3 PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

THEORY

3.1 INTRODUCTION

While solving the problems, in most cases, first we need to frame an equation. In this chapter, we learn how to frame and solve equations. There are given some methods to solve these equations. We will further study about word problems and application of simultaneous equations.

Equation: A statement in which two algebraic expressions are equal is known as equation.

For example: $2x + 3 = 0$, $\frac{2y}{3} + 1 = \frac{y}{3}$

Algebraic expressions: Algebraic expessions are made of numbers, symbols and the basic arithmetical operations. E.g., $x + 3y$, $4x^2 - 2x + 5\sqrt{x}$ are algebraic expressions.

Linear equation: An equation involving linear polynomials is called a linear equation. For example

$\frac{3x}{2} + 4 = 2x - 3$

Remark: A linear equation in one variable has the standard form

$ax + b = 0, \quad a \neq 0, a, b \in R$

Solution (root) of a linear equation: The value of the variable which makes the two sides of the equation equal and satisfies the equation is called the solution of the equation.

Rules for solving an equation:

(i) The same number is added or subtracted to both sides of an equation, the resulting equation is equivalent to the first.

(ii) If both sides of an equation are multiplied by the same non-zero number the resulting equation is equivalent to the first.

Remark: Every linear equation in one variable has only one (unique) solution.

In this chapter we shall study about system of linear equation in two variables, solution of a system of linear equations in two variables.

3.2 LINEAR EQUATION IN TWO VARIABLES

Definition: $ax + by + c = 0$, is called the standard form of a linear equation in two variables if $a \neq 0$, $b \neq 0$ and $a, b, c \in R$.

Eg.: $3x + 2y = 7$; $2x - \sqrt{3}\,y = \sqrt{5}$

The equation is linear equation in two variables if

(i) neither x nor y is under a radical sign.

(ii) neither x nor y is in the denominator.

(iii) the exponent (power) of x and y in each term is one.

Simultaneous equation: A pair of linear equations in two variables is said to form a system of simultaneous equation.

Solution of a linear equation in two variables: The pair of values of x and y which satisfies the given equation is called a solution of the equation.

Graphical method of solution of pair (Simultaneous) of line:

Let us consider a linear equation $ax + by + c = 0$ where $a \neq 0$, $b \neq 0$

Step-I: Write down $y = \left(\frac{ax + c}{b}\right)$

Step-II: Substitute any arbitrary value of x in step-I and obtain the corresponding value of y.

Step-III: Plot these points on the graph paper

Step-IV: Join these points. The line thus obtained is the required graph of $ax + by + c = 0$

Consistent system: A system of simultaneous linear equations is said to be consistent, if it has at least one solution.

Inconsistent system: A system of simultaneous linear equations is said to be in-consistent if it has no solution.

Some Important points:

(i) The graph of $x = a$ is a straight line parallel to y-axis.

(ii) The graph of $y = b$ is a straight line parallel to x-axis

(iii) The graph of $y = 0$ is x-axis and graph of $x = 0$ is y-axis.

(iv) If there is no constant term in the equation then the graph of $ax + by = 0$ will pass through the origin.

3.3 ALGEBRAIC METHODS OF SOLVING A PAIR OF LINEAR EQUATIONS

There are four methods for solving a pair of linear equations

(i) Substitution method

(ii) Elimination method

(iii) Cross-multiplication method

(iv) Graphical Method

3.3.1 Substitution method

Step–I: Obtain the two equations: $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Step–II: Find the value of one variable, say y in terms of the other variable.

Step–III: Substitute this value of y in the other equation and reduce it to an equation in one variable.

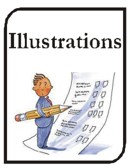


Illustration 1

Solve the following pair of equations by substitution method:

$2x + 3y = 9$; $3x + 4y = 5$

Solution

Step–I: $2x + 3y = 9$ ....(i) or $y = \left(\frac{9-2x}{3}\right)$ ...(ii)

Step–II: Substituting this value of y in equation $3x + 4y = 5$ from equation (ii)

$$\Rightarrow \quad 3x + 4\left(\frac{9-2x}{3}\right) = 5$$

$$9x + 36 - 8x = 15$$

$$x = 15 - 36$$

$$x = -21$$

Step–III: Putting the value of x in equation (ii),

$$y = \left(\frac{9-2(-21)}{3}\right)$$

$$y = \frac{9+42}{3} \quad \Rightarrow \quad y = \frac{51}{3} \Rightarrow y = 17$$

Therefore the solution of the given system of equations is $x = -21$, $y = 17$.

3.3.2 Elimination Method

Step–I: Obtain the two equations

Step–II: First multiply both the equations by some suitable non-zero constant to make the coefficient of one variable (either x or y) numerically equal.

Step–III: Add or subtract one equation from the other, then one variable gets eliminated.

Step–IV: Solve the equation in one variable.

Step–V: Substitute the value of x (or y) in any one of the given equations and find the value of another variable.

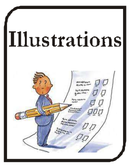


Illustration 2

Solve the system of linear equations by Elimination method

(i) $4x - y = 5$ (ii) $3x + 2y = 12$

Solution

Equation (i) is multiplied by '2' and adding to the equation (ii) then

$8x - 2y = 10$

$3x + 2y = 12$

$11x = 22 \Rightarrow x = 2$

Putting the value of x in equation (ii)

$3 \times 2 + 2y = 12 \Rightarrow 6 + 2y = 12$

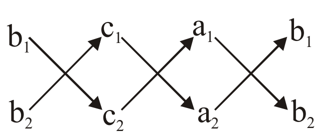
$2y = 6 \Rightarrow y = 3$

Hence the solution of the given system is $x = 2, y = 3$

3.3.3 Cross Multiplication Method

Step–I: Obtain the two equations : $a_1x + b_1y + c_1 = 0;\ a_2x + b_2y + c_2 = 0$

Step–II:



The down arrows (↘) show the term with a plus sign and up arrows (↗) show the term with a negative sign.

Then the solution is given by

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}\ ;\ \frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - c_1a_2} = \frac{1}{a_1b_2 - a_2b_1}$$

Illustrations

Illustration 3

Solve the system of equation by cross multiplication method

$$\frac{x}{a}+\frac{y}{b}=a+b,\ \frac{x}{a^2}+\frac{y}{b^2}=2$$

Solution

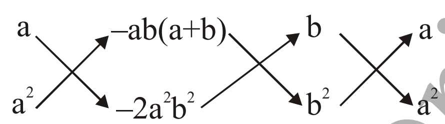
The given equations are

$$\frac{x}{a}+\frac{y}{b}=a+b \quad \Rightarrow \quad bx+ay=ab(a+b);\quad bx+ay-ab(a+b)=0 \qquad .....(i)$$

$$\frac{x}{a^2}+\frac{y}{b^2}=2 \quad \Rightarrow \quad b^2x+a^2y=2a^2b^2;\quad b^2x+a^2y-2a^2b^2=0 \qquad .....(ii)$$

From equation (i) and (ii),

$bx + ay - ab(a+b) = 0;\quad b^2x + a^2y - 2a^2b^2 = 0$



$$\frac{x}{-2a^3b^2+a^3b(a+b)}=\frac{y}{-ab^3(a+b)+2a^2b^3}=\frac{1}{a^2b-ab^2}$$

or $$\frac{x}{-2a^3b^2+a^4b+a^3b^2}=\frac{y}{ab^3\{2a-a-b\}}=\frac{1}{ab\,(a-b)}$$

or $$\frac{x}{a^3b(a-b)}=\frac{y}{ab^3(a-b)}=\frac{1}{ab(a-b)}$$

or $$x=\frac{a^3b\,(a-b)}{ab\,(a-b)}=a^2,\ y=\frac{ab^3\,(a-b)}{ab\,(a-b)}=b^2$$

Hence the solution of the given system of linear equation be $x = a^2$ and $y = b^2$.

3.3.4 Graphical Method:

In graphical method, we draw the graph of both equations using same pair of horizontal and vertical lines called X-axis and Y-axis respectively. Coordinates of the point(s) of intersection of the two lines is/are the solution.

Nature of solutions:

When we try of solve a pair of equations we could arrive at three possible results. They are, having

(a) a unique solution

(b) an infinite number of solutions

(c) no solution

Let the pair of equations be $a_1x + b_1y + c_1 = 0$ and $a_2 x + b_2y + c_2 = 0$, where a_1 and a_2 are the coefficients of x; b_1 and b_2 are the coefficients of y; while c_1 and c_2 are the known constant quantities.

(i) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ then graph will represent two intersecting lines and system has **unique solution** and is known as **consistent pair of linear equations**.

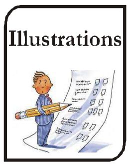


Illustration 4

4x + 5y = 37 and 5x + 4y = 35, for these two equations, $a_1 = 4$, $a_2 = 5$, $b_1 = 5$, $b_2 = 4$, $c_1 = -37$, $c_2 = -35$.

Solution

$$\therefore \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \quad \Rightarrow \quad \frac{4}{5} \neq \frac{5}{4}.$$

The above pair of equations has a unique solution.

(ii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ then graph will represent two parallel lines and system has **no common solution** and is known as **inconsistent pair of linear equations.**

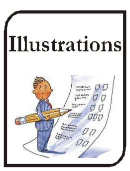


Illustration 5

6x + 5y = 16 and 12x + 10y = 50, for these two equations $a_1 = 6$, $a_2 = 12$, $b_1 = 5$, $b_2 = 10$, $c_1 = -16$, $c_2 = -50$.

Solution

Therefore, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad \Rightarrow \quad \frac{6}{12} = \frac{5}{10} \neq \frac{-16}{-50}$

The above pair of equations has no common solution.

(iii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ then graph will represent two coinciding lines and system has **infinitely many solutions** and is known as **dependant consistent pair of linear equations.**

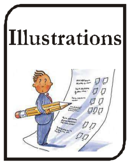


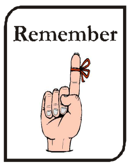
Illustration 6

4x + 7y = 29 and 12x + 21y = 87, for these two equations, $a_1 = 4$, $a_2 = 12$, $b_1 = 7$, $b_2 = 21$, $c_1 = -29$, $c_2 = -87$.

Solution

Therefore, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{4}{12} = \frac{7}{21} = \frac{-29}{-87}$

The above pair of equations has infinitely many solutions.



A pair of equations is said to be consistent if equations have a solution (finite or infinite).

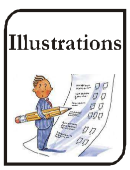


Illustration 7

Draw the graph of the equation x + y = –1 and x – y = 5.

Solution

(i) $x + y = -1$

x	–3	–2	–1	0	1	2	3
y = –1 – x	2	1	0	–1	–2	–3	–4

Some of the ordered pairs which satisfy the equation x + y = –1 are

(–3, 2), (–2, 1), (–1, 0), (0, –1), (1, –2), (2, – 3), (3, –4)

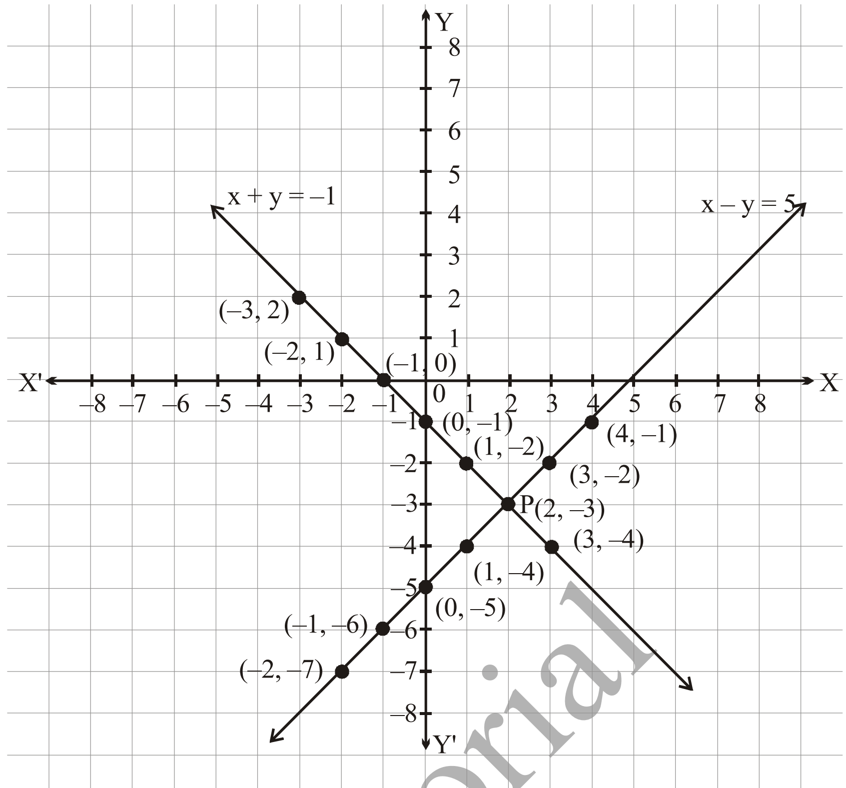
(ii) $x - y = 5$

x	–2	–1	0	1	2	3	4
y = x – 5	–7	–6	–5	–4	–3	–2	–1

∴ Some of the ordered pairs which satisfy the equation x – y = 5 are

(–2, –7), (–1, –6), (0, –5), (1, –4), (2, –3), (3, –2), (4, –1)

The ordered pairs which satisfy the equations x + y = –1 and x – y = 5 are plotted on a graph paper. We find that each equation represents a line.



From the graph, we notice that the two given lines intersect at the point (2, –3).

That is, lines $x + y = -1$ and $x - y = 5$ have a common point P(2, –3). Therefore (2, –3) is the solution of the equations $x + y = -1$ and $x - y = 5$.

Verification

$x + y = -1$ .....(1)

$x - y = 5$ .....(2)

Solving (1) and (2), we get,

$x = 2$ and $y = -3$

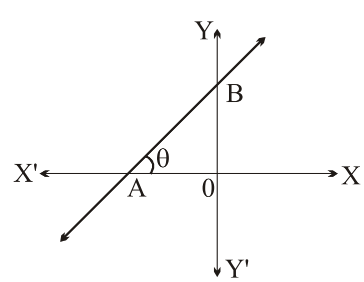
$\therefore$ (2, –3) is the solution of $x + y = -1$ and $x - y = 5$.

Note : From the above example, we notice that we can find the solution for simultaneous equations by representing them in graph i.e. by using the graphical method.

3.4 DEFINITION OF A STRAIGHT LINE

A straight line is the simplest geometric curve such that every point on the line segment joining any two points on it lies on it.

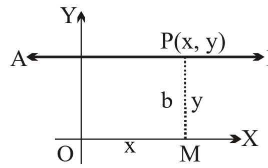
Slope (Gradient) of a line : The trigonometrical tangent of the angle that a line makes with the positive direction of the x-axis in anticlockwise sense is called the slope or gradient of the line.



The slope of a line is generally denoted by m. Thus, $m = \tan\theta$.

(i) A straight line parallel to x-axis at a given distance from it.

AB is a straight line parallel to x-axis at a distance b from it. Draw PM perpendicular to the x-axis then OM = x and MP = y



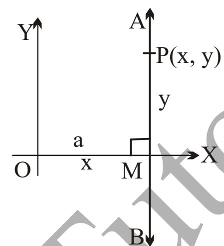
The straight line can be considered as the locus of a moving point P(x, y) whose distance from x-axis is equal to b from all positions of P.

So, MP = b, we get y = b

The equation of the x-axis is y = 0. Since b = 0 in this case.

(ii) A straight line parallel to the y-axis at a given distance from it.

AB is a straight line parallel to y axis at a distance a from it



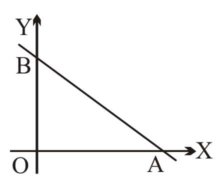
Let PM be the perpendicular from P to the x-axis. Then OM = x and MP = y. Using condition OM = a, we get x = a.

The equation of the y-axis is x = 0 since a = 0 in this case.

Important: The equation of a line parallel to the x-axis does not contain x and the equation of a line parallel to y-axis does not contain y.

3.4.1 Intercept Definition

If a straight line meets the x-axis at A and y axis at B then



(i) OA (i.e. the distance of A from the origin) is called the intercept made by the line on the x-axis or simply x-intercept.

(ii) OB (i.e. the distance of B from the origin) is called the intercept made by the line on the y-axis or simply y-intercept.

(iii) The two together OA and OB are called the intercepts made by the line on the coordinate axes.

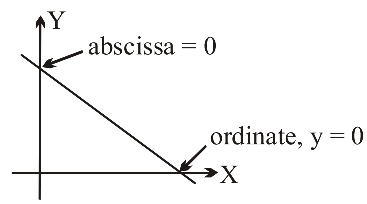
Rules for the signs of the intercepts

(i) The intercept on the x-axis is positive if measured to the right of the origin and negative if measured to the left of the origin.

(ii) The intercept on the y-axis is positive if it is measured above the origin and negative if measured below the origin

(iii) Rule to find intercepts of a line on the axis.

We know that a line cuts the x-axis at a point whose ordinate (y-coordinate) is zero and cuts the y-axis at a point whose abscissa (x-coordinate) is zero. Hence,

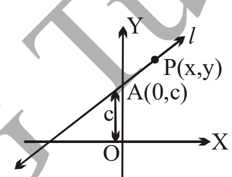


(a) To get the x-intercept we put y = 0 in the equation of the line and find value of x.

(b) To get the y-intercept we put x = 0 in the equation of the line and find the value of y.

3.4.2 Special forms of equation of the straight line

(i) **Slope intercept form:** To find the equation of the straight line whose gradient or slope is m and whose intercept on the y axis is c.



Let l be the line whose intercept OA on the y-axis is c and whose slope is m.

Since, intercept on y-axis = AO = c

Therefore, the coordinates of A are (0,c)

Let P(x, y) be any point on the line

Then, Slope, $AP = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y - c}{x - 0}$

$m = \frac{y - c}{x} \quad \Rightarrow \quad y = mx + c$

which is the required equation

As it invovles slope and y intercept of the straight line. It is some times referred to as slope intercept form.

For example, The equation of the line having slope = –3 and intercept on y axis is = 7 is

$y = mx + c;\ y = -3x + 7$

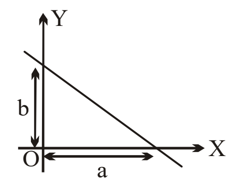
(ii) **Point Slope form:** The equation of the straight line passing through a given point (x_1, y_1) and having a gradient m is

$$y - y_1 = m(x - x_1)$$

(iii) **Two points form:** The equation of the straight line passing through two given points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$ which is required equation

(iv) **The Intercept form of a line:** A line which cuts off intercept a and b respectively from x-axis and y-axis is

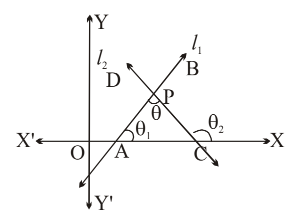


$$\frac{x}{a} + \frac{y}{b} = 1$$

This is the equation of the line in the intercept form.

3.4.3 Angle between two lines:

Angle between two lines : The acute angle θ between the lines having slopes m_1 and m_2 is given by



$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

or $$\tan\theta = \pm\frac{m_2 - m_1}{1 + m_1 m_2}$$

$$m_1 = \tan\theta_1,\ m_2 = \tan\theta_2$$

Condition for two lines to be perpendicular:

If two lines $y = m_1x + c_1$ and $y = m_2x + c_2$ are perpendicular then

i.e. $\theta = 90°$

$\tan\theta = \tan 90°$

or $\tan\theta = \infty$

$$\left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \infty$$

$$\mathbf{m_1 m_2 = -1}$$

When two lines are perpendicular the product of their slopes is –1.

Condition for two lines to be parallel:

If two lines $y_1 = m_1x + c_1$ and $y_2 = m_2x + c_2$ are parallel then their slopes will be equal. In this case angle between these two lines is 0°.

i.e. $\theta = 0°$

$\tan\theta = \tan 0$

or $$\left|\frac{m_2 - m_1}{1 + m_1m_2}\right| = 0$$

or $m_1 = m_2$

Thus when two lines are parallel their slopes are equal.

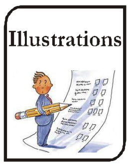


Illustration 8

State the equation of the line which has y-intercept –1 and parallel to y = 5x – 7.

Solution

Intercept = –1, m = 5 So equation, y = 5x –1

3.5 SOLVING THE WORD PROBLEMS

Many problems can be solved quickly and easily by converting them into a system of a pair of linear equations in two variables as follows:

(i) Represent the unknown quantities by variable x and y, which are to be determined.

(ii) Find the conditions given in the problem and translate the verbal conditions into a pair of simultaneous linear equations.

(iii) Solve these equations and obtain the required quantities with appropriate units.

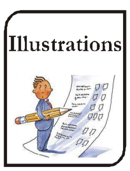


Illustration 9

The numerator of a fraction is 4 less than the denominator if the numerator is decreased by 2 and the denominator is increased by 1, then the denominator is eight times the numerator. Find the fraction.

Solution

Let the numerator and denominator of the fraction be x and y respectively.

Then required fraction = $\frac{x}{y}$

$\therefore \quad y - x = 4 \quad .....(1)$

and $\quad y + 1 = 8(x - 2)$

$\Rightarrow \quad y - 8x = -17 \quad .....(2)$

Subtracting (1) and (2),

$y - 8x - (y - x) = -17 - 4$

$$-7x = -21 \Rightarrow x = \frac{21}{7} = 3$$

$$\therefore \quad y = 4 + 3 = 7 \quad \therefore \quad \text{Required fraction} = \frac{3}{7}$$

3.6 SOLUTION OF A SYSTEM OF A PAIR OF EQUATIONS REDUCIBLE TO THE SYSTEM OF A PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

By using the suitable substitution or simplification first we convert the given system into the system of a pair of linear equations in two variables. Then after using any algebraic or graphical method we solve the system.

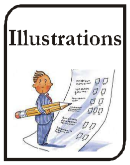


Illustration 10

Solve for x and y : $\frac{3}{x}+\frac{4}{y}=1;\ \frac{4}{x}+\frac{2}{y}=\frac{11}{12}$

Solution

$$\frac{3}{x}+\frac{4}{y}=1 \qquad .....(1)$$

$$\frac{4}{x}+\frac{2}{y}=\frac{11}{12} \qquad .....(2)$$

Multiplying (2) by 2

$$\Rightarrow \quad \frac{8}{x}+\frac{4}{y}=\frac{22}{12} \qquad .....(3)$$

Subtracting (1) from (3)

$$\Rightarrow \quad \frac{5}{x}=\frac{10}{12}$$

$$\therefore \quad x=\frac{5\times 12}{10}=6$$

Substituting $x = 6$ in (1)

$$\Rightarrow \quad \frac{3}{6}+\frac{4}{y}=1$$

$$\Rightarrow \quad \frac{4}{y}=1-\frac{1}{2}=\frac{1}{2}$$

$\therefore$ $y = 8$. Hence, $x = 6$ and $y = 8$

SOLVED EXAMPLES

Example 1

Solve for x and y : $152x - 378y = -74$ & $-378x + 152y = -604$

Solution

$152x - 378y = -74$ ...(i)

$-378x + 152y = -604$ ...(ii)

Add (i) & (ii)

$-226x - 226y = -678$

$\Rightarrow \quad x + y = 3$ ....(iii)

Subtract (ii) from (i)

$530x - 530y = 530$

$\Rightarrow \quad x - y = 1$ ....(iv)

Solving equation (iii) & (iv)

$x = 2$ and $y = 1$

Example 2

For what value of k, will the following pair of linear equations have no solution? $2x + 3y = 1$ and $(3k - 1)x + (1 - 2k)y = 2k + 3$.

Solution

From the given equations

$a_1 = 2$, $a_2 = 3k - 1$, $b_1 = 3$, $b_2 = 1 - 2k$, $c_1 = -1$ and $c_2 = -(2k + 3)$

The coordination for the given equations to have no solution is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Considering,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \quad \Rightarrow \quad \frac{2}{3k-1} = \frac{3}{1-2k}$$

$\Rightarrow \quad 2(1 - 2k) = 3(3k - 1) \quad \Rightarrow \quad 2 + 3 = 9k + 4k$

$\Rightarrow \quad 5 = 13k \quad \Rightarrow \quad k = \frac{5}{13}$

Example 3

For what value of k do the equations $3(k - 1)x + 4y = 24$ and $15x + 20y = 8(k + 13)$ have infinite solutions?

Solution

From the given pair of equations

$a_1 = 3(k - 1)$, $a_2 = 15$, $b_1 = 4$, $b_2 = 20$, $c_1 = -24$ and $c_2 = -8(k + 13)$

The condition for the given equations to have infinite solutions is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad \Rightarrow \quad \frac{3(k-1)}{15} = \frac{4}{20} = \frac{-24}{-8(k+13)}$$

Considering,

$$\frac{3(k-1)}{15} = \frac{4}{20} \quad \text{or} \quad 3k - 3 = \frac{4 \times 15}{20}$$

or $\quad 3k - 3 = 3 \quad$ or $\quad 3k = 6$

or $\quad k = 2$

Example 4

Solve the equations : $\frac{2x+1}{3}+\frac{3y+2}{5}=2$ **and** $\frac{2(2x+1)}{3}-\frac{3(3y+2)}{5}=-1$

Solution

Give equations are : $\frac{2x+1}{3}+\frac{3y+2}{5}=2$ .....(i)

and $\frac{2(2x+1)}{3}-\frac{3(3y+2)}{5}=-1$ .....(ii)

Let $\frac{2x+1}{3}=u$ and $\frac{3y+2}{5}=v$

Then, the equations become

$u+v=2$ .....(iii)

$2u-3v=-1$ .....(iv)

Multiplying (iii) by 3,

$3u+3v=6$ .....(v)

Adding (iv) and (v), $5u=5$

$\Rightarrow$ $u=1$

Substituting this value of u in (iii), $1+v=2 \Rightarrow v=2-1=1$

Then, $\frac{2x+1}{3}=u=1$ and $\frac{3y+2}{5}=v=1$

$\Rightarrow$ $2x+1=3$ and $3y+2=5$

$\Rightarrow$ $2x=3-1=2$ and $3y=5-2=3$

$\Rightarrow$ $x=1$ and $y=1$

Example 5

Solve the following pairs of equations by reducing them to a pair of linear equations

(i) 6x + 3y = 6xy; 2x + 4y = 5xy

(ii) $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4};\ \frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}$

Solution

(i) Given equations are $6x+3y=6xy$ and $2x+4y=5xy$

Dividing both the sides of both the equation by xy, we get $\frac{6}{y}+\frac{3}{x}=6$ and $\frac{2}{y}+\frac{4}{x}=5$

Let $\frac{1}{x}=u$ and $\frac{1}{y}=v$. Equations become $3u+6v=6$ and $4u+2v=5$

or $u+2v=2$ .....(i)

and $4u+2v=5$ .....(ii)

Subtracting (i) from (ii)

we get, $3u=3 \Rightarrow u=1$ and $1+2v=2 \Rightarrow v=\frac{1}{2}$

$u=1=\frac{1}{x} \Rightarrow x=1$ and $v=\frac{1}{2}=\frac{1}{y} \Rightarrow y=2$

So, $x=1$ and $y=2$

(ii) Given equation are $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$ and $\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}$

Let us take $u=\frac{1}{3x+y}$ and $\frac{1}{3x-y}=v$

So, equations become, $u+v=\frac{3}{4}$ and $\frac{u}{2}-\frac{v}{2}=\frac{-1}{8}$

or $4u+4v=3$ and $4u-4v=-1$

Adding both we get $8u=2 \Rightarrow u=\frac{1}{4}$ and $4\times\frac{1}{4}+4v=3 \Rightarrow v=\frac{2}{4}=\frac{1}{2}$

$u=\frac{1}{3x+y}=\frac{1}{4} \Rightarrow 3x+y=4$ .....(i)

and $v=\frac{1}{3x-y}=\frac{1}{2} \Rightarrow 3x-y=2$ .....(ii)

Adding (1) and (2) we get,

$6x=6 \Rightarrow x=1$

and putting $x=1$ in any equation, say equation (1),

$3\times 1+y=4 \Rightarrow y=1$

So, $x=1$, $y=1$

Example 6

Solve: $\mathbf{2x-\frac{1}{3}(2y-1)=3\frac{5}{24}+\frac{1}{4}(3x-2y);}$

$\mathbf{4y-\frac{1}{4}(5-2x)=6-\frac{1}{5}(3-2y)}$

Solution

$2x-\frac{1}{3}(2y-1)=3\frac{5}{24}+\frac{1}{4}(3x-2y)$

$2x-\frac{2y}{3}+\frac{1}{3}=\frac{77}{24}+\frac{3x}{4}-\frac{y}{2} \Rightarrow \frac{5x}{4}-\frac{y}{6}=\frac{69}{24}$

$\Rightarrow \frac{5x}{2}-\frac{y}{3}=\frac{69}{24} \Rightarrow 15x-2y=\frac{69}{2}$ ....(i)

$\Rightarrow 4y-\frac{1}{4}(5-2x)=\frac{6-1}{5}(3-2y) \Rightarrow 4y-\frac{5}{4}+\frac{x}{2}=6-\frac{3}{5}+\frac{2}{5}y$

$\Rightarrow \frac{18}{5}y+\frac{x}{2}=\frac{133}{20} \Rightarrow \frac{36y+5x}{10}=\frac{133}{20}$ ....(ii)

By Mlutiplying (ii) by 3 we get, $15x+108y=\frac{399}{2}$. ....(iii)

Ans. $x=2\frac{1}{2}$, $y=1\frac{1}{2} \Rightarrow -13y=13 \Rightarrow y=\frac{13}{-13} \Rightarrow y=-1$

Example 7

Solve the following system of linear equations for x and y

$a(x + y) + b(x - y) - (a^2 - ab + b^2) = 0$ and $a(x + y) - b(x - y) - (a^2 + ab + b^2) = 0$

Solution

The given system of equations is

$a(x + y) + b(x - y) - (a^2 - ab + b^2) = 0$ and $a(x + y) - b(x - y) - (a^2 + ab + b^2) = 0$

This can be written as

$(a + b)x + (a - b)y - (a^2 - ab + b^2) = 0$ and $(a - b)x + (a + b)y - (a^2 + ab + b^2) = 0$

Here $a_1 = a + b$, $b_1 = a - b$

$a_2 = a - b$, $b_2 = a + b$

and $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e. $\frac{a+b}{a-b} \neq \frac{a-b}{a+b}$

Also, $a_1b_2 - a_2b_1 = (a + b)(a + b) - (a - b)(a - b) = (a + b)^2 - (a - b)^2 = 4ab \neq 0$

Therefore, the given system of equations has a unique solution.

Now, we can solve this system of equations by using cross-multiplication method which gives:

$$\Rightarrow \quad \frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow \quad \frac{x}{-(a-b)(a^2+ab+b^2)+(a+b)(a^2-ab+b^2)}$$

$$= \frac{y}{-(a-b)(a^2-ab+b^2)+(a+b)(a^2+ab+b^2)} = \frac{1}{(a+b)(a+b)-(a-b)(a-b)}$$

$$\Rightarrow \quad \frac{x}{-(a^3-b^3)+(a^3+b^3)} = \frac{y}{2b(2a^2+b^2)} = \frac{1}{4ab}$$

$$\Rightarrow \quad \frac{x}{2b^3} = \frac{y}{2b(2a^2+b^2)} = \frac{1}{4ab}$$

$$\Rightarrow \quad x = \frac{2b^3}{4ab} = \frac{b^2}{2a} \text{ and } y = \frac{2b(2a^2+b^2)}{4ab} = \frac{2a^2+b^2}{2a}$$

Hence, the solution of the system is $x = \frac{b^2}{2a}$ and $y = \frac{2a^2+b^2}{2a}$

Example 8

The difference between two numbers is 2. Their product is 84 greater than the square of the smaller number. What is the sum of numbers.

Solution

smaller = x; larger = y

Let $y - x = 2$ ...(i)

$xy = 84 + x^2$ ...(ii)

From equation (i) & (ii)

$x(x + 2) = 84 + x^2$

$\Rightarrow$ $x = 42$

$\therefore$ from (i) $y = 44$

$\therefore$ $x + y = 86$

Example 9

Astha and Saumya each have certain number of oranges. Astha says to Saumya, "If you give me 10 of your orianges, I will have twice the number of oranges left with you. "Saumya replies, "If you give me 10 of your oranges, I will have the same number of orianges as left with you." Find the number of oranges with Astha and Saumya respectively.

Solution

Let Astha will have x number of oranges and Saumya will have y number of oranges. Then,

$(x + 10) = 2(y - 10) \Rightarrow x - 2y = -30$ ...(i)

and $(x - 10) = (y + 10) \Rightarrow x - y = 20$ ...(ii)

Subtracting equation (ii) from equation (i),

We get, $-y = -50 \Rightarrow y = 50$

$x = 70$

Example 10

A test has 50 questions. A student scores 1 mark for a correct answer, –1/3 for a wrong answer and – 1/6 for not attempting a question. If the net score of a student is 32, find the number of questions answered wrongly by that student cannot be less than.

Solution

Let the number of correct answers be 'x', number of wrong answers by 'y' and number of questions not attempted by 'z'.

Thus, $x + y + z = 50$ ...(i)

And $x - \frac{y}{3} - \frac{z}{6} = 32$

The above equation can be written as,

$6x - 2y - z = 192$ ...(ii)

Adding the two equations we get, $7x - y = 242$ or $x = \frac{242}{7} + \frac{y}{7}$

Since, x and y are both integers, y cannot be 1 or 2.

The minimum value that y can have is 3.

Example 11

At a certain fast food restaurant, Amit can buy 3 burgers, 7 shakes and one order of fries for Rs. 120. At the same place, it would cost Rs. 164.50 for 4 burgers,10 shakes and one order of fries. How much would it cost for an ordinary meal of one burger, one shake and one order of fries ?

Solution

Let the cost of 1 burger, 1 shake and one order of fries be x, y and z. Then

$3x + 7y + z = 120$ ...(i)

$4x + 10y + z = 164.50$ ...(ii)

Subtracting (i) from (ii)

$x + 3y = 44.50$ ...(iii)

Multiplying (iii) by 4 and subtracting (ii) from it,

we find, $2y - z = 13.5$ ...(iv)

Subtracting (iv) from (iii),

we get $x + y + z = 31$.

CONCEPT APPLICATION LEVEL - I [NCERT Questions]

Q.1 On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:

(i) $5x - 4y + 8 = 0$; $7x + 6y - 9 = 0$

(ii) $9x + 3y + 12 = 0$; $18x + 6y + 24 = 0$

(iii) $6x - 3y + 10 = 0$; $2x - y + 9 = 0$

Sol. (i) $5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

Here, $a_1 = 5, b_1 = -4, c_1 = 8$

$a_2 = 7, b_2 = 6, c_1 = -9$

We see that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the lines representing the given pair of linear equations intersect at a point.

(ii) $9x + 3y + 12 = 0$

$18x + 6y + 24 = 0$

Here, $a_1 = 9, b_1 = 3, c_1 = 12$

$a_2 = 18, b_2 = 6, c_2 = 24$

We see that $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the lines representing the given pair of linear equations are coincident.

(iii) $6x - 3y + 10 = 0$

$2x - y + 9 = 0$

Here $a_1 = 6, b_1 = -3, c_1 = 10$

$a_2 = 2, b_2 = -1, c_1 = 9$

We see that $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the lines representing the given pair of linear equations are parallel.

Q.2 On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines equations are consistent, or inconsistent:

(i) 3x + 2y = 5; 2x – 3y = 7 (ii) 2x – 3y = 8; 4x – 6y = 9

Sol. (i) $3x + 2y = 5 ; 2x - 3y = 7$

Here, $a_1 = 3, b_1 = 2, c_1 = -5$

$a_2 = 2, b_2 = -3, c_2 = -7$

$$\therefore \quad \frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{-2}{3} \text{ and } \frac{c_1}{c_2} = \frac{-5}{-7}$$

We see that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the given lines are intersecting.

So, the given pair of linear equations has exactly one solution and therefore it is consistent.

(ii) $2x - 3y = 8; 4x - 6y = 9$

Here, $a_1 = 2, b_1 = -3, c_1 = -8$

$a_2 = 4, b_2 = -6, c_2 = -9$

$$\therefore \quad \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-8}{-9}$$

We see that $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the given lines are parallel. So, the given pair of linear equations has no solution and therefore it is inconsistent.

Q.3 Draw the graphs of the equations x – y + 1 = 0 and 3x + 2y – 12 = 0. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

Sol. The given equations are

$x - y + 1 = 0$ ......(1)

$3x + 2y - 12 = 0$ ......(2)

Let us draw the graphs of equation (1) and (2) by finding two solutions for each of the equation. These two solutions of the equations (1) and (2) are given below in table 1 and table 2 respectively.

For equation (1)

$$x - y + 1 = 0$$

$$\Rightarrow \quad y = x + 1$$

Table 1 of solutions

x	0	–1
y	1	0

For equation (2)

$$3x + 2y - 12 = 0$$

$$\Rightarrow \quad 2y = 12 - 3x$$

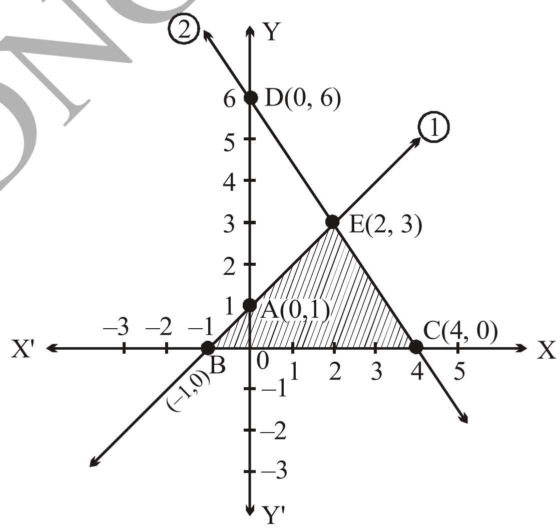
$$\Rightarrow \quad y = \frac{12 - 3x}{2}$$

Table 2 of solutions

x	4	0
y	0	6

We plot the points A(0, 1) and B(–1, 0) on a graph paper and join these points to form the line AB representing the equation (1) as shown in the figure. Also, we plot the points C(4, 0) and D(0, 6) on the same graph paper and join these points to form the line CD representing the equation (2) as shown in the same figure.

In the figure we observe that the coordinates of the vertices of the triangle formed by these given lines and the x-axis are E(2, 3), B(–1, 0) and C(4, 0).



The triangular region EBC has been shaded.

Q.4 Solve the following pair of linear equations by the substitution method

(i) 0.2x + 0.3y = 1.3; 0.4x + 0.5y = 2.3 **(ii)** $\sqrt{2}x + \sqrt{3}y = 0$; $\sqrt{3}x - \sqrt{8}y = 0$

Sol. (i) The given system of linear equation is

$$0.2x + 0.3y = 1.3 \quad .....(1)$$

$$0.4x + 0.5y = 2.3 \quad .....(2)$$

For equation (1),

$$0.3y = 1.3 - 0.2x$$

$$\Rightarrow \quad y = \frac{1.3 - 0.2x}{0.3} \quad .....(3)$$

Substituting this value of y in equation (2), we get

$$0.4x + 0.5\left(\frac{1.3 - 0.2x}{0.3}\right) = 2.3$$

$$\Rightarrow \quad 0.12x + 0.65 - 0.1x = 0.69$$

$$\Rightarrow \quad 0.12x - 0.1x = 0.69 - 0.65$$

$$\Rightarrow \quad 0.02x = 0.4$$

$$\Rightarrow \quad x = \frac{0.04}{0.02} = 2$$

Substituting this value of x in equation (3), we get

$$y = \frac{1.3 - 0.2(2)}{0.3} = \frac{1.3 - 0.4}{0.3} = \frac{0.9}{0.3} = 3$$

Therefore the solution is x = 2, y = 3.

Verification : Substituting x = 2 and y = 3, we find that both the equations (1) and (2) are satisfied as shown below:

$0.2x + 0.3y = (0.2)(2) + (0.3)(3) = 0.4 + 0.9 = 1.3$

$0.4x + 0.5y = (0.4)(2) + (0.5)(3) = 0.8 + 0.15 = 2.23$

This verifies the solution

(ii) The given pair of linear equation is

$$\sqrt{2}x + \sqrt{3}y = 0 \quad .....(1)$$

$$\sqrt{3}x - \sqrt{8}y = 0 \quad .....(2)$$

For equation (2),

$$\sqrt{3}x = \sqrt{8}y \quad \Rightarrow \quad x = \frac{\sqrt{8}}{\sqrt{3}}y \quad .....(3)$$

Substituting this value of x in equation (1), we get

$$\sqrt{2}\cdot\frac{\sqrt{8}}{\sqrt{3}}y+\sqrt{3}y=0 \Rightarrow \frac{4}{\sqrt{3}}y+\sqrt{3}y=0$$

$$\Rightarrow \left(\frac{4}{\sqrt{3}}+\sqrt{3}\right)y=0 \Rightarrow y=0$$

Substituting this value of y in equation (3), we get

$$x=\frac{\sqrt{8}}{\sqrt{3}}(0)=0$$

Therefore the solutions is x = 0, y = 0

Verification : Substituting x = 0 and y = 0, we find that both the equations (1) and (2) are satisfied as shown below:

$$\sqrt{2}x+\sqrt{3}y = \sqrt{2}(0)+\sqrt{3}(0)=0$$

$$\sqrt{3}x-\sqrt{8}y = \sqrt{3}(0)-\sqrt{8}(0)=0$$

This verifies the solution.

Q.5 Form the pair of linear equations for the following problems and find their solution by substitution method.

(i) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

(ii) The coach of a cricket team buys 7 bats and 6 balls for Rs. 3800. Later, she buys 3 bats and 5 balls for Rs.1750. Find the cost of each bat and each ball.

(iii) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is Rs. 105 and for a journey of 15 km, the charge paid is Rs.155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?

Sol. (i) Let the larger and the smaller of two supplementary angles be x° and y° respectively.

Then, according to the question.

The pair of linear equations formed is

$$x^\circ = y^\circ + 18^\circ \quad .....(1)$$

$$x^\circ + y^\circ = 180^\circ \quad .....(2) \quad (\because \text{The two angles are supplementary})$$

Substitute the value of x° from equation (1) in equation (2), we get

$$y^\circ + 18^\circ + y^\circ = 180^\circ$$

$$\Rightarrow 2y^\circ + 18^\circ = 180^\circ \Rightarrow 2y^\circ = 180^\circ - 18^\circ$$

$$\Rightarrow 2y^\circ = 162^\circ \Rightarrow y^\circ = \frac{162^\circ}{2} = 81^\circ$$

Substituting this value of y° in equation (1), we get

$$x^o = 81° + 18° = 99°$$

Hence, the larger and the smaller of the two supplementary angles are 99° and 81° respectively.

Verification : Substituting x° = 99° and y° = 81°, we find that both the equations (1) and (2) are satisfies as shown below:

$$y^o + 18^o = 81^o + 18^o = 99^o = x^o$$

$$x^o + y^o = 99^o + 81^o = 180^o$$

This verifies the solution.

(ii) Let the cost of each bat and each ball be Rs. x and Rs. y respectively.

Then, according to the question,

The pair of linear equations formed is

$$7x + 6y = 3800 \quad .....(1)$$

$$3x + 5y = 1750 \quad .....(2)$$

From equation (2),

$$5y = 1750 - 3x$$

$$y = \frac{1750 - 3x}{5} \quad .....(3)$$

Substitute this value of y in equation (1), we get

$$7x + 6\left(\frac{1750 - 3x}{5}\right) = 3800$$

$$\Rightarrow \quad 35x + 10500 - 18x = 19000$$

$$\Rightarrow \quad 17x + 10500 = 19000$$

$$\Rightarrow \quad 17x = 19000 - 10500$$

$$\Rightarrow \quad 17x = 8500$$

$$\Rightarrow \quad x = \frac{8500}{17} = 500$$

Substituting this value of x in equation (3), we get

$$y = \frac{1750 - 3(500)}{5} = \frac{1750 - 1500}{5} = \frac{250}{5} = 50$$

Hence, the cost of each bat and each ball is Rs. 500 and Rs.50 respectively.

Verification : Substituting x = 500 and y = 50, we find that both the equations (1) and (2) are satisfied as shown below :

$$7x + 6y = 7(500) + 6(50) = 3500 + 300 = 3800$$

$$3x + 5y = 3(500) + 5(50) = 1500 + 250 = 1750$$

This is verifies the solution.

(iii) Let the fixed charges be Rs. x and the charge per kilometer be Rs. y.

Then, according to the question

The pair of linear equations formed is

$x + 10y = 105$ .....(1)

$x + 15y = 155$ .....(2)

From equation (1),

$x = 105 - 10y$ .....(3)

Substitute this value of x in equation (2), we get

$105 - 10y + 15y = 155$

$\Rightarrow$ $05 + 5y = 155$ $\Rightarrow$ $5y = 155 - 105$

$\Rightarrow$ $5y = 50$ $\Rightarrow$ $y = \frac{50}{5} = 10$

Substituting this value of y in equation (3), we get

$x = 105 - 10(10) = 105 - 100 = 5$

Hence, the fixed charges are Rs.5 and the charge per kilometer is Rs.10.

Verification: Substituting x = 5 and y = 10, we find that both the equation (1) and (2) are satisfied as shown below:

$x + 10y = 5 + 10(10) = 5 + 100 = 105$

$x + 15y = 5 + 15(10) = 5 + 150 = 155$

This verifies the solution.

Again, for travelling a distance of 25 km, a person will have to pay = 5 + 10(25)

= 5 + 250 = 255 Rs.

Q.6 Solve the following pair of linear equations by the elimination method and the substitution method:

(i) $3x - 5y - 4 = 0$ and $9x = 2y + 7$ **(ii) $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$**

Sol. (i) **(I) By Elimination method**

The given system of equations is

$3x - 5y - 4 = 0$ . ....(1)

$9x = 2y + 7$

$\Rightarrow$ $9x - 2y - 7 = 0$ .....(2)

Multiplying equation (1) by 3, we get

$9x - 15y - 12 = 0$ .....(3)

Substracting equation (3) from equation (2), we get

$13y + 5 = 0$ $\Rightarrow$ $13y = -5$ $\Rightarrow$ $y = \frac{-5}{13}$

Substituting this value of y in equation (1), we get

$$3x - 5\left(\frac{-5}{13}\right) - 4 = 0$$

$$\Rightarrow \quad 3x - \frac{27}{13} = 0 \quad \Rightarrow \quad 3x = \frac{27}{13} \quad \Rightarrow \quad x = \frac{9}{13}$$

So, the solution of the given system of equations is $x = \frac{9}{13}, y = \frac{-5}{13}$.

(II) By Substitution method

The given system of equations is

$$3x - 5y - 4 = 0 \qquad .....(1)$$

$$9x = 2y + 7 \qquad .....(2)$$

From equation (2)

$$x = \frac{2y+7}{9} \qquad .....(3)$$

Substitute this value of x in equation (1), we get

$$3\left(\frac{2y+7}{9}\right) - 5y - 4 = 0$$

$$\Rightarrow \quad \frac{2y+7}{9} - 5y - 4 = 0 \quad \Rightarrow \quad 2y + 7 - 15y - 12 = 0$$

$$\Rightarrow \quad -13y - 5 = 0 \quad \Rightarrow \quad 13y = -5 \quad \Rightarrow \quad y = \frac{-5}{13}$$

Substituting this value of y in equation (3), we get

$$x = \frac{2\left(\frac{-5}{13}\right)+7}{9} = \frac{-\frac{10}{13}+7}{9} = \frac{-10+91}{117} = \frac{81}{117} = \frac{9}{13}$$

So, the solution of the given system of equations is

$$x = \frac{9}{13}, y = \frac{-5}{13}$$

Verification : Substituting $x = \frac{9}{13}$, $y = \frac{-5}{13}$, we find that both the equation (1) and (2) are satisfied as shown below:

$$3x - 5y - 4 = 3\left(\frac{9}{13}\right) - 5\left(\frac{-5}{13}\right) - 4 = \frac{27}{13} + \frac{25}{13} - 4 = \frac{52}{13} - 4 = 4 - 4 = 0$$

$$2y + 7 = 2\left(\frac{-5}{13}\right) + 7 = \frac{-10}{13} + 7 = \frac{81}{13} = 9\left(\frac{9}{13}\right) = 9x$$

Hence, the solution is correct.

(ii) **(I) By Elimination method**

The given system of equations as

$$\frac{x}{2}+\frac{2y}{3}=-1 \qquad .....(1)$$

$$x-\frac{y}{3}=3 \qquad .....(2)$$

Multiplying equation (2) by 2, we get

$$2x-\frac{2y}{3}=6 \qquad .....(3)$$

Adding equation (1) and equation (3), we get

$$\frac{5}{2}x=5 \quad \Rightarrow \quad x=\frac{5\times 2}{5} \quad \Rightarrow \quad x=2$$

Subtracting this value of x in equation (2), we get

$$2-\frac{y}{3}=3 \quad \Rightarrow \quad \frac{y}{3}=2-3=-1 \quad \Rightarrow \quad y=-3$$

So, the solution of the given system of equations is x = 2, y = –3.

(II) By Substitution method

The given system of equations is

$$\frac{x}{2}+\frac{2y}{3}=-1 \qquad .....(1)$$

$$x-\frac{y}{3}=3 \qquad .....(2)$$

From equation (2)

$$x=\frac{y}{3}+3 \qquad .....(3)$$

Substitute this value of x in equation (1), we get

$$\frac{1}{2}\left(\frac{y}{3}+3\right)+\frac{2y}{3}=-1$$

$$\Rightarrow \quad \frac{y}{6}+\frac{3}{2}+\frac{2y}{3}=-1 \quad \Rightarrow \quad \frac{5y}{6}=-1-\frac{3}{2}$$

$$\Rightarrow \quad \frac{5y}{6}=\frac{-5}{2} \quad \Rightarrow \quad y=-3$$

Substituting this value of y in equation (3), we get

$$x = \frac{-3}{3} + 3 = -1 + 3 = 2$$

So, the solution of the given system of equations

$x = 2, y = -3$

Verification : Substituting x = 2, y = –3, we find that both the equations (1) and (2) are satisfied as shown below:

$$\frac{x}{2} + \frac{2y}{3} = \frac{2}{2} + \frac{2(-3)}{3} = 1 - 2 = -1$$

$$x - \frac{y}{3} = 2 - \frac{(-3)}{3} = 2 + 1 = 3$$

Hence, the solution is correct.

Q.7 Form the pair of linear equation in the following problems, and find their solutions (if they exist) by the elimination method:

(i) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

(ii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

Sol. (i) Let Nuri and Sonu be x years and y years old respectively at present.

Then, according to the question,

$x - 5 = 3(y - 5)$

$x + 10 = 2(y + 10)$

$\Rightarrow$ $x - 5 = 3y - 15$

$x + 10 = 2y + 20$

$\Rightarrow$ $x - 3y = -10$ .....(1)

$x - 2y = 10$ .....(2)

Substracting equation (2) from equation (1), we get

$-y = -20 \quad \Rightarrow \quad y = 20$

Substituting this value of y in equation (2), we get

$x - 2(20) = 10 \quad \Rightarrow \quad x - 40 = 10$

$\Rightarrow \quad x = 40 + 10 \quad \Rightarrow \quad x = 50$

Hence, Nuri and Sonu are 50 years and 20 years old respectively at present.

Verification : Substituting the values of x = 50 and y = 20, we find that both the equation (1) and (2) are satisfied as shown below:

$x - 3y = 50 - 3(20) = 50 - 60 = -10$

$x - 2y = 50 - 2(20) = 50 - 40 = 10$

Hence, the solution is correct.

(ii) Let the unit's digit and the ten's digit in the two-digit number be x and y respectively.

Then the number = 10 y + x

Also, the number obtained by reversing the order of the digits = 10x + y

According to the question,

$x + y = 9$ .....(1)

$9(10y + x) = 2(10x + y)$

$\Rightarrow 90y + 9x = 20x + 2y \Rightarrow 11x - 88y = 0 \Rightarrow x - 8y = 0$

Subtracting equation (2) from equation (1), we get

$x + 1 = 9 \Rightarrow x = 9 - 1 = 8$

Hence, the required number is 18.

Verification : Substituting x = 8 and y = 1, we find that both the equations (1) and (2) are satisfied as shown below.

$x + y = 8 + 1 = 9$

$x - 8y = 8 - 8(1) = 0$

Hence, the solution is correct.

Q.8 (i) For which values of a and b does the following pair of linear equations have an infinite number of solutions?

$2x + 3y = 7$; $(a - b)x + (a + b)y = 3a + b - 2$

(ii) For which value of k will the following pair of linear equations have no solution?

$3x + y = 1$; $(2k - 1)x + (k - 1)y = 2k + 1$

Sol. (i) The given pair of linear equation is

$$2x + 3y = 7$$

$$(a - b)x + (a + b)y = 3a + b - 2$$

Here $a_1 = 2, b_1 = 3, c_1 = -7$

$a_2 = a - b, b_2 = a + b,$

$c_2 = -(3a + b - 2)$

For having an infinite number of solutions, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{2}{a-b} = \frac{3}{a+b}$$

$\Rightarrow 2(a + b) = 3(a - b) \Rightarrow 2a + 2b = 3a - 3b$

$\Rightarrow a - 5b = 0$ .....(1)

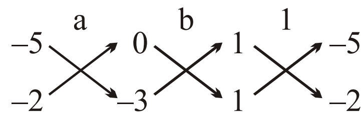
For last two,

$$\frac{3}{a+b} = \frac{7}{3a+b-2} \Rightarrow 3(3a + b - 2) = 7(a + b)$$

$\Rightarrow 9a + 3b - 6 = 7a + 7b \Rightarrow 2a - 4b - 6 = 0$

$\Rightarrow a - 2b - 3 = 0$ .....(2) (Dividing throughout by 2)

To solve the equation (1) and (2) by cross-multiplication method, we draw the diagram below:



Then, $\frac{a}{(-5)(-3)-(-2)(0)} = \frac{b}{(0)(1)-(-3)(1)} = \frac{1}{(1)(-2)-(1)(-5)}$

$\Rightarrow \quad \frac{a}{15} = \frac{b}{3} = \frac{1}{3}$

$\Rightarrow \quad a = \frac{15}{3} = 5 \quad$ and $\quad b = \frac{3}{3} = 1$

Hence, the required values of a and b are 5 and 1 respectively.

(ii) The given pair of linear equations is

$$3x + y = 1$$

$$(2k - 1)x + (k - 1)y = 2k + 1$$

$\Rightarrow \quad 3x + y - 1 = 0$

Here, $(2k - 1)x + (k - 1)y - (2k + 1) = 0$

$a_1 = 3, b_1 = 1, c_1 = -1$

$a_2 = 2k - 1, b_2 = k - 1,$

$c_2 = -(2k + 1)$

For having no solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{-1}{-(k+1)}$

From above, we have

$$\frac{3}{2k-1} = \frac{1}{k-1}$$

$\Rightarrow \quad 3(k - 1) = 2k - 1$

$\Rightarrow \quad 3k - 3 = 3 - 1$

$\Rightarrow \quad k = 2$

Hence , the required value of k is 2.

Q.9 Solve the following pair of linear equations by the substitution and cross-multiplication methods:
8x + 5y = 9; 3x + 2y = 4

Sol. The given pair of linear equations

$8x + 5y = 9$ .....(1)

$3x + 2y = 4$ .....(2)

(I) By Substitution method

From equation (2),

$2y = 4 - 3x \Rightarrow y = \frac{4-3x}{2}$ .....(3)

Substitute this value of y in equation (1), we get

$$8x + 5\left(\frac{4-3x}{2}\right) = 9$$

$\Rightarrow \quad 16x + 20 - 15x = 18 \quad \Rightarrow \quad x + 20 = 18$

$\Rightarrow \quad x = 18 - 20 \quad \Rightarrow \quad x = -2$

Substituting this value of x in equation (3), we get

$$y = \frac{4-3(-2)}{2} = \frac{4+6}{2} = \frac{10}{2} = 5$$

So the solution of the given pair of linear equations is $x = -2$, $y = 5$

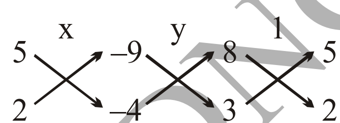
(II) By Cross-multiplication method

Let us write the given pair of linear equations is

$8x + 5y - 9 = 0$ .....(1)

$3x + 2y - 4 = 0$ .....(2)

To solve the equations (1) and (2) by cross multiplication method, we draw the diagram below:



Then, $$\frac{x}{(5)(-4)-(2)(-9)} = \frac{y}{(-9)(3)-(-4)(8)} = \frac{1}{(8)(2)-(3)(5)}$$

$$= \frac{x}{-20+18} = \frac{y}{-27+32} = \frac{1}{16-15} = \frac{x}{-2} = \frac{y}{5} = \frac{1}{1}$$

$\Rightarrow \quad x = -2$ and $y = 5$

Hence, the required solution of the given pair of linear equations is

$x = -2, y = 5$

Verification: Substituting $x = -2$, $y = 5$, we find that both the equations (1) and (2) are satisfied as shown below:

$8x + 5y = 8(-2) + 5(5) = -16 + 25 = 9$

$3x + 2y = 3(-2) + 2(5) = -6 + 10 = 4$

Hence, the solution is correct.

Q.10 Form the pair of linear equations in the following problem and find their solution (if they exist) by any algebraic method.

Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

Sol. Let the speeds of two cars be x km/hour and y km/hour respectively.

Case I: When the cars travel in the same direction

Let they meet at P.

Distance travelled by the car starting from A in 5 hours = AP = 5x km

(Distance = speed × time)

Distance travelled by the car starting from B in 5 hours = BP = 5y km

(Distance = speed × time)

Now, $AP - BP = 100$

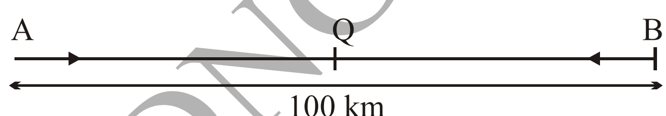
$\Rightarrow \quad 5x - 5y = 100$

$\Rightarrow \quad x - y = 20$ .....(1)

(Dividing throughout by 5)

Case II: When the cars travel towards each other

Let they meet at Q.



Distance travelled by the car starting from A in 1 hour = AQ = x km

Distance travelled by the car starting from B in 1 hour = BQ = y km

Now, $AQ + BQ = 100$

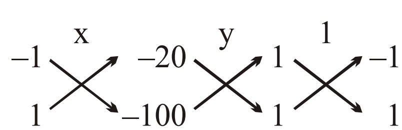
$\Rightarrow \quad x + y = 100$ .....(2)

Equation (1) and (2) can be re-written as

$x - y - 20 = 0$ .....(3)

$x + y - 100 = 0$ .....(4)

To solve the equations (3) and (4) by cross-multiplication method, we draw the diagram below:



Then, $\frac{x}{(-1)(-100)-(1)(-20)} = \frac{y}{(-20)(1)-(-100)(1)} = \frac{1}{(1)(1)-(1)(-1)}$

$\Rightarrow \frac{x}{100+20} = \frac{y}{-20+100} = \frac{1}{1+1}$

$\Rightarrow \frac{x}{120} = \frac{y}{80} = \frac{1}{2}$

$\Rightarrow x = \frac{120}{2} = 60$ and $y = \frac{80}{2} = 40$

Hence, the speeds of the two cars are 60 km/hour and 40 km/hour respectively.

Verification: Substituting x = 60, y = 40, we find that both the equations (1) and (2) are satisfied as shown below:

$x - y = 60 - 40 = 20$

$x + y = 60 + 40 = 100$

Hence, the solution we have got is correct.

Q.11 Solve the following pairs of equations by reducing them to a pair of linear equations:

(i) $\frac{1}{2x} + \frac{1}{3y} = 2$; $\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$ **(ii)** $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$; $\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$

(iii) $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$; $\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$

Sol. (i) The given pair of equations is

$\frac{1}{2x} + \frac{1}{3y} = 2$ .....(1)

$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$ .....(2)

Put $\frac{1}{x} = X$ .....(3)

and $\frac{1}{y} = Y$ .....(4)

Then the equations (1) and (2) can be rewritten as

$$\frac{1}{2}X + \frac{1}{3}Y = 2 \qquad .....(5)$$

$$\frac{1}{3}X + \frac{1}{2}Y = \frac{13}{6} \qquad .....(6)$$

$\Rightarrow \quad 3X + 2Y = 12 \qquad .....(7)$

$2X + 3Y = 13 \qquad .....(8)$

Multiplying equation (7) by 3 and equation (8) by 2, we get

$9X + 6Y = 36 \qquad .....(9)$

$4X + 6Y = 26 \qquad .....(10)$

Subtracting equation (10) from equation (9), we get

$9(2) + 6Y = 36 \qquad .....(11)$

$\Rightarrow \quad 18 + 6Y = 36$

$\Rightarrow \quad 6Y = 36 - 18 = 18$

$$\Rightarrow \quad Y = \frac{18}{6} = 3 \qquad .....(12)$$

From equation (3) and equation (11), we get

$$\frac{1}{x} = 2 \quad \Rightarrow \quad x = \frac{1}{2}$$

From equation (4) and equation (12), we get

$$\frac{1}{y} = 3 \quad \Rightarrow \quad y = \frac{1}{3}$$

Hence, the solution of the given pair of equations is

$$x = \frac{1}{2}, y = \frac{1}{3}$$

Verification : Substituting $x = \frac{1}{2}, y = \frac{1}{3}$, we find that both the equations (1) and (2) are satisfied as shown below:

$$\frac{1}{2x} + \frac{1}{3y} = \frac{1}{2 \cdot \frac{1}{2}} + \frac{1}{3 \cdot \frac{1}{3}} = 1 + 1 = 2$$

$$\frac{1}{3x} + \frac{1}{2y} = \frac{1}{3 \cdot \frac{1}{2}} + \frac{1}{2 \cdot \frac{1}{3}} = \frac{2}{3} + \frac{3}{2} = \frac{13}{6}$$

Hence, the solution we have got is correct.

(ii) The given pair of equations is

$$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2 \quad .....(1)$$

$$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1 \quad .....(2)$$

Put $\frac{1}{\sqrt{x}} = u$ .....(3)

and $\frac{1}{\sqrt{y}} = v$ .....(4)

Then equations (1) and (2) can be rewritten as

$2u + 3v = 2$ .....(5)

$4u - 9v = -1$ .....(6)

Multiplying equations (5) by 3, we get

$6u + 9v = 6$ .....(7)

Adding equation (6) and equation (7), we get

$10u = 5$

$\Rightarrow \quad u = \frac{5}{10} = \frac{1}{2}$ .....(8)

Substituting the value of x in equation (5), we get

$$2\left(\frac{1}{2}\right) + 3v = 2$$

$\Rightarrow \quad 1 + 3v = 2$

$\Rightarrow \quad 3v = 2 - 1 = 1$

$\Rightarrow \quad v = \frac{1}{3}$ .....(9)

From equation (3) and equation (8), we get

$$\frac{1}{\sqrt{x}} = \frac{1}{2} \quad \Rightarrow \quad \sqrt{x} = 2 \quad \Rightarrow \quad x = 4 \qquad \text{(Squaring)}$$

From equation (4) equation (9), we get

$$\frac{1}{\sqrt{y}} = \frac{1}{3} \quad \Rightarrow \quad \sqrt{y} = 3 \quad \Rightarrow \quad x = 9 \qquad \text{(Squaring)}$$

Hence, the solution if the given pair of equation is

$x = 4, y = 9$

Verification : Substituting x = 4, y = 9, we find that both the equations (1) and (2) are satisfied as shown below.

$$\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}} = \frac{2}{\sqrt{4}}+\frac{3}{\sqrt{9}} = \frac{2}{2}+\frac{3}{3} = 1 + 1 = 2$$

$$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}} = \frac{4}{\sqrt{9}}-\frac{9}{\sqrt{9}} = \frac{4}{2}-\frac{9}{3} = 2 - 3 = -1$$

Hence, the solution we have got is correct.

(iii) The given pair of equations is

$$\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4} \quad .....(1)$$

$$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8} \quad .....(2)$$

Put $$\frac{1}{3x+y}=u \quad .....(3)$$

and $$\frac{1}{3x-y}=u \quad .....(4)$$

Then equations (1) and (2) can be rewritten as

$$u+v=\frac{3}{4} \quad .....(5)$$

$$\frac{1}{2}u-\frac{1}{2}v=\frac{-1}{8} \quad .....(6)$$

(6) gives, $$u-v=\frac{-1}{4} \quad .....(7)$$ (Multiplying both sides by 2)

Adding equation (5) and equation (7), we get

$$2u=\frac{3}{4}-\frac{1}{4}=\frac{1}{2}$$

$\Rightarrow$ $$u=\frac{1}{4} \quad .....(8)$$

Subtracting equation (7) from equation (5), we get

$$2v = \frac{3}{4} + \frac{1}{4} = 1 \qquad .....(9)$$

From equation (3) and equation (8), we get

$$\frac{1}{3x+y} = \frac{1}{4}$$

$$\Rightarrow \qquad 3x + y = 4 \qquad .....(10)$$

From equation (4) and equation (9), we get

$$\frac{1}{3x+y} = \frac{1}{2}$$

$$\Rightarrow \qquad 3x - y = 2 \qquad .....(11)$$

Adding equation (10) and equation (11), we get

$$6x = 6 \quad \Rightarrow \quad x = \frac{6}{6} = 1$$

Substituting this value of x in equation (10), we get

$$3(1) + y = 4 \quad \Rightarrow \quad 3 + y = 4 \quad \Rightarrow \quad y = 4 - 3 = 1$$

Hence, the solution of the given pair of equation is

$x = 1, y = 1$

Verification : Substituting x = 1, y = 1, we find that both the equations (1) and (2) are satisfied as shown below :

$$\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{1}{3(1)+1} + \frac{1}{3(1)-1} = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$

$$\frac{1}{2(3x+y)} + \frac{1}{2(3x-y)} = \frac{1}{2(3\times1+1)} + \frac{1}{2(3\times1-1)} = \frac{1}{8} - \frac{1}{4} = \frac{-1}{8}$$

Q.12 Formulate the following problems as a pair of equations, and hence find their solutions:

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find the speed of rowing in still water and the speed of the current.

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

(iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Sol. (i) Let her speed of rowing in still water be x km/hour and the speed of the current by y km/hour.
Then, her speed of rowing downstream = (x + y) km/hour.
and, her speed of rowing upstream = (x – y) km/hour.

Also, $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

In the first case, when she goes 20 km downstream, then the time taken is 2 hours.

$$\therefore \quad \frac{20}{x+y} = 2 \quad \Rightarrow \quad x + y = 10 \quad .....(1)$$

In the second case, when she goes 4 km upstream, then the time taken is 2 hours.

$$\therefore \quad \frac{4}{x-y} = 2 \quad \Rightarrow \quad x - y = 2 \quad .....(2)$$

Adding equation (1) and equation (2), we get

$$2x = 12 \quad \Rightarrow \quad x = \frac{12}{2} = 6$$

Substituting this value of x in equation (1), we get

$$6 + y = 10 \quad \Rightarrow \quad y = 10 - 6 = 4$$

Hence, the speed of her rowing in still water is 6 km/hour and the speed of the current is 4 km/hour.

Verification : Substituting x = 6, y = 4 we find that both the equations (1) and (2) are satisfied as shown below:

$$x + y = 6 + 4 = 10$$
$$x - y = 6 - 4 = 2$$

Hence, the solution we have got is correct.

(ii) Let the time taken by 1 woman alone to finish the embroidery by x days and the time taken by 1 man alone to finish the embroidery be y days. Then

$$\text{1 woman's 1 day's work} = \frac{1}{x}$$

and $$\text{1 man's 1 day's work} = \frac{1}{y}$$

$$\therefore \quad \text{2 women's 1 day's work} = \frac{2}{x}$$

and $$\text{2 men's 1 day's work} = \frac{5}{y}$$

$\because$ 2 women and 5 men can together finish a piece of embroidery in 4 days.

$$\therefore \quad 4\left(\frac{2}{x}+\frac{5}{y}\right) = 1 \quad \Rightarrow \quad \frac{2}{x}+\frac{5}{y} = \frac{1}{4} \quad .....(1)$$

Again, 3 women's 1 day's work $= \frac{3}{x}$

and 6 men's 1 day's work $= \frac{6}{y}$

$\because$ 3 women and 6 men can together finish a piece of embroidery in 3 days.

$$\therefore \quad 3\left(\frac{3}{x}+\frac{6}{y}\right) = 1 \quad \Rightarrow \quad \frac{3}{x}+\frac{6}{y} = \frac{1}{3} \quad .....(2)$$

Put $\frac{1}{x} = u$ .....(3)

and $\frac{1}{y} = v$ .....(4)

Then equations (1) and (2) can be rewritten as

$$2u + 5v = \frac{1}{4} \quad .....(5)$$

$$3u + 6v = \frac{1}{3} \quad .....(6)$$

Multiplying equation (5) by 3 and equation (6) by 2, we get

$$6u + 15v = \frac{3}{4} \quad .....(7)$$

$$6u + 12v = \frac{2}{3} \quad .....(8)$$

Subtracting equation (8) from equation (7), we get

$$3v = \frac{3}{4} - \frac{2}{3} = \frac{1}{12} \quad \Rightarrow \quad v = \frac{1}{36} \quad .....(9)$$

Substituting this value of v in equation (5), we get

$$2u + 5\left(\frac{1}{36}\right) = \frac{1}{4} \quad \Rightarrow \quad 2u + \frac{5}{36} = \frac{1}{4}$$

$\Rightarrow \quad 2u = \frac{1}{4} - \frac{5}{36} \qquad \Rightarrow \quad 2u = \frac{9}{36} - \frac{5}{36}$

$\Rightarrow \quad 2u = \frac{4}{36} = \frac{1}{9} \qquad \Rightarrow \quad u = \frac{1}{18} \qquad .....(10)$

From equation (3) and equation (10), we get

$\frac{1}{x} = \frac{1}{18} \qquad \Rightarrow \quad x = 18$

From equation (4) and equation (9), we get

$\frac{1}{y} = \frac{1}{36} \qquad \Rightarrow \quad y = 36$

Hence, the time taken by 1 woman alone to finish the embroidery is 18 days and the time taken by 1 man alone to finish the embroidery is 36 days.

Verification : Substituting x = 18, y = 36 we find that both the equation (1) and (2) are satisfied as shown below:

$$\frac{2}{x} + \frac{5}{y} = \frac{2}{18} + \frac{5}{36} = \frac{1}{9} + \frac{5}{36} = \frac{1}{4}$$

$$\frac{3}{x} + \frac{6}{y} = \frac{3}{18} + \frac{6}{36} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

This verifies the solution.

(iii) Let the speed of the train and the bus be x km/hour and y km/hour respectively.

Case I When she travels 60 km by train and the remaining (300 – 60) km, i.e. 240 km by bus, the time taken is 4 hours.

$\therefore \quad \frac{60}{x} + \frac{240}{y} = 4 \qquad \left(\because \text{ Time} = \frac{\text{Distance}}{\text{Speed}}\right)$

$\Rightarrow \quad \frac{1}{x} + \frac{4}{y} = \frac{1}{15} \qquad .....(1) \qquad \text{(Dividing by 60)}$

Case II When she travels 100 km by train and the remaining (300 – 400) km i.e. 200 km by bus, the time taken is 4 hours 10 minutes, i.e. $\frac{25}{6}$ hours.

$\therefore \quad \frac{100}{x} + \frac{200}{y} = \frac{25}{6} \qquad \Rightarrow \quad \frac{4}{x} + \frac{8}{y} = \frac{1}{6} \qquad .....(2) \qquad \text{(Dividing by 25)}$

Multiplying equation (1) by 2, we get

$$\frac{2}{x}+\frac{8}{y}=\frac{2}{15} \quad .....(3)$$

Subtracting equation (3) from equation (2), we get

$$\frac{2}{x}=\frac{1}{6}-\frac{2}{15}=\frac{1}{30} \quad \Rightarrow \quad x=60$$

Substituting this value of x in equation (3) we get

$$\frac{2}{60}+\frac{8}{y}=\frac{2}{15} \quad \Rightarrow \quad \frac{1}{30}+\frac{8}{y}=\frac{2}{15}$$

$$\Rightarrow \quad \frac{8}{y}=\frac{2}{15}-\frac{1}{30}=\frac{1}{10} \quad \Rightarrow \quad y=80$$

So, the solution of the equations (1) and (2) is x = 60 and y = 80.

Hence, the speed of the train is 60 km/hour and the speed of the bus is 80 km/hour.

Verification : Substituting x = 60, y = 80, we find that both the equations (1) and (2) and satisfied as shown below :

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{60}+\frac{4}{80}=\frac{1}{60}+\frac{1}{20}=\frac{1}{15}$$

$$\frac{4}{x}+\frac{8}{y}=\frac{4}{60}+\frac{8}{80}=\frac{1}{15}+\frac{1}{10}=\frac{1}{6}$$

This verifies the solution.

Q.13 The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years. Find the ages of Ani and Biju.

Sol. Let the ages of Ani and Biju be x years and y years respectively. Then, according to the question,

$$x-y=\pm 3 \quad .....(1)$$

Age of Ani's father Dharm = 2x years

Age of Biju's sister = $\frac{y}{2}$ years

According to the question,

$$2x-\frac{y}{2}=30 \quad \Rightarrow \quad 4x-y=60 \quad .....(2)$$

Case I When $x - y = 3$

Then, we have

$$x - y = 3 \quad .....(1)$$

$$4x - y = 60 \quad .....(2)$$

Subtracting equation (1) from equation (2)

$$3x = 57 \quad .....(3)$$

$$x = \frac{57}{3} = 19 \text{ years}$$

Substituting the value of x is equation (1)

$$19 - y = 3 \quad \Rightarrow \quad y = 19 - 3 = 16$$

Ani's age = 19 years

Biju's age = 16 year

Verification :

$$x - y = 19 - 16 = 3$$

$$4x - \frac{y}{4} = 4 \times 19 - 16 = 76 - 16 = 60$$

This verifies the solution.

Case II When $x - y = -3$

Then, we have

$$x - y = -3 \quad .....(1)$$

$$4x - y = 60 \quad .....(2)$$

Subtracting equation (1) from equation (2)

$$3x = 63$$

$$x = \frac{63}{3} = 21 \text{ years}$$

Substituting the value of x is equation (1), we get

$$21 - y = -3 \quad \Rightarrow \quad y = 24$$

Ani's age = 21 years

Biju's age = 24 year

$$x - y = 21 - 24 = 3$$

$$4x - y = 4(21) - 24 = 84 - 24 = 60$$

This satisfied the solution.

Q.14 A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 km/hr, it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

Sol. Let the actual speed of the train be x km/hour and the actual time taken by y hours. Then,

$$\text{Distance} = (xy)\text{ km} \qquad .....(1) \quad (\text{Distance} = \text{speed} \times \text{time})$$

According to the equation,

$$xy = (x + 10)(y - 2)$$

$$\Rightarrow \quad xy = xy - 2x + 10y - 20$$

$$\Rightarrow \quad 2x - 10y + 20 = 0$$

$$\Rightarrow \quad x - 5y + 10 = 0 \qquad (\text{Dividing throughout by } 2)$$

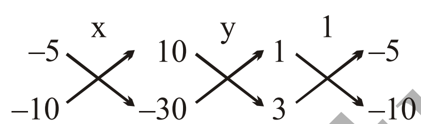
and

$$xy = (x - 10)(y + 3)$$

$$\Rightarrow \quad xy = xy + 3x - 10y - 30$$

$$\Rightarrow \quad 3x - 10y - 30 = 0 \qquad .....(2)$$

To solve the equations (1) and (2) by the cross-multiplication method, we draw the diagram below:



Then,
$$\frac{x}{(-5)(-30)-(-10)(10)} = \frac{y}{(10)(3)-(-30)(1)} = \frac{1}{(1)(-10)-(3)(-5)}$$

$$\Rightarrow \quad \frac{x}{150+100} = \frac{y}{30+30} = \frac{1}{-10+15}$$

$$\Rightarrow \quad \frac{x}{250} = \frac{y}{60} = \frac{1}{5}$$

$$\Rightarrow \quad x = \frac{250}{5} = 50$$

and
$$y = \frac{60}{5} = 12$$

So, the solution of the equations (1) and (2) is x = 50 and y = 12. Hence, the distance covered by the train is 50 × 12 = 600 km

Verification : Substituting x = 50, y = 12, we find that both the equations (1) and (2) are satisfied as shown below. Hence, the solution we have got is correct.

Q.15 Solve the following pair of linear equations

$(a - b)x + (a + b)y = a^2 - 2ab - b^2$; $(a + b)(x + y) = a^2 + b^2$

Sol. The given pair of linear equations is

$$(a - b)x + (a + b)y = a^2 - 2ab - b^2 \quad .....(1)$$

$$(a + b)(x + y) = a^2 + b^2$$

$$\Rightarrow \quad (a + b)x + (a + b)y = a^2 + b^2 \quad .....(2)$$

Subtracting equation (2) from equation (1), we get

$$-2bx = -2ab - 2b^2$$

$$\Rightarrow \quad x = \frac{-2ab - 2b^2}{-2b} = a\frac{-2b(a+b)}{-2b} = a + b$$

Subtracting this value of x is equation (1), we get

$$(a - b)(a + b) + (a + b)y = a^2 - 2ab - b^2$$

$$\Rightarrow \quad a^2 - b^2 + (a + b)y = a^2 - 2ab - b^2$$

$$\Rightarrow \quad (a + b)y = -2ab$$

$$\Rightarrow \quad y = \frac{-2ab}{a+b}$$

Hence, the solution of the given pair of linear equation is

$$x = a + b,\ y = \frac{-2ab}{a+b}$$

Verification : Substituting $x = a + b, y = \frac{-2ab}{a+b}$, we find that both the equation (1) and (2) are satisfied as shown below:

$$(a - b)x + (a + b)y = (a - b)(a + b) + (a + b)\left(\frac{-2ab}{a+b}\right) = a^2 - b^2 - 2ab$$

$$(a + b)(x + y) = (a + b)\left(a + b - \frac{2ab}{a+b}\right) = (a + b)^2 - 2ab = a^2 + b^2$$

This verifies the solution.

CONCEPT APPLICATION LEVEL - II [Previous Year Questions]

Q.1 If 6 years hence a man's age will be 3 times the age of his son and three years ago he was 9 times as old as his son, then what is the present age of the man? **[NSTSE-2013]**
(A) 25 years (B) 35 years (C) 15 years (D) 30 years

Q.2 If the system of equation $2x + 3y = 7$ and $2ax + (a + b)y = 28$ represents coincident lines, which of the condition holds true? **[NSTSE-2013]**
(A) $b = 2a$ (B) $a = 2b$ (C) $2a + b = 0$ (D) $a + 2b = 0$

Q.3 For which values of 'a' and 'b' does the following pair of linear equations have an infinite number of solutions **[NSTSE-2013]**
$2x + 3y = 7; (a - b)x + (a + b)y = 3a + b - 2$
(A) $a = 5, b = 1$ (B) $a = 4, b = 2$ (C) $a = 1, b = 5$ (D) $a = 2, b = 4$

Q.4 Find the equation of the line passing through (3, 7) whose slope is $\frac{-3}{2}$ **[NIMO]**
(A) $3x + 2y = 23$ (B) $3x + 3y = 24$ (C) $3x + 4y = 23$ (D) None of these

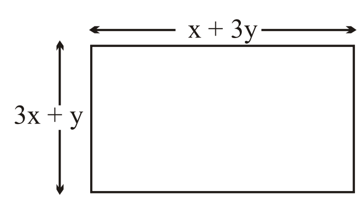
Q.5 Two lines with slopes m_1 and m_2 are parallel to each other if **[NIMO]**
(A) $m_1 = m_2$ (B) $m_1 m_2 = 1$ (C) $\frac{m_1}{m_2} = 1$ (D) $m_1 + m_2 = 1$

Q.6 Find the slope of a line whose inclination with the x-axis is 150° **[NIMO]**
(A) $\frac{1}{2}$ (B) $\sqrt{3}$ (C) $\frac{-1}{\sqrt{3}}$ (D) None of these

Q.7 If twice the area of a smaller square is subtracted from the area of a larger square, the result is 14 cm^2. However if twice the area of the large square is added to three times the area of the smaller square, the result is 203 cm^2. Determine the sides of the two squares. **[NIMO]**
(A) 7 cm, 4 cm (B) 6 cm, 3 cm (C) 8 cm, 5 cm (D) None of these

Q.8 The denominator of a rational number is greater than its numerator by 3. If 3 is subtracted from the numerator and 2 is added to the denominator, the new number becomes $\frac{1}{5}$. What was the original number? **[NIMO]**
(A) $\frac{5}{8}$ (B) $\frac{3}{5}$ (C) $\frac{7}{11}$ (D) $\frac{3}{8}$

Q.9 Find the values of x and y in the given rectangle if its length is cube root of 2197 and width is one less than the fourth multiple of first prime number
(A) 6, 1 (B) 3, 2
(C) 4, 1 (D) 13, 7



[5th IMO]

Q.10 The coach of a cricket team buys 7 bats and 6 balls for Rs.3800. Later he buys 3 bats and 5 balls for Rs.1750. Find the cost of each ball **[IOM-2012]**

(A) Rs.75 (B) Rs.50 (C) Rs.30 (D) Rs.80

Q.11 A number consists of two digits whose sum is 5. When the digits are reversed the number becomes greater by 9. Find the given number **[IMO-2012]**

(A) 23 (B) 21 (C) 19 (D) 25

Q.12 Five years age, Rani was thrice as old as Tanvi, ten years later, Rani will be twice as old as Tanvi. How old is Rani? **[IMO-2012]**

(A) 40 years (B) 50 years (C) 45 years (D) 52 years

Q.13 A boat covers 32 km upstream and 36 km downstream in 7 hours. Also, it covers 40 km upstream and 48 km downstream in 9 hours. Find the speed of boat in still water **[IOM-2012]**

(A) 10 km/h (B) 8 km/h (C) 5 km/h (D) 2 km/h

Q.14 Find the nature of solution of the system of linear equations give by $3x + 4y = 5$ and $4x - 6y = 8$

(A) unique solution (B) no solution

(C) infinitely many solutions (D) inadequate data **[IOM-2013]**

Q.15 Which one of the following is the condition for no solution? **[IOM-2013]**

(A) $a_1 a_2 = b_1 b_2$ (B) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (C) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (D) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Q.16 For which values of 'a' and 'b' does the following pair of linear equations have an infinite number of solutions ; $2x + 3y = 7$, $(a - b)x + (a + b)y = 3a + b - 2$ **[Raj.NTSE Stage-1_2013]**

(A) $a = 5, b = 1$ (B) $a = 4, b = 2$ (C) $a = 1, b = 5$ (D) $a = 2, b = 4$

Q.17 If the system of equations $kx + 3y - (k - 3) = 0$, $12x + ky - k = 0$ has infinitely many solution, then k = **[Raj. NTSE Stage-1_2014]**

(A) 6 (B) –6 (C) 0 (D) None of these

Q.18 Which of the following system of equations has no solution? **[IMO - 2016]**

(A) $3x - y = 2, 9x - 3y = 6$ (B) $4x - 7y + 28 = 0, 5y - 7x + 9 = 0$

(C) $3x - 5y - 11 = 0, 6x - 10y - 7 = 0$ (D) None of these

Q.19 In the equations $3x + 2y = 13xy$ and $4x - 5y = 2xy$, the values of x and y that satisfy the equations are **[NTSE - 2016]**

(A) (2, 3) (B) (3, 2) (C) $\left(\frac{1}{2}, \frac{1}{3}\right)$ (D) $\left(\frac{1}{3}, \frac{1}{2}\right)$

CONCEPT APPLICATION LEVEL - III

SECTION-A

- **Multiple choice questions with one correct answer**

Q.1 The pair of equations $3^{x+y} = 81, 81^{x-y} = 3$ has
(A) no solution (B) the solution x = 2½, y = 2½
(C) the solution x = 2, y = 2 (D) the solution $x = 2\frac{1}{8}, y = 1\frac{7}{8}$

Q.2 The points (7,2) and (–1,0) lie on a line
(A) 7y = 3x – 7 (B) 4y = x +1 (C) y = 7x + 7 (D) x = 4y +1

Q.3 The condition for which the system of equations kx – y = 2 and 6x–2y = 3 has a unique solution is
(A) k = 3 (B) k ≠ 3 (C) k ≠ 0 (D) k = 0

Q.4 The equations ax +b = 0 and cx + d = 0 are consistent if
(A) ad = bc (B) ad + bc = 0 (C) ab –cd = 0 (D) ab + cd = 0

Q.5 The solution to the system of equation |x+y| = 1 and x–y = 0 is given by
(A) x = y = 1/2 (B) x = y = –1/2
(C) x = 1, y = 0 (D) x = y = 1/2 or x = y = –1/2

Q.6 The value of x + y in the solution of equations $\frac{x}{4}+\frac{y}{3}=\frac{5}{12}$ and $\frac{x}{2}+y=1$ is
(A) 1/2 (B) 3/2 (C) 2 (D) 5/2

Q.7 Simplify $|x-3|+2|x+1|=4$
(A) 1 (B) –1 (C) 3 (D) many solution

Q.8 If $1+\frac{1}{x}=\frac{x+1}{x}$, what does x equal to ?
(A) 1 or 2 only (B) +1 only (C) +1 & -1 only (D) any number except.

Q.9 If $4x+5y=82$, $3x+2z=54$, $5y+4z=110$, what is the value of $5x+2y+z$?
(A) 50 (B) 65 (C) 75 (D) 100

Q.10 The total number of integer pairs (x,y) satisfying the equation x + y = xy is
(A) 0 (B) 1 (C) 2 (D) None of these

Q.11 x and y are 2 different digits. If the sum of the two numbers formed by using both digits is a perfect square, then find x + y
(A) 10 (B) 11 (C) 12 (D) 13

Q.12 Two candles of the same length are highted at the same time. the first is consumed in 6 hours and the second in 4 hours. Assuming each candle burns at a constant rate, in how many hours after being lighted was the first candle twice the length of the second.
(A) 1 hour (B) 2 hour (C) 3 hours (D) 5 hours

Q.13 A boat travels with speed of 15 km/hour in still water. In a river flowing at 5 km/hour the boat travels. Some distance downsteam and then returns. The ratio of average speed to the speed in still water is
(A) 8 : 3 (B) 3 : 8 (C) 8 : 9 (D) 9 : 8

Q.14 Evaluate: $|3| + |-2-3| - 3 - |-7|$
(A) –2 (B) 2 (C) 10 (D) –10

Q.15 Find the equation of the line which is parallel to $3x - 2y + 5 = 0$ and passes through point (5 –6)
(A) $3x-2y + 27 = 0$ (B) $2x-3y + 27 = 0$ (C) $3x-2y-27=0$ (D) $3x + y + 27 = 0$

Q.16 State whether the two lines through (5,6) and (2,3) through (9,–2) and (6–5) are
(A) parallel (B) perpendicular
(C) neither parallel nor perpendicular (D) none of these

Q.17 Find the equation of line which cuts off an intercept 4 on the positive direction of x-axis and an intercept 3 cm the negative direction of y-axis.
(A) $3x-4y = 12$ (B) $3x+4y = 12$ (C) $4x-3y = 12$ (D) $4x+3y = 12$

Q.18 What can be said regarding a line of its slop is ∞.
(A) The line is y-axis. (B) The line is x-axis (C) Parallel to x-axis (D) None of these

Q.19 Find the set of value of z satisfying $\left|\frac{5-x}{3}\right| < 2$:
(A) $1 < x < 11$ (B) $-1 < x < 11$ (C) $x < 11$ (D) None of these

Q.20 If a and b are real no's the equation $3x - 5 + a = bx + 1$ has a unique solution x
(A) For all a and b (B) if $a \neq 2b$ (C) if $a \neq b$ (D) if $b \neq 3$

SECTION-B

- **Multiple choice question with one or more than one correct answers**

Q.1 The equations $2x + y - 5 = 0$ and $6x + 3y - 15 = 0$ shows
(A) Coincident lines (B) Infinite number of solution
(C) Unique solution (D) no solution

Q.2 If $p > q$ and $r < 0$, which of the following is/are true:-
(A) $pr < qr$ (B) $p + r > q + r$ (C) $p - r < q - r$ (D) $2p + r < 2q + r$

Q.3 If u is between 0 and 1, but $u \neq 0$ or 1 which of the following increases as u increases ?
(A) $1-u^2$ (B) $u-1$ (C) $1/u^2$ (D) u^2

Q.4 Simplify $x^2 + |x-1| = 1$

(A) 1 (B) –1 (C) 0 (D) 2

Q.5 The simultaneous equations $2x + 3y = 5,\ 4x + 6y = 10$ represents.

(A) several solutions (B) only two solutions (C) parallel lines (D) coincident lines

Q.6 Consider the following statements : The system of equations $2x - y = 4$, $px - y = q$, which of the following statements is/are true for above system of equations:-

(A) has a unique solution if $p \neq 2$ (B) represents parallel lines if p = 2

(C) has infinitely many solutions if $p = 2, q = 4$ (D) has no solution if $p \neq 2$

SECTION-C

- **Assertion & Reason**

Instructions: In the following questions as Assertion (A) is given followed by a Reason (R). Mark your responses from the following options.

(A) Both Assertion and Reason are true and Reason is the correct explanation of 'Assertion'

(B) Both Assertion and Reason are true and Reason is not the correct explanation of 'Assertion'

(C) Assertion is true but Reason is false

(D) Assertion is false but Reason is true

Q.1 **A :** The graph of equation y + 8 = x + 8 passes through origin.

B : The graph of a linear equation with its constant term = 0 always passes through origin

Q.2 **A :** The equation 2x +3y = 3(2 + y) has a unique solution.

B : The linear equation in two variables has a unique solution.

SECTION-D

- **Comprehension**

Draw the graph of the linear function whose respective values of x and y are given below

x	**–3**	**–2**	**–1**	**0**	**1**
y	**5**	**—**	**3**	**–**	**1**

Q.1 Write down the linear relation between x and y

(A) x + y = 2 (B) x – y = 2 (C) 2x – y = 2 (D) x – 2y = 2

Q.2 Find the missing numbers from the graph

(A) (4,2) (B) (3,4) (C) (1,2) (D) None of these

Q.3 Find the slope of the above graph

(A) –2 (B) 1 (C) –1 (D) None of these

ANSWER KEY

CONCEPT APPLICATION LEVEL - II

Q.1	D	Q.2	A	Q.3	A	Q.4	A	Q.5	A	Q.6	C	Q.7	C
Q.8	A	Q.9	C	Q.10	B	Q.11	A	Q.12	B	Q.13	A	Q.14	A
Q.15	C	Q.16	A	Q.17	A	Q.18	C	Q.19	C				

CONCEPT APPLICATION LEVEL - III

SECTION-A

Q.1	D	Q.2	B	Q.3	B	Q.4	A	Q.5	D	Q.6	B	Q.7	C
Q.8	D	Q.9	C	Q.10	C	Q.11	B	Q.12	C	Q.13	C	Q.14	A
Q.15	C	Q.16	A	Q.17	A	Q.18	A	Q.19	B	Q.20	D		

SECTION-B

Q.1	(A,B)	Q.2	(A,B)	Q.3	(B,D)	Q.4	(A,C)	Q.5	(A,D)	Q.6	(A,C)

SECTION-C

Q.1	A	Q.2	C

SECTION-D

Q.1	A	Q.2	A	Q.3	C