
4. If the graph is tangent to the x – axis, the roots are real and equal.
5. If the graph cuts the x – axis, the roots of the equation will be real and unequal. Their values will be given by the abscissa of the points of intersection of the graph and the x – axis.

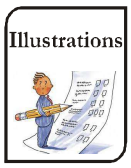


Illustration 7

Find the equation whose roots are the reciprocals of the roots of $3x^2 - 5x + 7 = 0$

Solution:

The equation whose roots are the reciprocals of the roots of $f(x) = 0$ is $f\left(\frac{1}{x}\right) = 0$

$\Rightarrow$ The required equation is $3\left(\frac{1}{x}\right)^2 - 5\left(\frac{1}{x}\right) + 7 = 0$

$\Rightarrow$ $\left(\frac{3}{x^2}\right) - \left(\frac{5}{x}\right) + 7 = 0$

$\Rightarrow$ $7x^2 - 5x + 3 = 0$

Illustration 8

If one root of $x^2 - x - k = 0$ is square that of other, then find the value of k.

Solution

Let one root be α

Other root $= \alpha^2$

Sum of the roots $= +1$

$\Rightarrow$ $\alpha + \alpha^2 = 1$ .....(i)

product of roots $= -k$

$\Rightarrow$ $\alpha \cdot \alpha^2 = -k$

$\Rightarrow$ $\alpha^3 = -k$

$\Rightarrow$ $\alpha = (-k)^{\frac{1}{3}}$ .....(ii)

$\because$ $\alpha^2 + \alpha = +1$

substituting $\alpha = (-k)^{\frac{1}{3}}$ from equation (2), we get

$$\left[(-k)^{\frac{1}{3}}\right]^2 + (-k)^{\frac{1}{3}} = +1$$

$$k^{\frac{2}{3}} - k^{\frac{1}{3}} = 1$$

$\Rightarrow$ $\left[k^{\frac{2}{3}} - k^{\frac{1}{3}}\right]^3 = (1)^3$ $\Rightarrow$ $k^2 - k - 3k = 1$

$\Rightarrow$ $k^2 - 4k = 1$ $\Rightarrow$ $k^2 - 4k - 1 = 0$

$\Rightarrow$ $k = \frac{4 \pm \sqrt{16+4}}{2}$ $\Rightarrow$ $k = \frac{4 \pm \sqrt{20}}{2}$

$$k = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

Illustration 9

Find the condition that the quadratic equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ may have a common root.

Solution:

Let α be a common root of the given equations.

Then $\alpha^2 + a\alpha + b = 0$

and $\alpha^2 + b\alpha + a = 0$

By the method of cross – multiplication, we get

$$\frac{\alpha^2}{a^2 - b^2} = \frac{\alpha}{b - a} = \frac{1}{b - a}$$

This gives $\alpha^2 = \dfrac{a^2 - b^2}{b - a} = -(a + b)$ and $\alpha = 1$

$\Rightarrow$ $(1)^2 = -(a + b)$ $\Rightarrow$ $1 = -a - b$

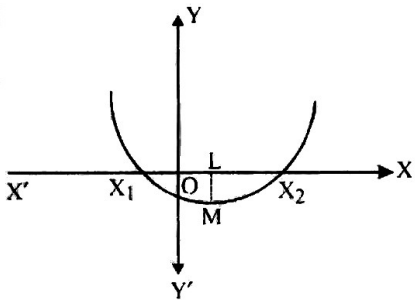
$\Rightarrow$ $a + b + 1 = 0$ is the required condition.

7.9 GRAPH OF QUADRATIC EXPRESSIONS

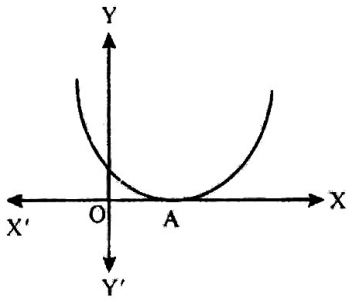
Consider the expression $y = ax^2 + bx + c (a \neq 0)$ and $a, b, c \in R$ then the graph between x, y is always a parabola if $a > 0$ then the shape of the parabola is concave upwards and if $a < 0$ then the shape of the parabola is concave downwards. There is only 6 possible graphs of a quadratic as given below:

Case – I (When a > 0)

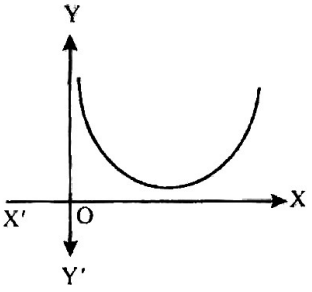
(i) If $D > 0$



(ii) If $D = 0$

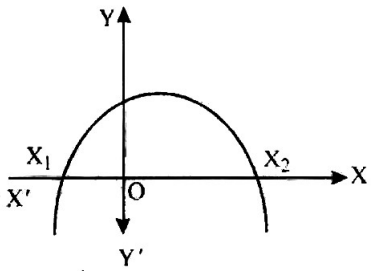


(iii) If $D < 0$

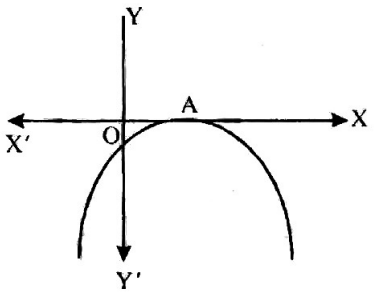


Case – II (When a < 0)

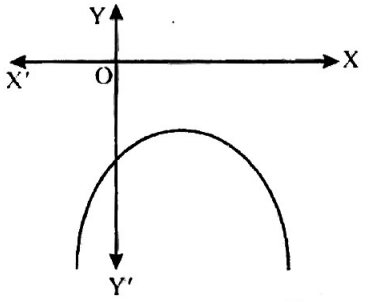
(i) If D > 0



(ii) If D = 0



(iii)) If D < 0



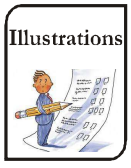


Illustration 10

Find the solution set of the equation $x + 5 - \frac{8}{x+5} = 7$

Solution:

The given equation is :

$$x + 5 - \frac{8}{x+5} = 7$$

Multiply both sides by (x + 5), we get

$(x + 5)^2 - 8 = 7\,(x + 5)$

i.e. $(x + 5)^2 - 7\,(x + 5) - 8 = 0$

Put $u = x + 5$

The equation reduces to $u^2 - 7u - 8 = 0$

i.e. $(u - 8)\,(u + 1) = 0$

$\therefore$ $u = 8$ or $u = -1$

$u = x + 5$

i.e. $x + 5 = 8 \Rightarrow x = 8 - 5 = 3$

$x + 5 = -1 \Rightarrow x = -1 - 5 = -6$

$\therefore$ roots are $x = 3$ and $x = -6$

The solution set = $\{-6, 3\}$

7.10 NATURE OF THE ROOTS:

A quadratic equation has real roots only if $b^2 - 4ac \geq 0$.

If $b^2 - 4ac < 0$, then the roots of the quadratic equation are complex conjugates.

The following table gives us a clear idea about the nature of the roots of a quadratic equation when a, b and c are all rational.

Condition	Nature of roots
when $b^2 - 4ac < 0$	the roots are complex conjugates
when $b^2 - 4ac = 0$	the roots are rational and equal
When $b^2 - 4ac > 0$ and a perfect square	the roots are rational and unequal
When $b^2 - 4ac > 0$ and not a perfect square	the roots are irrational and unequal

Note:

1. Whenever the roots of the quadratic equation are irrational, (a, b, c being rational), are of the form $a + \sqrt{b}$ and $a - \sqrt{b}$, i.e. whenever $a + \sqrt{b}$ is one root of a quadratic equation, $a - \sqrt{b}$ is the other root of quadratic equation and vice – versa. In other words, if the roots of a quadratic equation are irrational, then they are conjugate to each other.

2. If the sum of the coefficients of a quadratic equation, say $ax^2 + bx + c = 0$, is zero, then its roots are 1 and $\frac{c}{a}$. That is if $a + b + c = 0$, then the roots of $ax^2 + bx + c = 0$ are 1 and $\frac{c}{a}$.

7.11 SIGNS OF THE ROOTS :

We can comment on the signs of the roots, i.e. whether the roots are positive or negative, based on the sign of the sum of the roots and the product of the roots of the quadratic equation. The following table indicates the signs of the roots when the signs of the sum and the product of the roots are given.

Sign of Product of the roots	Sign of Sum of the roots	Sign of the roots
+ve	+ve	Both the roots are positive .
+ve	-ve	Both the roots are negative.
-ve	+ve	One root is positive and the other negative. The numerically greater root is positive.
-ve	-ve	One root is positive and the other negative. The numerically greater root is negative.

Constructing a new quadratic equation by changing the roots of a given quadratic equation

If we are given a quadratic equation, we can build a new quadratic equation by changing the roots of the equation in the manner specified to us.

For example, consider the quadratic equation $ax^2 + bx + c = 0$ and let its roots be α and β respectively. Then, we can build new quadratic equations as per the following points :

(i) A quadratic equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$, i.e. the roots are reciprocal to the roots of the given quadratic equation can be obtained by substituting $(1/x)$ for x in the given equation which gives us $cx^2 + bx + a = 0$, i.e. we get the equation required by interchanging the coefficient of x and the constant term.

(ii) A quadratic equation whose roots are $(\alpha + k)$ and $(\beta + k)$ can be obtained by substituting $(x - k)$ for x in the given equation.

(iii) A quadratic equation whose roots are $(\alpha - k)$ and $(\beta - k)$ can be obtained by substituting $(x + k)$ for x in the given equation.

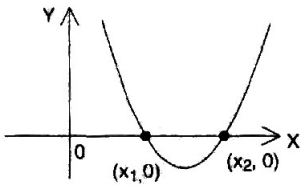
(iv) A quadratic equation whose roots are $(k\alpha)$ and $(k\beta)$ can be obtained by substituting with $\left[\frac{x}{k}\right]$ for x in the given equation.

(v) A quadratic equation whose roots are $\left[\frac{\alpha}{k}\right]$ and $\left[\frac{\beta}{k}\right]$ can be obtained by substituting (kx) for x in the given equation.

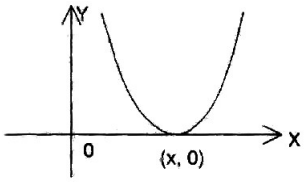
(vi) A quadratic equation whose roots are $(-\alpha)$ and $(-\beta)$ can be obtained by replacing x by $(-x)$ in the given equation.

Note:

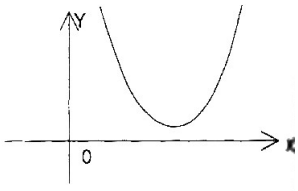
1. If the graph meets the x – axis at two distinct points, then the roots of the given equation are real and distinct.



2. If the graph touches the x – axis at the only one point, then the roots of the quadratic equation are real and equal.



3. If the graph does not meet the x – axis, then the roots of the quadratic equation are not real, i.e. they are complex.



7.12 MAXIMUM OR MINIMUM VALUE OF A QUADRATIC EXPRESSION.

The quadratic expression $ax^2 + bx + c$ takes different values as x takes different values.

For all the values of x, as x varies from $-\infty$ to $+\infty$ (i.e. when x is real), the quadratic expression $ax^2 + bx + c$

(i) has a minimum value if $a > 0$ (i.e. a is positive). The minimum value of the quadratic expression is $\frac{4ac - b^2}{4a}$ and it occurs at $x = \frac{-b}{2a}$

(ii) has a maximum value if $a < 0$ (i.e., a is negative). The maximum value of the quadratic expression is $\frac{(4ac - b^2)}{4a}$ and it occurs at $x = -\frac{b}{2a}$.

7.13 RELATION BETWEEN ROOTS AND COEFFICENTS OF CUBIC AND BIQUADRATIC EQUATIONS

If the equation is of a cubic form, $ax^3 + bx^2 + cx + d = 0$, and α, β and γ are the 3 roots of the cubic equation, then

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\alpha\beta\gamma = -\frac{d}{a}$$

If the equation of a biquadratic form, $ax^4 + bx^3 + cx^2 + dx + e = 0$ and α, β, γ, δ are the four roots of the biquadratic equation,

$$\alpha + \beta + \gamma + \delta = -\frac{b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\delta + \alpha\gamma + \beta\delta + \alpha\delta = \frac{c}{a}$$

$$\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = -\frac{d}{a}$$

$$\alpha\beta\gamma\delta = \frac{e}{a}$$

SOLVED EXAMPLES

Example 1

Determine the value of k, (k > 0) such that both the equations $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ will have real roots

Solution

$x^2 + kx + 64 = 0$

To have real roots

$b^2 - 4ac \geq 0$

$k^2 - 256 \geq 0$

$\Rightarrow (k+16)(k-16) \geq 0 \quad \Rightarrow \quad k \geq 16 \text{ or } k \geq -16 \quad .....(1)$

$x^2 - 8x + k = 0$

To have real roots

$b^2 - 4ac \geq 0$

$(-8)^2 - 4 \times 1 \times k \geq 0$

$64 - 4k \geq 0 \quad \Rightarrow \quad 4k \leq 64 \quad \Rightarrow \quad k \leq 16 \quad .....(2)$

since k > 0, therefore $0 < k \leq 16$ for both equations to have real roots both will satisfy k = 16

Example 2

Solve the equation for x: $3\sqrt{\frac{x}{5}} + 3\sqrt{\frac{5}{x}} = 10$

Solution

[Hint: Let $\sqrt{\frac{x}{5}} = y$.]

$3y + \frac{3}{y} = 10 \quad \Rightarrow \quad 3y^2 + 3 = 10y$

$\Rightarrow \quad 3y^2 - 10y + 3 = 0 \quad \Rightarrow \quad 3y^2 - 9y - y + 3 = 0$

$\Rightarrow \quad 3y(y-3) - 1(y-3) = 0 \quad \Rightarrow \quad (y-3)(3y-1) = 0$

$\Rightarrow \quad y - 3 = 0 \text{ or } 3y - 1 = 0 \quad \Rightarrow \quad y = 3 \text{ or } y = \frac{1}{3}$

Case : I If y = 3

$\Rightarrow \quad \sqrt{\frac{x}{5}} = 3 \Rightarrow \frac{x}{5} = 9 \quad \Rightarrow \quad x = 45$

Case : II if $y = \frac{1}{3}$

$\Rightarrow \quad \sqrt{\frac{x}{5}} = \frac{1}{3} \quad \Rightarrow \frac{x}{5} = \frac{1}{9} \quad \Rightarrow \quad x = \frac{5}{9}$

Ans. $\left\{\frac{5}{9}, 45\right\}$

Example 3

Find the number which exceeds its positive square root by 20.

Solution:

$x - \sqrt{x} = 20$ or $x - 20 = \sqrt{x}$

squaring on both sides, we get

or $(x-20)^2 = x$ or $x^2 - 40x + 400 = x$

or $x^2 - 41x + 400 = 0$ or $(x-25)(x-16) = 0$

This gives $x = 25$ or $x = 16$

But $x = 16$ does not satisfy the given equation. Hence the required number is 25.

Example 4

Evaluate $20 + \cfrac{1}{20 + \cfrac{1}{20.....}}$

Solution.

Let $x = 20 + \cfrac{1}{20 + \cfrac{1}{20}.....}$ .....(i)

or $x = 20 + \cfrac{1}{x}$ (From (i))

Therefore, $x^2 - 20x - 1 = 0$

This gives $x = \cfrac{20 + \sqrt{404}}{2} = 10 + \sqrt{101}$ or $x = \cfrac{20 - \sqrt{404}}{2} = 10 - \sqrt{101}$

Since, the given expression can not be negative therefore, we neglect the negative value $10 - \sqrt{101}$.

Hence, the desired value of the expression is $10 + \sqrt{101}$.

Example 5

Discuss the nature of the roots of the equation $4x^2 - 2x + 1 = 0$.

Solution:

Discriminant $= (-2)^2 - 4(4)(1) = 4 - 16 = -12 < 0$

Since the discriminant is negative, the roots are imaginary.

Example 6

If the sum of the roots of the equation $kx^2 - 3x + 9 = 0$ is $\cfrac{3}{11}$, then find the product of the roots of that equation.

Solution:

Sum of roots of the equation $= \cfrac{3}{k} = \cfrac{3}{11}$ (given) $\therefore$ $k = 11$

In the given equation, product of roots $= \cfrac{9}{k}$

As $k = 11$, product of roots $= \cfrac{9}{11}$

Example 7

Form a quadratic equation with rational coefficients one of whose root is $3 + \sqrt{5}$.

Solution

If $(3 + \sqrt{5})$ is one root, then the other root is $(3 - \sqrt{5})$

Sum of the roots = 6; Product of the roots = 4

Thus the required equation is $x^2 - 6x + 4 = 0$

Example 8

If α and β are the roots of the equation $x^2 - 6x + 8 = 0$, then find the values of

(i) $\alpha^2 + \beta^2$ **(ii) $\frac{1}{\alpha}+\frac{1}{\beta}$** **(iii) $\alpha - \beta$ $(\alpha > \beta)$**

Solution:

For the given equation, we get $\alpha + \beta = 6$ and $\alpha\beta = 8$

(1) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (6)^2 - 2(8) = 20$

(2) $\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{6}{8} = \frac{3}{4}$

(3) $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$\Rightarrow$ $(\alpha - \beta) = \pm\sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \sqrt{6^2 - 4(8)}$ $\Rightarrow$ $(\alpha - \beta) = \pm 2$

$\therefore$ $\alpha - \beta = 2$ $(\because \alpha > \beta)$

Example 9

Solve for x :

$3^{x+1} + 3^{2x+1} = 270$

Solution

$3^{x+1} + 3^{2x+1} = 270$

$\Rightarrow$ $3.3^x + 3^{2x}.3 = 270$ $\Rightarrow$ $3^x + 3^{2x} = 90$

Substituting $3^x = a$, we get

$a + a^2 = 90$

$\Rightarrow$ $a^2 + a - 90 = 0$ $\Rightarrow$ $a^2 + 10a - 9a - 90 = 0$

$\Rightarrow$ $(a + 10)(a - 9) = 0$ $\Rightarrow$ $a = 9$ or $a = -10$

If $3^x = 9$ then $x = 2$

If $3^x = -10$, which is not possible

$\therefore$ $x = 2$

Example 10

Solve $|x|^2 - 7|x| + 12 = 0$

Solution:

Given equation is $|x|^2 - 7|x| + 12 = 0$

$\Rightarrow$ $(|x| - 3)(|x| - 4) = 0$ $\Rightarrow$ $|x| = 3$ or $|x| = 4$ $\Rightarrow$ $x = \pm 3$ or $x = \pm 4$

Example 11

Solve $|x|^2 + 7|x| + 10 = 0$

Solution:

$\Rightarrow$ $(|x| + 2)(|x| + 5) = 0$ $\Rightarrow$ $|x| = -2$ or $|x| = -5$

But, absolute value of any number can never be negative.

$\therefore$ No roots are possible for the given equation.

CONCEPT APPLICATION LEVEL - I [NCERT Questions]

Q.1 Check whether the following are quadratic equations :

(i) $(x + 1)^2 = 2(x - 3)$ **(ii)** $x^2 - 2x = (-2)(3 - x)$

(iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$ **(iv)** $(x - 3)(2x + 1) = x(x + 5)$

(v) $(2x - 1)(x - 3) = (x + 5)(x - 1)$ **(vi)** $x^2 + 3x + 1 = (x - 2)^2$

(vii) $(x + 2)^3 = 2x(x^2 - 1)$ **(viii)** $x^3 - 4x^2 - x + 1 = (x - 2)^3$

Sol. (i) $(x + 1)^2 = 2(x - 3)$

The given equation is

$(x + 1)^2 = 2(x - 3)$

$\Rightarrow x^2 + 2x + 1 = 2x - 6$

$\Rightarrow x^2 + 7 = 0$

It is of the form $ax^2 + bx + c = 0$, $a \neq 0$.

Therefore, the given equation is a quadratic equation.

(ii) $x^2 - 2x = (-2)(3 - x)$

The given equation is

$x^2 - 2x = (-2)(3 - x)$

$\Rightarrow x^2 - 2x = -6 + 2x$

$\Rightarrow x^2 - 4x + 6 = 0$

The given equation is a quadratic equation.

(iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$

The given equation is

$(x - 2)(x + 1) = (x - 1)(x + 3)$

$\Rightarrow x^2 + x - 2x - 2 = x^2 + 3x - x - 3$

$\Rightarrow x^2 - x - 2 = x^2 + 2x - 3$

$\Rightarrow 3x - 1 = 0.$

It is a linear equation.

(iv) $(x - 3)(2x + 1) = x(x + 5)$

The given equation is

$(x - 3)(2x + 1) = x(x + 5)$

$\Rightarrow 2x^2 + x - 6x - 3 = x^2 + 5x$

$\Rightarrow 2x^2 - 5x - 3 = x^2 + 5x$

$\Rightarrow x^2 - 10x - 3 = 0.$

The given equation is a quadratic equation.

(v) $(2x - 1)(x - 3) = (x + 5)(x - 1)$

The given equation is

$(2x - 1)(x - 3) = (x + 5)(x - 1) \Rightarrow 2x^2 - 6x - x + 3 = x^2 - x + 5x - 5$

$\Rightarrow 2x^2 - 7x + 3 = x^2 + 4x - 5 \Rightarrow x^2 - 11x + 8 = 0$

The given equation is a quadratic equation.

(vi) $x^2 + 3x + 1 = (x-2)^2$

The given equation is

$x^2 + 3x + 1 = (x-2)^2$

$\Rightarrow x^2 + 3x + 1 = x^2 - 4x + 4$

$\Rightarrow 7x - 3 = 0$

It is a linear equation.

(vii) $(x + 2)^3 = 2x(x^2 - 1)$

The given equation is

$(x + 2)^3 = 2x(x^2 - 1)$

$\Rightarrow x^3 + 6x^2 + 12x + 8 = 2x^3 - 2x$

$\Rightarrow x^3 - 6x^2 - 14x - 8 = 0$

It is a cubic equation.

(viii) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

The given equation is

$x^3 - 4x^2 - x + 1 = (x - 2)^3$

$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 8 - 6x^2 + 12x$

$\Rightarrow 2x^2 - 13x + 9 = 0$

The given equation is a quadratic equation.

Q.2 Represent the following situation in the form of quadratic equations:

(i) The area of rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km / h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Sol. (i) Let the breadth of the rectangular plot be x metres.

Then the length of the rectangular plot =(2x + 1) metres.

Therefore, the area of the rectangular plot = Length × Breadth = (2x + 1) x m^2

According to the problem situation,

$(2x + 1)\,x = 528 \quad \Rightarrow \quad 2x^2 + x = 528$

$\Rightarrow 2x^2 + x - 528 = 0$

Therefore, the breadth of the rectangular plot satisfies the quadratic equation

$2x^2 + x - 528 = 0$.

(ii) Let the two consecutive positive integers be x and x + 1.

Then, their product = x (x + 1)

According to the problem situation.

$x(x + 1) = 306$

$\Rightarrow \quad x^2 + x = 306 \qquad \Rightarrow \qquad x^2 + x - 306 = 0$

Therefore, the smaller positive integer satisfies the quadratic equation.

$x^2 + x - 306 = 0$

(iii) Let Rohan's present age be x years.

Then present age of Rohan's mother = (x + 26) years

3 years from now, age of Rohan = (x + 3) years

age of Rohan's mother = (x + 26 + 3) years

Therefore, the product of their ages 3 years from now = (x + 3) (x + 29)

According to the problem situation,

$(x + 3)(x + 29) = 360$

$\Rightarrow \quad x^2 + 29x + 3x + 87 = 360$

$\Rightarrow \quad x^2 + 32x - 273 = 0$

Therefore, Rohan's present age satisfies the quadratic equation $x^2 + 32x - 273 = 0$

(iv) Let the uniform speed of the train be x km/hour.

Then, time taken by the train to cover a distance of 480 km = $\frac{480}{x}$ hours $\left[\text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$

If the speed had been 8 km/h less, then the speed of the train = (x – 8) km/h.

Therefore, time taken by the train to cover a distance of 480 km now = $\frac{480}{x-8}$ hours

$\left[\text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$

According to the problem situation,

$\frac{480}{x-8} = \frac{480}{x} + 3 \qquad \Rightarrow \qquad \frac{480}{x-8} - \frac{480}{x} = 3$

$\Rightarrow \quad 480\left(\frac{1}{x-8} - \frac{1}{x}\right) = 3 \qquad \Rightarrow \qquad \frac{1}{x-8} - \frac{1}{x} = \frac{3}{480}$

$\Rightarrow \quad \frac{x-(x-8)}{(x-8)x} = \frac{1}{160} \qquad \Rightarrow \qquad \frac{x-x+8}{x^2-8x} = \frac{1}{160}$

$\Rightarrow \quad \frac{8}{x^2-8x} = \frac{1}{160} \qquad \Rightarrow \qquad x^2 - 8x = (8)(160)$

$\Rightarrow \quad x^2 - 8x = 1280 \qquad \Rightarrow \qquad x^2 - 8x - 1280 = 0$

Therefore, the uniform speed of the train satisfies the quadratic equation $x^2 - 8x - 1280 = 0$.

Q.3 Find the roots of the following quadratic equations by factorisation

(i) $x^2 - 3x - 10 = 0$ **(ii) $2x^2 + x - 2 = 0$** **(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$**

(iv) $2x^2 - x + \frac{1}{8} = 0$ **(v) $100x^2 - 20x + 1 = 0$**

Sol. (i) We have, $x^2 - 3x - 10 = 0$, $x^2 - 5x + 2x - 10 = 0$

$\Rightarrow x(x-5) + 2(x-5) = 0 \Rightarrow (x-5)(x+2) = 0$

Either $x - 5 = 0 \Rightarrow x = 5$ or $x + 2 = 0 \Rightarrow x = -2$

Thus, the required roots are $x = 5$ and $x = -2$

(ii) We have, $2x^2 + x - 6 = 0 \Rightarrow 2x^2 + 4x - 3x - 6 = 0$

$\Rightarrow 2x(x+2) - 3(x+2) = 0 \Rightarrow (x+2)(2x-3) = 0$

Either $x + 2 = 0 \Rightarrow x = -2$ or $2x - 3 = 0 \Rightarrow x = \frac{3}{2}$

Thus, the required roots are $x = -2$ and $x = \frac{3}{2}$.

(iii) We have $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0 \Rightarrow \sqrt{2}x^2 + (\sqrt{2}.\sqrt{2})x + 5x + 5.\sqrt{2} = 0$

$\Rightarrow \sqrt{2}x[x + \sqrt{2}] + 5[x + \sqrt{2}] = 0 \Rightarrow (x + \sqrt{2})(\sqrt{2}x + 5) = 0$

Either $x + \sqrt{2} = 0 \Rightarrow x = -\sqrt{2}$ or $\sqrt{2}x + 5 = 0$

$\Rightarrow x = -\frac{5}{\sqrt{2}} \Rightarrow x = \frac{-5\sqrt{2}}{2}$

Thus, the required roots are $x = -\sqrt{2}$ or $x = \frac{-5\sqrt{2}}{2}$

(iv) We have, $2x^2 - x + \frac{1}{8} = 0$

$\Rightarrow 16x^2 - 8x + 1 = 0 \Rightarrow 16x^2 - 4x - 4x + 1 = 0$

$\Rightarrow 4x(4x-1) - 1(4x-1) = 0 \Rightarrow (4x-1)(4x-1) = 0$

$\Rightarrow x = \frac{1}{4}$ and $x = \frac{1}{4}$

Thus, the required roots are $x = \frac{1}{4}$ and $x = \frac{1}{4}$.

(v) We have, $100x^2 - 20x + 1 = 0$

$\Rightarrow 100x^2 - 10x - 10x + 1 = 0 \Rightarrow 10x(10x-1) - 1(10x-1) = 0$

$\Rightarrow (10x-1)(10x-1) = 0 \Rightarrow (10x-1) = 0$ and $(10x-1) = 0$

$\Rightarrow x = \frac{1}{10}$ and $x = \frac{1}{10}$

Thus, the required roots are $x = \frac{1}{10}$ and $x = \frac{1}{10}$.

Q.4 Solve the problems :

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have in 124. We would like to find out how many marbles they had to start with.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs. 750. We would like to find out the number of toys produced on that day.

Sol. (i) Let one of them had x marbles and the other one had (45 –x).

$\therefore$ $(x - 5) \times (45 - x - 5) = 124$

We have : $x^2 - 45x + 324 = 0$ $\Rightarrow$ $x^2 - 9x - 36x + 324 = 0$

$\Rightarrow$ $x(x - 9) - 36(x - 9) = 0$ $\Rightarrow$ $(x - 9)(x - 36) = 0$

Either $x - 9 = 0$ $\Rightarrow$ $x = 9$ or $x - 36 = 0$ $\Rightarrow$ $x = 36$

Thus, $x = 9$ and $x = 36$

(ii) Let the number of toys produced on that day be x.

Then cost of 1 toy $= \frac{750}{x}$

$\Rightarrow$ $\frac{750}{x} = 55 - x$ $\Rightarrow$ $x^2 - 55x = -750$

$\Rightarrow$ $x^2 - 55x + 750 = 0$

We have : $x^2 - 55x + 750 = 0$ $[(-30) \times (-25) = 750$ and $(-30) + (-25) = -55]$

$\Rightarrow$ $x^2 - 30x - 25x + 750 = 0$

$\Rightarrow$ $x(x - 30) - 25(x - 30) = 0$

$\Rightarrow$ $(x - 30)(x - 25) = 0$

Either $x - 30 = 0$

$\Rightarrow$ $x = 30$ or $x - 25 = 0$ $\Rightarrow$ $x = 25$

Thus, $x = 30$ and $x = 25$.

Q.5 Find two numbers whose sum 27, and product is 182.

Sol. Let one number be x.

Then the other number $= 27 - x$ ($\because$ Sum of two numbers is 27)

Therefore, their product $= x(27 - x)$

According to the question,

$x(27 - x) = 182$ $\Rightarrow$ $27x - x^2 = 182$

$\Rightarrow$ $x^2 - 27x + 182 = 0$ $\Rightarrow$ $x^2 - 13x - 14x + 182 = 0$

$\Rightarrow$ $x(x - 13) - 14(x - 13) = 0$ $\Rightarrow$ $(x - 13)(x - 14) = 0$

$\Rightarrow$ $x - 13 = 0$ or $x - 14 = 0$ $\Rightarrow$ $x = 13$ or $x = 14$

$\Rightarrow$ $x = 13, 14$ $\Rightarrow$ $27 - x = 14, 13$

So, the required two numbers are 13 and 14.

Q.6 Find two consecutive positive integers, sum of whose squares is 365.

Sol. Let the consecutive positive integers be x and x + 1. ($\because$ Two consecutive positive integers differ by 1)

Then, the sum of their squares

$= x^2 + (x + 1)^2 = x^2 + x^2 + 2x + 1 = 2x^2 + 2x + 1$

According to the question,

$2x^2 + 2x + 1 = 365$

$\Rightarrow$ $2x^2 + 2x - 364 = 0$ $\Rightarrow$ $x^2 + x - 182 = 0$ (Dividing throughout by 2)

$\Rightarrow$ $x^2 + 14x - 13x - 182 = 0$ $\Rightarrow$ $x(x + 14) - 13(x + 14) = 0$

$\Rightarrow$ $(x + 14)(x - 13) = 0$ $\Rightarrow$ $x + 14 = 0$ or $x - 13 = 0$

$\Rightarrow$ $x = -14, 13$

$\because$ x is a positive integer

$\therefore$ $x = -14$ is unadmissible

So, $x = 13$ $\Rightarrow$ $x + 1 = 13 + 1 = 14$

Hence, the required two consecutive positive integers are 13 and 14.

Q.7 The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Sol. Let the base of the right triangle be x cm. Then the height of the right triangle = (x –7) cm

By Pythagoras theorem,

$(\text{Base})^2 + (\text{Height})^2 = (\text{Hypotenuse})^2$

$\Rightarrow$ $x^2 + (x - 7)^2 = (13)^2$ $\Rightarrow$ $x^2 + x^2 - 14x + 49 = 169$

$\Rightarrow$ $2x^2 - 14x - 120 = 0$ $\Rightarrow$ $x^2 - 7x - 60 = 0$ (Dividing throughout by 2)

$\Rightarrow$ $x^2 - 12x + 5x - 60 = 0$ $\Rightarrow$ $x(x - 12) + 5(x - 12) = 0$

$\Rightarrow$ $(x - 12)(x + 5) = 0$ $\Rightarrow$ $x - 12 = 0$ or $x + 5 = 0$

$\Rightarrow$ $x = 12$ or $x = -5$ $\Rightarrow$ $x = 12, -5$

$x = -5$ is inadmissible

($\because$ x is the length of the base of the right triangle and length can not be negative)

$\therefore$ $x = 12$ $\Rightarrow$ $x - 7 = 12 - 7 = 5$

Therefore, the lengths of the other two sides are 5 cm and 12 cm.

Q.8 A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs. 90, find the number of articles produced and the cost of each article.

Sol. Let the number of articles produced on that day be x.

Then, the cost of production of each article on that day. = Rs. (2x + 3)

Therefore, total cost of production on that day

= (Number of articles producted on that day) × (cost of production of each article on that day)

= Rs. x(2x + 3)

According to the question,

$x(2x + 3) = 90$

$\Rightarrow \quad 2x^2 + 3x - 90 = 0 \quad \Rightarrow \quad 2x^2 + 15x - 12x - 90 = 0$

$\Rightarrow \quad x(2x + 15) - 6(2x + 15) = 0 \quad \Rightarrow \quad (2x + 15)(x - 6) = 0$

$\Rightarrow \quad 2x + 15 = 0 \text{ or } x - 6 = 0 \quad \Rightarrow \quad x = -\frac{15}{2} \text{ or } x = 6 \quad \Rightarrow \quad x = -\frac{15}{2}, 6$

Since, x is the number of articles, it cannot be negative. So, the number of articles produced on that day = 6

The cost of each article = $2x + 3 = 2 \times 6 + 3$ = Rs. 15.

Q.9 Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

(i) $2x^2 - 7x + 3 = 0$ (ii) $2x^2 + x - 4 = 0$ (iii) $4x^2 + 4\sqrt{3}x + 3 = 0$ (iv) $2x^2 + x + 4 = 0$

Sol. (i) We have, $2x^2 - 7x + 3 = 0$

Dividing throughout by the co-efficient of x^2, we get $x^2 - \frac{7}{2}x + \frac{3}{2} = 0$

$\Rightarrow \quad \left\{x^2 - \frac{7}{2}x + \left(\frac{7}{4}\right)^2\right\} - \left(\frac{7}{4}\right)^2 + \frac{3}{2} = 0$

$\Rightarrow \quad \left\{x - \frac{7}{4}\right\}^2 - \frac{49}{16} + \frac{3}{2} = 0 \quad \Rightarrow \quad \left\{x - \frac{7}{4}\right\}^2 - \frac{49}{16} + \frac{24}{16} = 0$

$\Rightarrow \quad \left\{x - \frac{7}{4}\right\}^2 - \frac{25}{16} = 0 \quad \Rightarrow \quad \left\{x - \frac{7}{4}\right\}^2 = \frac{25}{16} = \left(\frac{5}{4}\right)^2$

$\Rightarrow \quad x - \frac{7}{4} = \pm\frac{5}{4}$

Case I : When $\frac{5}{4}$ is positive, then $x - \frac{7}{4} = \frac{5}{4} \Rightarrow x = \frac{5}{4} + \frac{7}{4}$

$\Rightarrow \quad x = \frac{12}{4} = 3$

Case II : When $\frac{5}{4}$ is negative, then $x - \frac{7}{4} = -\frac{5}{4} \Rightarrow x = \frac{-5}{4} + \frac{7}{4} \Rightarrow x = \frac{2}{4} = \frac{1}{2}$

Thus, required roots are $x = 3$, and $x = \frac{1}{2}$

(ii) We have, $2x^2 + x - 4 = 0$

Dividing throughout by 2, $x^2 + \frac{x}{2} - 2 = 0$

$\Rightarrow \left\{x^2 + \frac{x}{2} + \left(\frac{1}{4}\right)^2\right\} - \left(\frac{1}{4}\right)^2 - 2 = 0 \quad \Rightarrow \quad \left(x + \frac{1}{4}\right)^2 - \frac{1}{16} - 2 = 0$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 - \frac{33}{16} = 0 \quad \Rightarrow \quad \left(x + \frac{1}{4}\right)^2 = \left(\frac{\sqrt{33}}{4}\right)^2 \quad \Rightarrow \left(x + \frac{1}{4}\right) = \pm\frac{\sqrt{33}}{4}$

Case I : When $\frac{\sqrt{33}}{4}$ is positive, then $x + \frac{1}{4} = \frac{\sqrt{33}}{4}$

$\Rightarrow \quad x = \frac{\sqrt{33}}{4} - \frac{1}{4} \quad \Rightarrow \quad x = \frac{\sqrt{33}-1}{4}$

Case II : When $\frac{\sqrt{33}}{4}$ is negative, then $x + \frac{1}{4} = -\frac{\sqrt{33}}{4}$

$\Rightarrow \quad x = \frac{-\sqrt{33}}{4} - \frac{1}{4} \quad \Rightarrow \quad x = \frac{-\sqrt{33}-1}{4}$

Thus, the required roots are $x = \frac{\sqrt{33}-1}{4}$ and $x = \frac{-\sqrt{33}-1}{4}$

(iii) We have $4x^2 + 4\sqrt{3}x + 3 = 0$

$\Rightarrow \quad \left\{x^2 + \sqrt{3}x + \left(\frac{\sqrt{3}}{2}\right)^2\right\} - \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{3}{4} = 0 \Rightarrow \quad \left[x + \frac{\sqrt{3}}{2}\right]^2 - \frac{3}{4} + \frac{3}{4} = 0$

$\Rightarrow \quad \left[x + \frac{\sqrt{3}}{2}\right]^2 = 0 \quad \Rightarrow \quad \left(x + \frac{\sqrt{3}}{2}\right)\left(x + \frac{\sqrt{3}}{2}\right) = 0$

$x = -\frac{\sqrt{3}}{2}$ and $x = -\frac{\sqrt{3}}{2}$

(iv) $2x^2 + x + 4 = 0$

Dividing throughout by 2, we have $x^2 + \frac{x}{2} + 2 = 0$

$\Rightarrow \quad \left[x + \frac{1}{4}\right]^2 - \left(\frac{1}{4}\right)^2 + 2 = 0 \quad \Rightarrow \quad \left[x + \frac{1}{4}\right]^2 - \frac{1}{16} + 2 = 0$

$\Rightarrow \quad \left[x + \frac{1}{4}\right]^2 + \frac{31}{16} = 0 \quad \Rightarrow \quad \left[x + \frac{1}{4}\right]^2 = -\frac{31}{16}$

But the square of a number cannot be negative.

$\therefore \quad \left[x + \frac{1}{4}\right]^2$ cannot give a real value.

$\Rightarrow$ There is no real value of x satisfy the given equation.

Q.10 Find the roots of the following quadratic equations, using the quadratic formula :

(i) $2x^2 - 7x + 3 = 0$ (ii) $2x^2 + x - 4 = 0$ (iii) $4x^2 + 4\sqrt{3}x + 3 = 0$ (iv) $2x^2 + x + 4 = 0$

Sol. (i) Comparing the given equation with $ax^2 + bx + c = 0$, we have

$a = 2, b = -7, c = 3$

$\therefore\ b^2 - 4ac = (-7)^2 - 4(2)(3) = 49 - 24 = 25 \geq 0$

Since $b^2 - 4ac > 0$

$\therefore$ The given equation has real roots. The roots are given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \dfrac{-(-7) \pm \sqrt{(25)}}{2(2)} = \dfrac{7 \pm 5}{4}$

Taking positive sign, $x = \dfrac{7+5}{4} = \dfrac{12}{4} = 3$; Taking negative sign, $x = \dfrac{7-5}{4} = \dfrac{2}{4} = \dfrac{1}{2}$

Thus, the roots of the given equation are $x = 3$ and $x = \dfrac{1}{2}$

(ii) Comparing the given equation with $ax^2 + bx + c = 0$, we have $a = 2, b = 1, c = -4$

$\therefore\ b^2 - 4ac = (1)^2 - 4(2)(-4) = 1 + 32 = 33 > 0$

Since $b^2 - 4ac > 0$

$\therefore$ The given equation has real roots. The roots are given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \dfrac{-1 \pm \sqrt{33}}{2(2)} = \dfrac{-1 \pm \sqrt{33}}{4}$

Taking positive sign, $x = \dfrac{-1+\sqrt{33}}{4}$; Taking negative sign, $x = \dfrac{-1-\sqrt{33}}{4}$

Thus, the required roots are $x = \dfrac{-1+\sqrt{33}}{4}$ and $x = \dfrac{-1-\sqrt{33}}{4}$

(iii) Comparing the given equation with $ax^2 + bx + c = 0$, we have $a = 4, b = 4\sqrt{3}, c = 3$

$\therefore\ b^2 - 4ac = (4\sqrt{3})^2 - 4(4)(3) = [16 \times 3] - 48 = 48 - 48 = 0$

Since $b^2 - 4ac = 0$

$\therefore$ The given equation has real and equal roots, which are given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \dfrac{-4\sqrt{3} \pm \sqrt{0}}{2(4)} = \dfrac{-4\sqrt{3} \pm 0}{8} = \dfrac{-\sqrt{3} \pm 0}{2}$

$\therefore\ x = \dfrac{-\sqrt{3}}{2}$ and $x = \dfrac{-\sqrt{3}}{2}$

(iv) Comparing the given equation with $ax^2 + bx + c = 0$, we have $a = 2, b = 1, c = 4$

$\therefore\ b^2 - 4ac = (1)^2 - 4(2)(4) = 1 - 32 = -31 < 0$

Since $b^2 - 4ac$ is less than 0, therefore the given equation does not have real roots.

Q.11 Find the roots of the following equations :

(i) $x - \frac{1}{x} = 3,\ x \neq 0$ **(ii)** $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30},\ x \neq -4, 7$

Sol. (i) Here, we have : $x - \frac{1}{x} = 3$

$\Rightarrow x^2 - 1 = 3x \Rightarrow x^2 - 3x - 1 = 0$ .....(1)

Comparing (1) with $ax^2 + bx + c = 0$, we have $a = 1, b = -3, c = -1$

$\therefore\ b^2 - 4ac = (-3)^2 - 4\,(1)\,(-1) = 9 + 4 = 13 > 0$

$\therefore\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\Rightarrow$ $x = \frac{-(-3) \pm \sqrt{13}}{2(1)} = \frac{3 \pm \sqrt{13}}{2}$

Now, taking positive sign, $x = \frac{3 + \sqrt{13}}{2}$;

Now, taking negative sign, $x = \frac{3 - \sqrt{13}}{2}$

Thus, the required roots of the given equation are $x = \frac{3 + \sqrt{13}}{2}$ and $x = \frac{3 - \sqrt{13}}{2}$.

(ii) We have $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

$\Rightarrow (x-7) - (x+4) = \frac{11}{30}(x+4)(x-7)$

$\Rightarrow x - 7 - x - 4 = \frac{11}{30}(x^2 - 3x - 28)$

$\Rightarrow -11 \times 30 = 11\,(x^2 - 3x - 28)$ $\Rightarrow -30 = x^2 - 3x - 28$

$\Rightarrow x^2 - 3x - 28 + 30 = 0$ $\Rightarrow x^2 - 3x + 2 = 0$ .....(1)

Comparing (1) with $ax^2 + bx + c = 0$, we have $a = 1, b = -3, c = 2$

$\therefore\ b^2 - 4ac = (-3)^2 - 4\,(1)\,(2) = 9 - 8 = 1 > 0$

$\therefore$ The given equation (1) has real roots, which are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \frac{-(-3) \pm \sqrt{1}}{2(1)} = \frac{3+1}{2}$

Taking positive sign, $x = \frac{3+1}{2} = \frac{4}{2} = 2.$; Taking negative sign, $x = \frac{3-1}{2} = 1$

Thus, the required roots are $x = 2$ and $x = 1$.

Q.12 The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Sol. Let the present age of Rehman = x

$\therefore$ 3 years ago Rehman's age = (x – 3) years

5 years later Rehman's age = (x + 5) years

Now, according to the condition, $\frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3}$

$\Rightarrow \frac{(x+5)+(x-3)}{(x-3)(x+5)}=\frac{1}{3} \quad \Rightarrow \quad 3[x+5+x-3]=(x-3)(x+5)$

$\Rightarrow 3[2x+2]=x^2+2x-15 \quad \Rightarrow \quad 6x+6=x^2+2x-15$

$\Rightarrow x^2-4x-21=0$ .....(1)

Now, comparing (1) with $ax^2+bx+c=0$, we have a = 1, b = –4, c = –21

$\therefore b^2-4ac=(-4)^2-4(1)(-21)=16+84=100$

Since, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \quad \Rightarrow \quad x=\frac{-(-4)\pm\sqrt{100}}{2(1)}=\frac{4\pm10}{2}$

Taking positive sign, we have $x=\frac{4+10}{2}=7$; Taking negative sign, we have $x=\frac{4-10}{2}=\frac{-6}{2}=-3$

Since age cannot be negative, $\therefore \quad x\neq -3 \Rightarrow \quad x=7$

So, the present age of Rehman = 7 years.

Q.13 In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210 ? Find her marks in the two subjects.

Sol. Let Shefali's marks in Mathematics = x

$\therefore$ Marks in English = (30 – x) ($\because$ Sum of the marks in English and Mathematics = 30)

Now, according to the condition $(x+2)\times[(30-x)-3]=210$

$\Rightarrow (x+2)\times(30-x-3)=210 \quad \Rightarrow \quad (x+2)(-x+27)=210$

$\Rightarrow -x^2+25x+54=210 \quad \Rightarrow \quad x^2+25x+54-210=0$

$\Rightarrow -x^2+25x-156=0 \quad \Rightarrow \quad x^2-25x+156=0$ . .....(1)

Now, comparing (1) with $ax^2+bx+c=0$, we have a = 1, b = –25, c = 156

$\therefore b^2-4ac=(-25)^2-4(1)(156)=625-624=1$

Since, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \quad \Rightarrow \quad x=\frac{-(-25)\pm\sqrt{1}}{2(1)} \quad \Rightarrow \quad x=\frac{25\pm1}{2}$

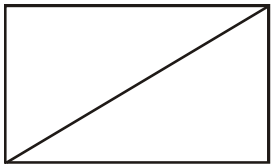
Taking positive sign, $x=\frac{25+1}{2}=\frac{26}{2}=13$; Taking negative sign, $x=\frac{25-1}{2}=\frac{24}{2}=12$

When x = 13, then 30 – 13 = 17; When x = 12, then 30 – 12 = 18

Thus, marks in Maths = 13, marks in English = 17

marks in Maths = 12, marks in English = 18.

Q.14 The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.



Sol. Let the shorter side (i.e., breadth) = x meters

$\therefore$ The longer side (length) = (x + 30) metres

In a rectangle, diagonal = $\sqrt{(\text{breadth})^2 + (\text{length})^2}$

$\Rightarrow x + 60 = \sqrt{x^2 + (x+30)^2}$ $\Rightarrow$ $x + 60 = \sqrt{x^2 + x^2 + 60x + 900}$

$\Rightarrow (x + 60)^2 = 2x^2 + 60x + 900$ $\Rightarrow$ $x^2 + 120x + 3600 = 2x^2 + 60x + 900$

$\Rightarrow 2x^2 - x^2 + 60x - 120x + 900 - 3600 = 0$

$\Rightarrow x^2 - 60x - 2700 = 0$ .....(1)

Comparing (1) with $ax^2 + bx + c = 0$, we have a = 1, b = –60, c = –2700

$\therefore b^2 - 4ac = (-60)^2 - 4(1)(-2700)$

$\Rightarrow b^2 - 4ac = 3600 + 10800$ $\Rightarrow$ $b^2 - 4ac = 14400$

Since, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\Rightarrow$ $x = \dfrac{-60 \pm \sqrt{14400}}{2(1)}$ $\Rightarrow$ $x = \dfrac{60 \pm 120}{2}$

Taking +ve sign, $x = \dfrac{60+120}{2} = \dfrac{180}{2} = 90$; Taking –ve sign, $x = \dfrac{60-120}{2} = \dfrac{-60}{2} = -30$

Since breadth cannot be negative, so,

$x \neq -30 \Rightarrow x = 90$ $\therefore x + 30 = 90 + 30 = 120$

Thus, the shorter side = 90 m. The longer side = 120 m.

Q.15 The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Sol. Let the larger number be x.

Since, $(\text{smaller number})^2 = 8\,(\text{larger number})$

$\Rightarrow (\text{smaller number})^2 = 8x$ $\Rightarrow$ $\text{smaller number} = \sqrt{8x}$

Now, according to the condition,

$x^2 - (\sqrt{8x})^2 = 180$ $\Rightarrow$ $x^2 - 8x = 180$ $\Rightarrow$ $x^2 - 8x - 180 = 0$

Comparing (1) with $ax^2 + bx + c = 0$, we have a = 1, b = –8, c = –180

$\therefore b^2 - 4ac = (-8)^2 - 4(1)(-180) = 64 + 720 = 784$

Since, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \dfrac{-(-8) \pm \sqrt{784}}{2(1)} \Rightarrow x = \dfrac{8 \pm 28}{2}$ $[\because \sqrt{784} = 28]$

Taking positive sign, $x = \dfrac{8+28}{2} = \dfrac{36}{2} = 18$; Taking negative sign, $x = \dfrac{8-28}{2} = \dfrac{-20}{2} = -10$

But x = –10 is not admissible, $\therefore$ The smaller number = 18

$\therefore \sqrt{18 \times 18} = \sqrt{144} = \pm 12$

Thus, the larger number = 12 or –12

Thus, the two numbers are 18 and 12, 18 and –12.

Q.16 A train travels 360 km at a uniform speed, if the speed had been 5 km/hr more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Sol. Let the uniform speed of the train be x km/hr.

Since, time taken by the train $= \frac{\text{Distance}}{\text{Speed}}$ $\Rightarrow$ time $= \frac{360}{x}$ h

When speed is 5 km/hr more, then time taken is 1 hour less.

$\Rightarrow \frac{360}{x+5} - \frac{360}{x} = -1$ $\Rightarrow$ $360\left[\frac{1}{x+5} - \frac{1}{x}\right] = -1$

$\Rightarrow \frac{1}{x+5} - \frac{1}{x} = \frac{-1}{360}$ $\Rightarrow$ $\frac{x-(x+5)}{x(x+5)} = \frac{-1}{360}$

$\Rightarrow x - x - 5 = \frac{-1(x+5)x}{360}$ $\Rightarrow$ $-5 \times 360 = -1\,(x^2 + 5x)$

$\Rightarrow -5 \times 360 = -x^2 - 5x$ $\Rightarrow$ $x^2 + 5x - 1800 = 0$ .....(1)

Comparing (1) with $ax^2 + bx + c = 0$, we have $a = 1, b = 5, c = -1800$

$\therefore b^2 - 4ac = (5)^2 - 4(1)(-1800) = 25 + 7200 = 7225$

Since, $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\therefore$ $x = \frac{-5 \pm \sqrt{7225}}{2(1)}$ $\Rightarrow$ $x = \frac{-5 \pm 85}{2(1)}$

Taking positive sign, $x = \frac{-5+85}{2} = \frac{80}{2} = 40$; Taking negative sign, $x = \frac{-5-85}{2} = \frac{-90}{2} = -45$

Since, the speed of a vehicle cannot be negative,

So, $x = -45$ (rejected)

Thus, $x = 40 \Rightarrow$ speed of the train = 40 km/hr.

Q.17 Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Sol. Let the smaller tap fills the tank in x hours.

$\therefore$ The larger tap fills the tank in (x – 10) hours.

Amount of water flowing through both tapes in one hour $= \frac{1}{x} + \frac{1}{x-10} = \frac{x-10+x}{x(x-10)} = \frac{2x-10}{x^2-10x}$

Now, according to the condition,

$\frac{8}{75} = \left(\frac{2x-10}{x^2-10x}\right)$ $\Rightarrow$ $\frac{75(2x-10)}{8(x^2-10x)} = 1$

$\Rightarrow 10x\ \frac{150x-750}{8x^2-80x} = 1$ $\Rightarrow$ $8x^2 - 80x = 150x - 750$

$\Rightarrow 8x^2 - 80x - 150x + 750 = 0$ $\Rightarrow$ $8x^2 - 230x + 750 = 0$ .....(1)

Comparing (1) with $ax^2 + bx + c = 0$, we get

$a = 8, b = -230, c = 750$

$\therefore\ b^2 - 4ac = (-230)^2 - 4(8)\ 750 = 52900 - 24000 = 28900 > 0$

Since, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \dfrac{-(-230) \pm \sqrt{28900}}{2(8)}$ $\Rightarrow x = \dfrac{230 \pm 170}{16}$ $[\because \sqrt{28900} = 170]$

Taking positive sign, $x = \dfrac{230 + 170}{16} = \dfrac{400}{16} = 25$; Taking negative sign, $x = \dfrac{230 - 170}{16} = \dfrac{60}{16} = \dfrac{15}{4}$

For $x = \dfrac{15}{4}$, $(x - 10) = \dfrac{15}{4} - 10 = \dfrac{-25}{4}$ [$\because$ Time cannot be negative]

which is not possible,

$\therefore\ x = 25 \Rightarrow x - 10 = 25 - 10 = 15$

Thus, time to fill the tank by the smaller tap alone = 25 hours

Larger tap alone = 15 hours.

Q.18 An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11 km/hr more than that of the passenger train, find the average speed of the two trains.

Sol Let the average speed of the passenger train be x km/hr.

$\therefore$ Average speed of the express train = (x + 11) km/hr

Total distance covered = 132 km

Also, $\text{Time} = \dfrac{\text{Distance}}{\text{speed}}$

Time taken by the passenger train = $\dfrac{132}{x}$ hours; Time taken by the express train = $\dfrac{132}{x+11}$ hours

According to the condition, we get

$\dfrac{132}{x+11} = \left(\dfrac{132}{x}\right) - 1$ $\Rightarrow$ $\dfrac{132}{x+11} - \dfrac{132}{x} = -1$

$\Rightarrow 132\left[\dfrac{1}{x+11} - \dfrac{1}{x}\right] = -1$ $\Rightarrow$ $132\left[\dfrac{x - x - 11}{x(x+11)}\right] = -1$

$\Rightarrow 132\left[\dfrac{-11}{x^2 + 11x}\right] = -1$ $\Rightarrow$ $-11\ (132) = -1(x^2 + 11x)$

$\Rightarrow -1452 = -1\ (x^2 + 11x)$ $\Rightarrow$ $x^2 + 11x - 1452 = 0$ .....(1)

Comparing (1) with $ax^2 + bx + c = 0$, we get $a = 1, b = 11, c = -1452$

$\therefore b^2 - 4ac = (11)^2 - 4(1)(-1452) \Rightarrow b^2 - 4ac = 121 + 5808 = 5929$

Since, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-11 \pm \sqrt{5929}}{2(1)} = \frac{-11 \pm 77}{2}$

$\therefore$ Taking +ve sign, $x = \frac{-11 + 77}{2} = \frac{66}{2} = 33$; Taking –ve sign, $x = \frac{-11 - 77}{2} = \frac{-88}{2} = -44$

But average speed cannot be negative so, $x \neq -44$ $\therefore x = 33$

$\Rightarrow$ Average speed of the passenger train = 33 km/hr

$\therefore$ Average speed of the express train = (x + 11) km/hr = (33 + 11) km/hr = 44 km/h

Q.19 Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the side of the two squares.

Sol. Let the side of the smaller square be x m.

$\Rightarrow$ Perimeter of the smaller square = 4x m

$\therefore$ Perimeter of the larger square = (4x + 24) m

$\Rightarrow$ side of the larger square = $\frac{\text{Perimeter}}{4} = \frac{4x + 24}{4}$ m = (x + 6) m

$\therefore$ Area of the smaller square = $(\text{side})^2 = x^2 m^2$

Area of the larger square = $(x + 6)^2 m^2$

According to the condition,

$x^2 + (x + 6)^2 = 468 \Rightarrow x^2 + x^2 + 12x + 36 = 468$

$\Rightarrow 2x^2 + 12x - 432 = 0 \Rightarrow x^2 + 6x - 216 = 0$ .....(1)

Comparing (1) with $ax^2 + bx + c = 0$, we get

$\therefore a = 1, b = 6, c = -216$

$\therefore b^2 - 4ac = (6)^2 - 4(1)(-216) = 36 + 864 = 900$

Since, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-6 \pm \sqrt{900}}{2(1)} = \frac{-6 \pm 30}{2}$

$\therefore$ Taking positive sign, we have $x = \frac{-6 + 30}{2} = \frac{24}{2} = 12$

Taking negative sign, we get $x = \frac{-6 - 30}{2} = \frac{-36}{2} = -18$

But the length of a square cannot be negative

$\therefore x = 12$

$\Rightarrow$ length of the smaller square = 12 m

Thus the length of the larger square = x + 6 = 12 + 6 = 18 m

Q.20 Find the nature of the roots of the following quadratic equations. If the real roots exist, find them :

(i) $2x^2 - 3x + 5 = 0$ (ii) $3x^2 - 4\sqrt{3x} + 4 = 0$ (iii) $2x^2 - 6x + 3 = 0$

Sol. (i) Comparing the given quadratic equation with $ax^2 + bx + c = 0$, we have

$a = 2, b = -3, c = 5$

$\therefore$ The discriminant $= b^2 - 4ac = (-3)^2 - 4(2)(5) = 9 - 40 = -31 < 0$

Since $b^2 - 4ac$ is negative

$\therefore$ The given quadratic equation has no real roots.

(ii) Comparing the given quadratic equation with $ax^2 + bx + c = 0$, we get

$a = 3, b = -4\sqrt{3}, c = 4$

$\therefore$ $b^2 - 4ac = [-4\sqrt{3}]^2 - 4(3)(4) = (16 \times 3) - 48 = 48 - 48 = 0$

Thus, the given quadratic equation has two real roots which are equal. Hence, the roots are $\frac{-b}{2a}$ and $\frac{-b}{2a}$

i.e., $\frac{-(-4\sqrt{3})}{2\times 3}$ and $\frac{-(-4\sqrt{3})}{2\times 3}$ $\Rightarrow$ $\frac{4\sqrt{3}}{2\sqrt{3}\times\sqrt{3}}$ and $\frac{4\sqrt{3}}{2\sqrt{3}\times\sqrt{3}}$

$\Rightarrow$ Thus, $x = \frac{2}{\sqrt{3}}$ and $x = \frac{2}{\sqrt{3}}$

(iii) Comparing it with the general quadratic equation, we have

$a = 2, b = -6, c = 3$

$\therefore$ $b^2 - 4ac = (-6)^2 - 4(2)(3) = 36 - 24 = 12 > 0$

$\therefore$ The given quadratic equation has two real and distinct roots, which are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\Rightarrow$ $x = \frac{-(-6) \pm \sqrt{12}}{2\times 2} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm 2\sqrt{3}}{2}$

Thus, the roots are $x = \frac{3+\sqrt{3}}{2}$ and $x = \frac{3-\sqrt{3}}{2}$

Q.21 Find the values of k for each of the following quadratic equations so that they have two equal roots

(i) $2x^2 + kx + 3 = 0$ (ii) $kx(x - 2) + 6 = 0$

Sol. (i) Comparing the given quadratic equation with $ax^2 + bx + c = 0$ we get

$a = 2, b = k, c = 3$

$\therefore\ b^2 - 4ac = (k)^2 - 4(2)(3) = k^2 - 24$

$\because$ For a quadratic equation to have equal roots, $b^2 - 4ac = 0$

$\therefore\ k^2 - 24 = 0 \quad \Rightarrow \quad k = \pm\sqrt{24}$

$\Rightarrow k \pm 2\sqrt{6}$

Thus, the required values of k are $2\sqrt{6}$ and $-2\sqrt{6}$.

(ii) Comparing $kx(x - 2) + 6 = 0$ i.e., $kx^2 - 2kx + 6 = 0$ with $ax^2 + bx + c = 0$, we get

$a = k, b = -2k, c = 6$

$\therefore\ b^2 - 4ac = (-2k)^2 - 4(k)(6) = 4k^2 - 24k$

Since, the roots are real and equal,

$\therefore\ b^2 - 4ac = 0$

$\Rightarrow\ 4k^2 - 24k = 0$

$\Rightarrow\ 4k(k - 6) = 0$

$\Rightarrow\ 4k = 0$ or $k - 6 = 0$

$\Rightarrow\ k = 0$ or $k = 6$

But k cannot be 0, otherwise, the given equation is no more quadratic. Thus, the required value of $k = 6$.

Q.22 Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 sq. metre ? If so, find its length and breadth.

Sol. Let the breadth be x metres.

$\therefore$ Length $= 2x$ metres

Now, Area = Length × Breadth $= 2x \times x$ metres$^2 = 2x^2$ sq. metre.

According to the given condition, $2x^2 = 800$

$\Rightarrow\ x^2 = \dfrac{800}{2} = 4$

$\Rightarrow\ x = \pm\sqrt{400} = \pm 20$

Therefore, $x = 20$ and $x = -20$

But $x = -20$ is not possible [$\because$ breadth cannot be negative]

$\therefore\ x = 20$

$\Rightarrow\ 2x = 2 \times 20 = 40$

Thus, length = 40 m and breadth = 20 m

Q.23 Is the following situation possible ? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Sol. Let the age of one friend = x years

$\therefore$ The age of the other friend = (20 – x) years [$\because$ Sum of their ages is 20 years]

Four years ago,

Age of one friend = (x – 4) years

Age of other friend = (20 – x – 4) years = (16 –x) years

According to the condition, $(x - 4) \times (16 - x) = 48$

$\Rightarrow$ $16x - 64 - x^2 + 4x = 48$

$\Rightarrow$ $-x^2 + 20x - 64 - 48 = 0$

$\Rightarrow$ $-x^2 + 20x - 112 = 0$

$\Rightarrow$ $x^2 - 20x + 112 = 0$ .....(1)

Here, a = 1, b = –20 and c = 112

$\therefore$ $b^2 - 4ac = (-20)^2 - 4(1)(112) = 400 - 448 = -48 < 0$

Since $b^2 - 4ac$ is less than 0.

$\therefore$ The quadratic equation (1) has no real roots.

Thus, the given equation is not possible.

Q.24 Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2 ? If so, find its length and breadth.

Sol. Let the breadth of the rectangle be x m.

Since, the perimeter of the rectangle = 80 m

$\therefore$ 2 [Length + breadth] = 80

2 [Length + x] = 80

$\Rightarrow$ $\text{Length} + x = \frac{80}{2} = 40$

$\Rightarrow$ Length = (40 – x) m

$\therefore$ Area = $(40 - x) \times x$ sq.m = $40x - x^2$

Now, according to the given condition,

Area of the rectangle = 400 m^2

$\therefore$ $40x - x^2 = 400$

$\Rightarrow$ $-x^2 + 40x - 400 = 0$

$\Rightarrow$ $x^2 - 40x + 400 = 0$ .....(1)

Comparing (1) with $ax^2 + bx + c = 0$, we get

a = 1, b = –40, c = 400

$\therefore$ $b^2 - 4ac = (-40)^2 - 4(1)(400) = 1600 - 1600 = 0$

Thus, the equation (1) has two equal and real roots.

$\because$ $x = \frac{-b}{2a}$ and $x = \frac{-b}{2a}$

$\therefore$ $\text{breadth} = \frac{-(-40)}{2(1)} = \frac{40}{2} = 20$

$\therefore$ breadth, x = 20 m

$\therefore$ Length = (40 – x) = (40 – 20) m = 20 m

Since Length = Breadth

$\therefore$ This rectangle is a square.

CONCEPT APPLICATION LEVEL - II [Previous Year Questions]

Q.1 Number of real solutions of $(x^2 - 7x + 11)^{x^2 - 11x + 30} = 1$ is **[NTSE Chandigarh]**

(A) 4 (B) 5 (C) 6 (D) No solution

Q.2 The roots of $(x + a)(x + b) - 8K = (K - 2)^2$ are real and equal, where a, b, $K \in R$, then **[NTSE Chandigarh]**

(A) $a + b = 0$ (B) $a = b, K = -2$ (C) $K = -3$ (D) $K = 0$

Q.3 If $b^2 - 4ac \geq 0$ then the roots of quadratic equation $ax^2 + bx + c = 0$ is **[NTSE Rajasthan]**

(A) $\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$ (B) $-\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$ (C) $\frac{b}{2a} \pm \frac{\sqrt{b^2 + 4ac}}{2a}$ (D) $-\frac{b}{2a} \pm \frac{\sqrt{b^2 + 4ac}}{2a}$

Q.4 If 'r' and 's' are the roots of the equation $ax^2 + bx + c = 0$, then the value of $\frac{1}{r^2} + \frac{1}{s^2}$ is : **[NTSE Andhra Pradesh]**

(A) $b^2 - 4ac$ (B) $\frac{b^2 - 4ac}{2a}$ (C) $\frac{b^2 - 4ac}{c^2}$ (D) $\frac{b^2 - 2ac}{c^2}$

Q.5 If the sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to product of their reciprocals, then : **[NTSE Andhra Pradesh]**

(A) $a^2 + bc = 0$ (B) $b^2 + ca = 0$ (C) $c^2 + ab = 0$ (D) $b + c = 0$

Q.6 The quadratic equation $ax^2 + bx + c = 0$ will have real and distinct roots if **[NTSE Mizoram]**

(A) $b^2 - 4ac < 0$ (B) $b^2 - 4ac > 0$ (C) $b^2 - 4ac = 0$ (D) all the above

Q.7 Which of the following is not a quadratic equation ? **[NTSE Mizoram]**

(A) $2(x - 1)^2 = 4x^2 - 2x + 1$ (B) $(x^2 + 1)^2 = x^2 + 3x + 9$

(C) $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$ (D) $x^2 + 9 = 3x^2 - 5x$

Q.8 If the sum of the two roots of the equation $\frac{1}{x + a} + \frac{1}{x + b} = \frac{1}{c}$ is zero, then the product of two roots is _____ .

(A) 0 (B) $\frac{a^2 + b^2}{2}$ (C) $\frac{a + b}{2}$ (D) $-\frac{(a^2 + b^2)}{2}$

Q.9 If $x^2 - 6x + 5 < 0$, then real values of x satisfies :

(A) $1 < x < 5$ (B) $3 < x < 7$ (C) $4 < x < 5$ (D) $7 < x < 10$

Q.10 A group of girls planned a picnic. The budget for food was Rs. 2400. Due to illness, 10 girls could not go to picnic and cost of food for each girl increased by Rs. 8. How many girls had planned the picnic ? **[IMO]**

(A) 60 (B) 50 (C) 65 (D) 57

Q.11 If roots of the equation $(a^2+b^2)x^2-2(ac+bd)x+(c^2+d^2)=0$ are equal, then $bc-ad=$ _____ **[IMO]**

(A) 1 (B) 0 (C) –1 (D) 2

Q.12 If $2\left(x^2+\frac{1}{x^2}\right)-9\left(x+\frac{1}{x}\right)+14=0$, then x = _____ . **[NSO]**

(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) All of these

Q.13 The positive value of k for which the equations $x^2+kx+64=0$ and $x^2-8x+k=0$ will have real roots, is _____. **[NSO]**

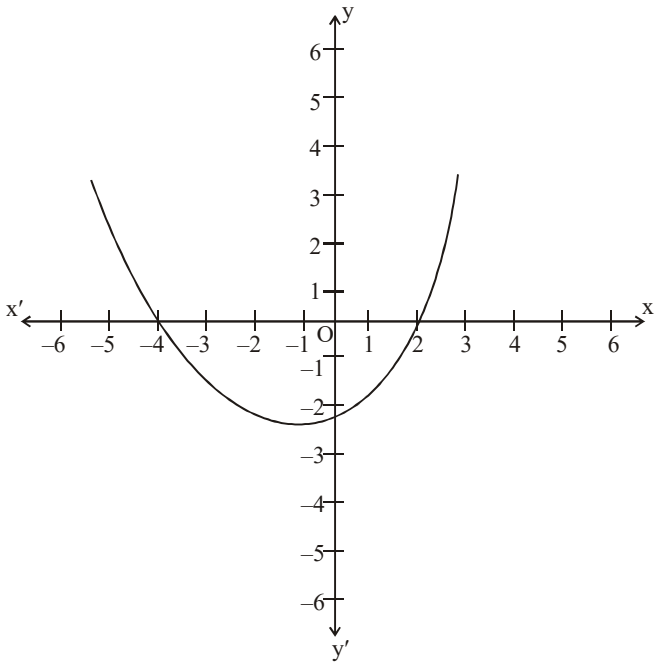
(A) 16 (B) 17 (C) 20 (D) 112

Q.14 If α, β are the roots of the equation $(x-a)(x-b)+k=0$, then a, b will be the roots of the equation :

(A) $(x-\alpha)(x-\beta)-k=0$ (B) $(x-\alpha)(x-\beta)+k=0$ **[NSO]**

(C) $(x-a)(x-b)-k=0$ (D) $(x-\alpha)(x-b)+k=0$

Q.15 What are the roots of the function graphed below ? **[NSO]**



(A) (1, 9) and (0, 8) (B) (0, 4) and (2, 0) (C) (–4, 0) and (2, 0) (D) (0, 2) and (0, 4)

Q.16 What will be the difference of the roots of quadratic equation $4y^2-4y+1=0$? **(IMO)**

(A) 0 (B) 1 (C) $-\frac{1}{2}$ (D) 2

Q.17 If α, β be the roots of equation $4x^2 - 7x + 3 = 0$ then the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ will be **[IMO]**

(A) 25/12 (B) 23/8 (C) 24/25 (D) 24/23

Q.18 Which of the statements given below is correct, if discriminant for equation $5x^2 - 4x + 2 = 0$ is D ? **[IMO]**

(A) $D > 0$ (B) $D = 0$ (C) $D < 0$ (D) D is not defined

Q.19 If one root of a quadratic equation is $\frac{1}{\sqrt{4} - \sqrt{3}}$, then the quadratic equation can be : **[IMO]**

(A) $x^2 - 2\sqrt{4}\,x + 1 = 0$ (B) $x^2 - \sqrt{4}x + 1 = 0$

(C) $x^2 + 2\sqrt{4}x + 1 = 0$ (D) $x^2 - 2\sqrt{3}x + 1 = 0$

Q.20 The number of roots satisfying the equation $\sqrt{5 - x} = x\sqrt{5 - x}$ is/are :

(A) 1 (B) 2 (C) 3 (D) 0

Q.21 If one root is $3 + \sqrt{5}$, then quadratic equation will be **[Raj. NTSE Stage-1 2006]**

(A) $x^2 + 6x - 4 = 0$ (B) $x^2 + 6x + 4 = 0$ (C) $x^2 - 6x + 4 = 0$ (D) $x^2 - 6x - 4 = 0$

Q.22 Graph drawn from the equation $y = x^2 - 3x - 4$ will be **[Raj. NTSE Stage-1 2013]**

(A) Circle (B) Parabola (C) Straight line (D) Hyperbola

Q.23 If the sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to product of their reciprocal, then **[M.P. NTSE Stage-1 2013]**

(A) $a^2 + bc = 0$ (B) $b^2 + ca = 0$ (C) $c^2 + ab = 0$ (D) $b + c = 0$

Q.24 If one root of $x^2 - 4x + k = 0$ is 6 then the value of k is **[M.P. NTSE Stage-1 2013]**

(A) –12 (B) 2 (C) –2 (D) 12

Q.25 If 2 is a root of the equation $x^2 + bx + 12 = 0$ and the equation $x^2 + bx + q = 0$ has equal roots, then q = **[Raj. NTSE Stage-1 2014]**

(A) 8 (B) –8 (C) 16 (D)–16

Q.26 If the roots of $px^2 + 2qx + r = 0$ and $qx^2 - 2\sqrt{pr}x + q = 0$ are simultaneously real, then **[Harayana NTSE Stage-1 2014]**

(A) $p = q, r \neq 0$ (B) $2q = \sqrt{pr}$ (C) $pr = q^2$ (D) $2p = \sqrt{qr}$

Q.27 The number of solution of the equation $\sqrt{6-4x-x^2} = x+4$ is **[W.Bengal NTSE Stage-1 2014]**

(A) 0 (B) 1 (C) 2 (D) 4

Q.28 If the roots of a quadratic equation $2x^2 + 3kx + 8 = 0$ are equal, the value of k is **[Rajasthan NTSE Stage-1 2016]**

(A) $\pm\frac{2}{3}$ (B) $\pm\frac{3}{2}$ (C) $\pm\frac{3}{8}$ (D) $\pm\frac{8}{3}$

Q.29 If α and β are the roots of the quadratic equation $x^2 - 6x - 2 = 0$ and if $a_n = \alpha^n - \beta^n$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is **[NTSE Stage-2 2016]**

(A) 6.0 (B) 5.2 (C) 5.0 (D) 3.0

Q.30 Which of the following options hold? **[IMO-2016]**

Statement I : The equation $\sqrt{3}x^2 - 2\sqrt{2x} - 2\sqrt{3} = 0$ has real and equal roots.

Statement II : If the difference between the roots of the quadratic equation $x^2 + kx + 12 = 0$ is 1, then the value of k is 6.

(A) Both statement I and Statement II are true.

(B) Statement I is true but Statement II is false.

(C) Statement I is the false but Statement II is true

(D) Both statement I and Statement II are false

Q.31 Which one of the following is not a quadratic equation? **[IOM-2016]**

(A) $3(x-1)^2 = x^2 - x$ (B) $(x^2 + 2x)^2 = x^4 + 4x^3 + 3$

(C) $3x - x^2 = x^2 + 2$ (D) $(\sqrt{3}x + \sqrt{2})^2 + x^2 = 4x - 6x$

Q.32 If p and q are the zeroes of the quadratic polynomial $bx^2 + cx + a$, then the value of $\frac{1}{p^3} + \frac{1}{q^3}$ is **[IOM-2016]**

(A) $\frac{3abc - c^3}{ab^2}$ (B) $\frac{3abc + c^3}{ab^2}$ (C) $\frac{3abc - c^3}{a^2b}$ (D) None of these

CONCEPT APPLICATION LEVEL - III

SECTION-A

- **Multiple choice questions with one correct answer**

Q.1 Roots of the quadratic equation $x^2 - 5x - 6 = 0$ are

(A) equal but negative (B) unequal but of same signs

(C) unequal but of opposite signs (D) equal but positive

Q.2 The quadratic equation where one root is $3 + 2\sqrt{3}$ is

(A) $x^2 - 6x - 3 = 0$ (B) $x^2 + 6x - 3 = 0$ (C) $x^2 + 6x + 3 = 0$ (D) $x^2 - 6x + 3 = 0$

Q.3 If one root of the equation $ax^2 + bx + c = 0$ is the reciprocal of other then

(A) a = b (B) b = c (C) a = c (D) a = –c

Q.4 Given that $f(x) = 3x^4 - 5x^3 + 8x^2 - 6x + 8$ and $g(x) = x^2 - 2x + 2$. Then how many real roots does the equation $\frac{f(x)}{g(x)} = 0$ have?

(A) 0 (B) 2 (C) 3 (D) 4

Q.5 The number of real roots of equation

$(a^2 + b^2)x^2 + 2a(\sqrt{b^2 + c^2})x + a^2 + c^2 = 0$

where, a, b and c are non zero is

(A) 0 (B) 1 (C) 2 (D) 4

Q.6 If the roots of the equation $x^2 - 4x + 1 = 0$ are in the ratio p : q then the value of $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}}$ is

(A) 0 (B) 4 (C) $2\sqrt{3}$ (D) Cannot be determined

Q.7 How are the roots of the quadratic equations $ax^2 + bx + c = 0$ and $cx^2 + bx + a = 0$ are related?

(A) No definite relation exist between the roots.

(B) The roots of second equation are the sum and the difference of the roots of the first equation.

(C) The roots of the one equation are the reciprocals of the roots of the other equation

(D) The roots of the first and the second equations are equal in magnitude and opposite in sign.

Q.8 The condition that the equation $x^2 + px + q = 0$ whose one root is the cube of the other root is

(A) $p = q^{1/4}[1 - q^{1/2}]$ (B) $-p = q^{1/2}[1 - q^{1/4}]$

(C) $-p = q^{1/4}[1 + q^{1/2}]$ (D) $p = q^{1/2}[1 + q^{1/4}]$

Q.9 If the quadratic equation $x^2 + ax + b = 0$ and $x^2 + bx + a = 0 (a \neq b)$ have a common root, then the numerical value of a + b is

(A) 1 (B) 2 (C) –1 (D) None

Q.10 The sum of all the real roots of the equation $|x-2|^2 + |x-2| - 2 = 0$

(A) 2 (B) 3 (C) 4 (D) 1

Q.11 Both the roots of the equation
(x– b) (x – c) + (x– a) (x – c) + (x– a) (x – b) are always (a, b, c should be positive)

(A) positive (B) negative (C) real (D) None

Q.12 The number of real solutions of the equation $|x|^2 - 3|x| + 2 = 0$

(A) 4 (B) 1 (C) 3 (D) 2

Q.13 The equation has $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$

(A) no root (B) one root (C) two equal roots (D) Infinitely many solution

Q.14 Let α, β be the roots of the equation (x – a) (x – b) + k = 0. Find the roots of the equation $(x - \alpha)(x - \beta) + c = 0$ are

(A) a and c (B) b and c (C) a and b (D) a + c, b + c

Q.15 The equation has $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$

(A) no solution (B) one solution (C) two solutions (D) None

Q.16 The roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3 then

(A) $a < 2$ (B) $2 \leq a \leq 3$ (C) $3 \leq a \leq 4$ (D) $a > 4$

Q.17 One root is square of the other root of the equation $x^2 + px + q = 0$, then the relation between p and q is

(A) $p^3 - q(3p - 1) + q^2 = 0$ (B) $p^3 - q(3p + 1) + q^2 = 0$
(C) $p^3 + q(3p - 1) + q^2 = 0$ (D) $p^3 + q(3p + 1) + q^2 = 0$

Q.18 If p, q, r are +ve and in A.P. Show that the roots of quadratic equation $px^2 + qx + r = 0$ are all real for

(A) $\left|\frac{r}{p} - 7\right| \geq 4\sqrt{3}$ (B) $\left|\frac{p}{r} - 7\right| \leq 4\sqrt{3}$ (C) All p & r (D) no p & r

Q.19 If $\left(x - \frac{1}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2 = x + 2$, then x = ?

(A) 3 (B) 2 (C) 4 (D) None of these

Q.20 If α, β, γ are the roots of the equation $2x^3 - 3x^2 + 6x + 1 = 0$, then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :
(A) –15/4 (B) 15/4 (C) 9/4 (D) 4

Q.21 The value of m for which the equation $\frac{a}{x+a+m} + \frac{b}{x+b+m} = 1$ has roots equal in magnitude but opposite in sign is

(A) $\frac{a+b}{a-b}$ (B) 0 (C) $\frac{a-b}{a+b}$ (D) $\frac{2(a-b)}{a+b}$

Q.22 If the ratio of the roots of the equation $x^2 + bx + c = 0$ is the same as that of $x^2 + qx + r = 0$, then
(A) $r^2b = qc^2$ (B) $r^2c = qb^2$ (C) $c^2r + q^2b$ (D) $b^2r = q^2c$

Q.23 If a + b + c = 0 and a, b, c are rational, then the roots of the equation :
$(b + c - a)\, x^2 + (c + a - b)\, x + (a + b - c) = 0$ are :
(A) rational (B) irrational (C) imaginary (D) equal

Q.24 The ratio of the roots of $lx^2 + nx + n = 0$ is p : q then

(A) $\sqrt{\frac{q}{p}} + \sqrt{\frac{p}{q}} + \sqrt{\frac{l}{n}} = 0$ (B) $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$

(C) $\sqrt{\frac{q}{p}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{l}{n}} = 0$ (D) $\sqrt{\frac{p}{q}} - \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$

Q.25 The real roots of the equation $x^{2/3} + x^{1/3} - 2 = 0$ are
(A) 1, 8 (B) –1, –8 (C) –1, 8 (D) 1, –8

Q.26 Which of the following is not a quadratic equation ?
(A) $x^2 - 2x + 2\,(3 - x) = 0$ (B) $x(x + 1) + 1 = (x - 2)\,(x - 5)$
(C) $(2x - 1)\,(x - 3) = (x + 5)\,(x - 1)$ (D) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

Q.27 If the equation $(m^2 + n^2)\, x^2 - 2\,(mp + nq)\, x + p^2 + q^2 = 0$ has equal roots, then
(A) mp = nq (B) mq = np (C) mn = pq (D) $mq = \sqrt{np}$

Q.28 If one root of $ax^2 + bx + c = 0$ is equal to n^{th} power of the other, then $(ac^n)^{1/(n+1)} + (a^nc)^{1/(n+1)}$:
(A) 1 (B) 1 (C) b (D) –b

Q.29 The quadratic equation $ax^2 - 4ax + 2a + 1 = 0$ has repeated root if a =

(A) 0 (B) $\frac{1}{2}$ (C) 2 (D) 4

Q.30 If α, β are the roots of the equation $ax^2 + bx + c = 0$, then $\frac{\alpha}{a\beta + b} + \frac{\beta}{a\alpha + b} =$

(A) $\frac{2}{a}$ (B) $\frac{2}{b}$ (C) $\frac{2}{c}$ (D) $-\frac{2}{a}$

Q.31 If α and β are the roots of the equation $x^2 + bx + c = 0$, then the roots of the equation $cx^2 + (b^2 - 2c)x + c = 0$ are :

(A) α^2, β^2 (B) $\frac{1}{\alpha}, \frac{1}{\beta}$ (C) $-\frac{\alpha}{\beta}, -\frac{\beta}{\alpha}$ (D) $2\alpha, 2\beta$

Q.32 The roots of the equation $x^2 - 2\sqrt{2}\,x + 1 = 0$ are

(A) real and distinct (B) not real (C) real and equal (D) rational and distinct

Q.33 If $x = \sqrt{7 + 4\sqrt{3}}$, then $x + \frac{1}{x} =$

(A) 4 (B) 6 (C) 3 (D) 2

Q.34 If the equation $x^2 + 4x - k = 0$ has real and distinct roots, then

(A) $k < -4$ (B) $k > -4$ (C) $k \le -4$ (D) $k \ge -4$

Q.35 If $ax^2 + bx + c = 0$ has equal roots, then $c =$

(A) $\frac{-b}{2a}$ (B) $\frac{b}{2a}$ (C) $\frac{-b^2}{4a}$ (D) $\frac{b^2}{4a}$

Q.36 If the equation $ax^2 + 2x + a = 0$ has two equal roots, if

(A) $a = \pm 1$ (B) $a = 0$ (C) $a = 0, 1$ (D) $a = -1, 0$

Q.37 The value of $\sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots}}}$ is

(A) 4 (B) 3 (C) – 2 (D) 3.5

Q.38 If the equation $x^2 - bx + 1 = 0$ does not possess real roots, then

(A) $-3 < b < 3$ (B) $-2 < b < 2$ (C) $b > 2$ (D) $b < -2$

Q.39 If $x = 1$ is a common root of the equations $ax^2 + ax + 3 = 0$ and $x^2 + x + b = 0$, then $ab =$

(A) 3 (B) 3.5 (C) 6 (D) –3

Q.40 If a and b can take values 1, 2, 3, 4. Then the number of the equations of the form $ax^2 + bx + 1 = 0$ having real roots is :

(A) 10 (B) 7 (C) 6 (D) 12

Q.41 A chess board contains 64 equal squares and the area of each square is 6.25 cm^2. A border round the board is 2 cm wide. Find the length of the side of chess board.

(A) 36 cm (B) 24 cm (C) 16 cm (D) 32 cm

SECTION-B

- **Multiple choice questions with one or more than one correct answer**

Q.1 Consider the following statements.
If the quadratic equations, $x^2 + ax + 2 = 0$ and $x^2 + x + b = 0$ have a common root $x = 1$, then

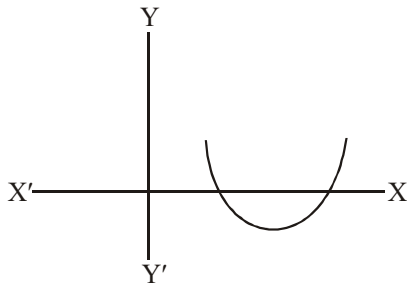
(A) a + b =5 (B) ab = 6 (C) $\frac{a}{b} = \frac{3}{2}$ (D) a – b = 1

Q.2 If α,β are roots of the equation $x^2 - 5x + 6 = 0$ then the equation whose roots are $\alpha + 3$ and $\beta + 3$ is
(A) $2x^2 - 11x + 30 = 0$ (B) $-x^2 + 11x = 0$ (C) $x^2 - 11x + 30 = 0$ (D) $2x^2 - 22x + 60 = 0$

Q.3. Two numbers whose sum is 8 and the absolute value of whose difference is 10 are roots of the equation
(A) $x^2 - 8x + 9 = 0$ (B) $x^2 - 8x - 9 = 0$ (C) $x^2 + 8x - 9 = 0$ (D) $-(-x^2 + 8x + 9) = 0$

Q.4 The inequality $b^2 + 5 > 9b + 12$ is satisfied if
(A) b > 10 (B) b < 1 (C) b > 0 (D) b < –1

Q.5 For the below figure of $ax^2 + bx + c = 0$



(A) a < 0 (B) b > 0 (C) D > 0 (D) a > 0

Q.6 Which of the following is/are quadratic equations :

(A) (x + 1) (x + 3) – x + 7 = 0 (B) $x^2 + 2x + \frac{1}{x} = 0$

(C) $y(8y + 5) = y^2 + 3$ (D) $y(2y + 15) = 2(y^2 + y + 8)$

Q.7 The roots of $\sqrt{2}x^2 - 3x + \sqrt{2} = 0$ are

(A) $-\frac{1}{\sqrt{2}}$ (B) $-\sqrt{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) $\sqrt{2}$

Q.8 Which of the following satisfy the equation $a^2b^2x^2 + b^2x - a^2x - 1 = 0$?

(A) $\frac{1}{a^2}$ (B) $\frac{1}{b^2}$ (C) $-\frac{1}{a^2}$ (D) $-\frac{1}{b^2}$

Q.9 The equation $x^2 + ax - 4 = 0$, for all real values of a has :
(A) real roots (B) equal roots (C) unequal roots (D) none of these

Q.10 If the roots of equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$ are equal, then

(A) $a = 0$ (B) $a^3 + b^3 + c^3 = 3abc$

(C) $a^2 + b^2 + c^2 = 2abc$ (D) none of these

Q.11 Two numbers differ by 3 and their product is 504. The numbers are

(A) 21, 24 (B) 24, 27 (C) –24, 21 (D) –24, –21

SECTION-C

Assertion & Reason

Instructions: In the following questions as Assertion (A) is given followed by a Reason (R). Mark your responses from the following options.

(A) Both Assertion and Reason are true and Reason is the correct explanation of 'Assertion'

(B) Both Assertion and Reason are true and Reason is not the correct explanation of 'Assertion'

(C) Assertion is true but Reason is false

(D) Assertion is false but Reason is true

Q.1 **Assertion :** $2x^2 - 4x + 3 = 0$ is a quadratic equation.

Reason : All polynomials of degree n, when n is a whole number can be treated as quadratic equation.

Q.2 **Assertion :** $9x^2 - 3x - 20 = 0 \Rightarrow (3x - 5)(3x + 4) = 0$ the roots are calculated by splitting the middle term.

Reason : To factorise $ax^2 + bx + c = 0$, we write in the form $ax^2 + b_1x + b_2x + c = 0$ such that $b_1 + b_2 = b$ and $b_1b_2 = ac$.

Q.3 **Assertion :** $5x^2 + 14x + 10 = 0$ has no real roots.

Reason : $ax^2 + bx + c = 0$ has no real roots if $b^2 < 4ac$.

Q.4 **Assertion :** $4x^2 - 12x + 9 = 0$ has repeated roots.

Reason : The quadratic equation $ax^2 + bx + c = 0$ have repeated roots if $D > 0$.

Q.5 **Assertion :** $3y^2 + 17y - 30 = 0$ have distinct roots.

Reason : The quadratic equation $ax^2 + bx + c = 0$ have distinct roots (real roots) if $D > 0$.

SECTION-D

- **Comprehension**

The distance of road between the towns A and B is 216 km and by rail is 208 km. A car at a speed of x km/hr and the train travels at a speed which is 16 km/hr faster than the car. Calculate.

Q.1 The time taken by the car to reach town B from A, in terms of x

(A) $\frac{216}{x}$ hrs. (B) $\frac{216}{x^2}$ hrs (C) $\frac{206}{x^2}$ (D) None

Q.2 The time taken by the train to reach town B from A in terms of x.

(A) $\frac{208}{16+x^2}$ hrs. (B) $\frac{208}{16+x}$ hrs (C) $\frac{206}{16+x}$ (D) None

SECTION-E

- **Match the following (one to one)**

Column-I and **column-II** contains **four** entries each. Entries of column-I are to be matched with some entries of column-II. Only One entries of column-I may have the matching with the one entry of column-II and one entry of column-II may have Only one matching with entries of column-I

Q.1

	Column–I (Quadratic equation)		**Column–II (Pair of roots)**
(A)	$2x^2 + x - 6 = 0$	(p)	(2,3)
(B)	$\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$	(q)	$\left(-\frac{3}{2}, \frac{3}{5p}\right)$
(C)	$10px^2 - 6x + 15px - 9 = 0;\ p \neq 0$	(r)	$\left(-4\sqrt{3}, \frac{2}{\sqrt{3}}\right)$
(D)	$x^2 - 5x + 6 = 0$	(s)	$\left(\frac{3}{2}, -2\right)$

Q.2 Match the quadratic equations formed in column I to that in column II.

	Column-I		**Column-II**
(A)	The product of two consecutive even integers is 528	(p)	$z^2 - 45z + 500 = 0$
(B)	Megha and Latika have 45 chocolates both of them lost 3 each, and the product of the chocolates now is 374.	(q)	$x^2 - 5x - 5 = 0$
(C)	The hypotenuse of right-angled triangle is 6 more that the shortest side and third side 3 less than the hypotenuse.	(r)	$n^2 + 2n - 528 = 0$
(D)	Difference between two numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$.	(s)	$y^2 - 6y - 27 = 0$

Q.3 Column-II gives roots of quadratic equations given in column-I, match them correctly.

	Column-I		**Column-II**
(A)	$6x^2 + x - 12 = 0$	(p)	(–6, 4)
(B)	$8x^2 + 16x + 10 = 202$	(q)	(9, 36)
(C)	$x^2 - 45x + 324 = 0$	(r)	(3, – 1/2)
(D)	$2x^2 - 5x - 3 = 0$	(s)	(–3/2, 4/3)

SECTION-F

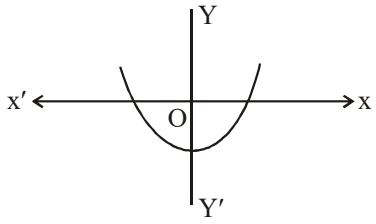
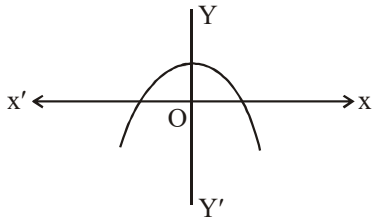
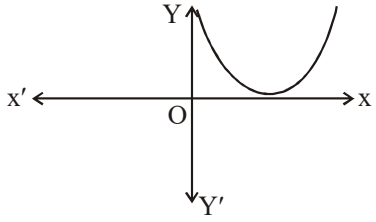
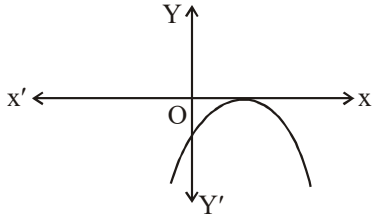
- **Match the following (one to many)**

Column-I and **column-II** contains **four** entries each. Entries of column-I are to be matched with some entries of column-II. One or more than one entries of column-I may have the matching with the same entries of column-II and one entry of column-II may have one or more than one matching with entries of column-I

Q.1

	Column–I *(find k so that roots of equation)*		**Column–II** *(k)*
(A)	$3kx^2 - 4kx + 4$ are real & equal	(p)	0
(B)	$(k-2)x^2+2(2k-3)x+5k-6=0$ have repeated roots	(q)	3
(C)	$9x^2 + 8kx + 16 = 0$ are real & equal	(r)	–3
(D)	$x^2-6x + 3k = 0$ has coincident roots	(s)	1

Q.2 D be the discriminant of the quadratic equation $ax^2 + bx + c = 0$

	Column-I		**Column-II**
(A)		(p)	$a < 0$
(B)		(q)	$a > 0$
(C)		(r)	$D < 0$
(D)		(s)	$D > 0$
		(t)	$D = 0$

ANSWER KEY

CONCEPT APPLICATION LEVEL - II

Q.1	B	Q.2	B	Q.3	B	Q.4	D	Q.5	A	Q.6	B	Q.7	B
Q.8	D	Q.9	A	Q.10	A	Q.11	B	Q.12	D	Q.13	A	Q.14	A
Q.15	C	Q.16	A	Q.17	A	Q.18	C	Q.19	A	Q.20	C	Q.21	C
Q.22	B	Q.23	A	Q.24	A	Q.25	C	Q.26	C	Q.27	B	Q.28	D
Q.29	D	Q.30	D	Q.31	D	Q.32	D						

CONCEPT APPLICATION LEVEL - III

SECTION-A

Q.1	C	Q.2	A	Q.3	C	Q.4	A	Q.5	A	Q.6	B	Q.7	C
Q.8	C	Q.9	C	Q.10	C	Q.11	A	Q.12	A	Q.13	A	Q.14	C
Q.15	A	Q.16	A	Q.17	A	Q.18	B	Q.19	C	Q.20	A	Q.21	B
Q.22	D	Q.23	A	Q.24	B	Q.25	D	Q.26	B	Q.27	B	Q.28	D
Q.29	B	Q.30	D	Q.31	C	Q.32	A	Q.33	A	Q.34	B	Q.35	D
Q.36	A	Q.37	B	Q.38	B	Q.39	A	Q.40	B	Q.41	B		

SECTION-B

Q.1	B, C	Q.2	C, D	Q.3.	B,D	Q.4	A	Q.5	C,D	Q.6	A,C	Q.7	C,D
Q.8	B,C	Q.9	A,C	Q.10	A,B	Q.11	A,D						

SECTION-C

Q.1	C	Q.2	A	Q.3	A	Q.4	C	Q.5	A

SECTION-D

Q.1	A	Q.2	B

SECTION-E

Q.1 (A)–(s), (B)–(r), (C)–(q), (D)–(p)

Q.2 (A) → r ; (B) → p ; (C) → s ; (D) → q

Q.3 (A) → s ; (B) → p ; (C) → q ; (D) → r

SECTION-F

Q.1 (A)-(q), (B)-(q, s), (C)-(r, q), (D)-(q)

Q.2 (A) → q, s ; (B) → p, s ; (C) → q, r ; (D) → p, t