2

POLYNOMIALS

THEORY

2.1 INTRODUCTION :

In earlier classes, we have learnt about polynomials in one variable, their degrees, factors, multiples and zeros (or roots). In this chapter, we will study about the geometrical representation of linear quadratic and cubic polynomials and geometrical meaning of their zeros. We will also study about the relationship between the zeros and coefficients of a polynomial. LCM and HCF of two or more polynomials, rational expressions, basic operation on polynomials and concept of square root of polynomials.

2.2 POLYNOMIALS :

An algebraic expression f(x) of the form $f(x) = a_0 + a_1x + a_2x^2 + ...+ a_nx^n$; where $a_0, a_1, a_2,.......,a_n$ are real numbers and all the indices of variable x are non-negative integers, is called a polynomial in variable x and the highest indices n is called the degree of the polynomial, if $a_n \neq 0$. Here, a_0, a_1x, a_2x^2,........and a_nx^n are called the terms of the polynomial and $a_0, a_1, a_2...............a_n$ are called various coefficients of the polynomial f(x). A polynomial in x is said to be in standard form when the terms are written either in increasing order or in decreasing order of the indices of x in various terms.

For example : $x^2 - a^2$, $ax^2 + bx + c, x^3 + 3x^2 + 3x + 1$, $y^3 - 7y + 6$ etc. are the polynomials written in their standard form.

Illustrations

Illustration 1 : Which of the following expressions are polynomials?

(i) $\frac{1}{x^{-2}} + \frac{1}{x^{-1}} + \frac{1}{2}$ **(ii)** $\frac{1}{x}(x-1)(x-2)$

(iii) $\frac{(x^2+x+1)(x+1)}{(1+x)}$ **(iv)** $x^2 + \frac{1}{x^2}$

Solution : **(i)** $\frac{1}{x^{-2}} + \frac{1}{x^{-1}} + \frac{1}{2} = x^2 + x^1 + \frac{1}{2}\ x^0$ [It is a polynomial]

(ii) $\frac{1}{x}(x-1)(x-2) = x^{-1}(x^1-1)(x^1-2)$

$= x^{-1}(x^2 - 3x + 2)$

$= x - 3 + 2x^{-1}$

$= x^1 - 3x^0 + 2x^{-1}$ [It is not a polynomial]

(iii) $\frac{(x^2+x+1)(x+1)}{(x+1)} = x^2 + x + 1 = x^2 + x^1 + 1 \cdot x^0$ [It is a polynomial]

(iv) $x^2 + \frac{1}{x^2} = x^2 + x^{-2}$ [It is not a polynomial]

Illustration 2. **Rewrite the following polynomials in the standard form :**

(i) $x - 7 + 8x^2 + 9x^3$ **(ii) $-5x^2 + 6 - 3x^3 + 4x$**

Solution : **(i)** $9x^3 + 8x^2 + x - 7$ is the standard form of $x - 7 + 8x^2 + 9x^3$.

(ii) $-3x^3 - 5x^2 + 4x + 6$ is the standard form of $-5x^2 + 6 - 3x^3 + 4x$.

- **Degree of a Polynomial in One Variable :** In case of a polynomial in one variable, the highest power of the variable is called the degree of the polynomial.
  For example : the degree of $9x^3 + 8x^2 + x - 7$ is 3.

- **Degree of a Polynomial in Two or More Variables :** In case of polynomials in more than one variable, the sum of the powers of the variables in each term is taken up and the highest sum so obtained is called the degree of the polynomial.
  For example : the degree of $5x^3 + 6x^2y^2 + 12y^3$ is $2 + 2 = 4$. Because the power of the variables in first and third terms is 3 but the sum of power of variables in second term is 4 and $4 > 3$. Hence the degree is 4.

- **Additive Inverse of a Polynomial :** A polynomial Q is the additive inverse of a polynomial P if the sum of Q and P is zero, e.g. $3x^2 - 5x + 7$ is the additive inverse of $-3x^2 + 5x - 7$.
  The additive inverse of a polynomial is obtained by reversing the sign of each of the coefficients of the polynomial.

- **Value of a polynomial :** The value of a polynomial f(x) at x = a is obtained by substituting x = a in the given polynomial and is denoted by f(a).
  Consider the polynomial : $p(x) = 6x^2 + 7x - 2$
  If we replace x by 1 everywhere in p(x), we get
  $$p(1) = 6(1)^2 + 7(1) - 2$$
  $$= 6 + 7 - 2 = 11$$
  So, we say that the value of p(x) at x = 1 is 11.

- **Zero(es) / Root(s) of polynomials :** x = r is a root or zero of a polynomial p(x), if p(r) = 0.
  In other words, x = r is a root or zero of a polynomial p(x), if it is a solution to the equation p(x) = 0.
  The process of finding the zeros of p(x) is nothing more than solving to the equation p(x) = 0.

- **Geometrical meaning of the zero(es) of a polynomial :** Zero(es) of a polynomial is/are the x-coordinate of the point(s) where the graph of y = f(x) intersects the x-axis.

- **Graphs of polynomial :** In algebraic language the graph of a polynomial f(x) is the collection of all points (x,y), where y = f(x). In geometrical or in graphical language the graph of a polynomial f(x) is a smooth free hand curve passing through point (x_1, y_1), (x_2, y_2), (x_3, y_3)...........etc, where y_1, y_2, y_3 .... are the values of the polynomial f(x) at x_1, x_2, x_3 ....respectively. In order to draw the graph of a polynomial f(x), we may follow the following algorithm.

Algorithm

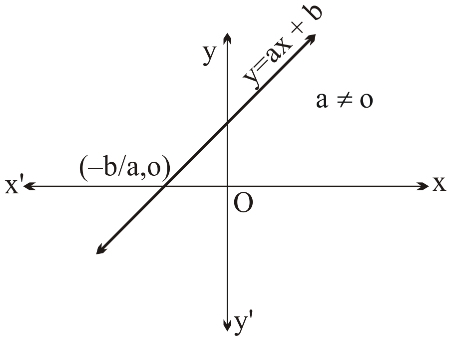
Step - 1:- Find the values y_1, y_2...... y_n of polynomial f(x) at different points x_1, x_2......x_n and prepare a table that gives values of y or f(x) for various values of x.

Step - 2:- Plot the points (x_1y_1), (x_2,y_2), (x_3, y_3)........ (x_n, y_n) on rectangular coordinate system. In plotting these points we may use different scales on the x and y axes.

Step -3:- Draw a free hand smooth curve passing through points plotted in step 2 to get the graph of the polynomial f(x).

x	x_1	x_2	....	x_n
y = f(x)	$y_1 = f(x_1)$	$y_2 = f(x_2)$	....	$y_n = f(x_n)$

- **Graph of a Linear Polynomial :-** Consider a linear polynomial f(x) = ax + b, a ≠ 0. We know that the graph of polynomial y = ax + b is a straight line, so it is called a linear polynomial. Since a straight line can be determined by two points, so only two points need to be poltted to draw the graph of y = ax +b. The graph of y = ax + b crosses the x axis at exactly one point namely $\left(\frac{-b}{a},0\right)$.



- **Graph of a Quadratic Polynomial :-** Consider a quadratic polynomial f(x) = y = ax^2 + bx + c. Where a, b and c be real numbers and a ≠ 0. We know that the graph of quadratic polynomial is a cup shaped graph known as parabola.

In order to draw the graph of a quadratic polynomial f(x), we may follow the following algorithm.

Algorithm to draw the graph of quadratic polynomial :

Step -1 : Write the given quadratic polynomial f(x) = y = ax^2 + bx + c.

Step -2 : Determine the zero of the polynomial , if they exist. This can be done by putting y = 0 i.e. $ax^2 + bx + c = 0$.

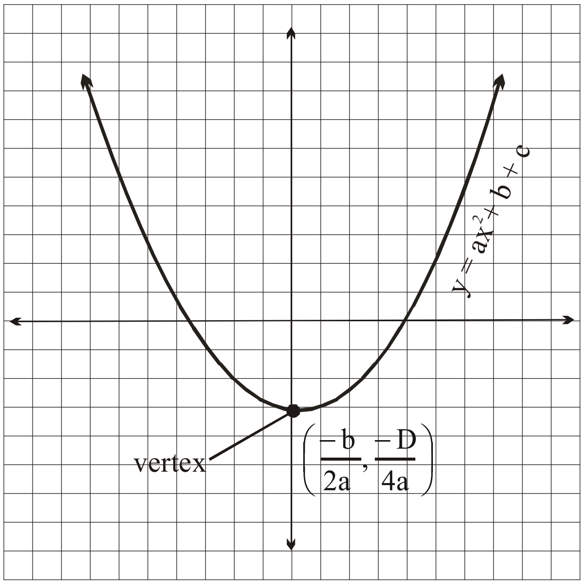
Step -3 : Calculate the discriminant $D = b^2 - 4ac$

Step - 4 : Determine the point where the curve intersects y - axis. This can be done by putting x = 0 in the given function and calculating the value of y.

Step - 5 : Determine the vertex i.e., $\left(\frac{-b}{2a},\frac{-D}{4a}\right)$.

Step - 6 : Prepare a table of selected values of x and corresponding values of y (generally two or three points on the left and two or three points on the right of the vertex.)

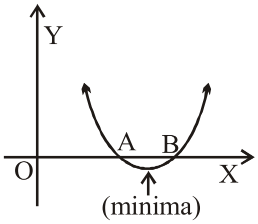
Step - 7 : Draw a smooth curve, through these points.



Graph of Quadratic Polynomial : The graph of a quadratic polynomial $ax^2 + bx + c$, $a \neq 0$ is a parabola which opens upward or downward as $a > 0$ or $a < 0$, we have the following possibilities :-

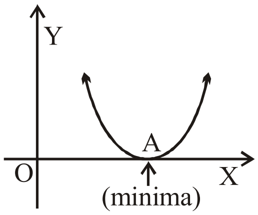
Case-I : If **a > 0**, then graph of a quadratic polynomial looks similar to one of the graphs in Figure (i), (ii) and (iii). In these figures parabola is opening upward.

(i) When $b^2 - 4ac > 0$ ⇒



The graph $y = ax^2 + bx + c$, $a \neq 0$ cuts the x-axis at two distinct points A and B. The x-coordinates of these points are the two zeroes of the polynomial $y = ax^2 + bx + c$, $a \neq 0$.

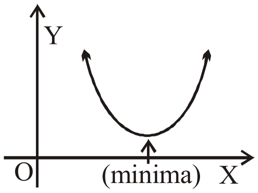
(ii) When $b^2 - 4ac = 0$ ⇒



In this case, the graph of polynomial $y = ax^2 + bx + c$, $a \neq 0$, touches the x-axis at exactly one point A and whose coordinates are $\left(\frac{-b}{2a}, 0\right)$. So, in this case the x-coordinates of point A gives two equal zeros of the polynomial $y = ax^2 + bx + c$, $a \neq 0$.

(iii) When $b^2 - 4ac < 0$ (In this case polynomial $ax^2 + bx + c$ is not factorizable.)

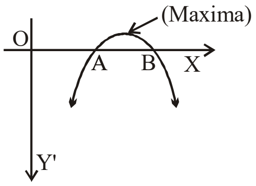
⇒



The graph of polynomial $y = ax^2 + bx + c$, $a \neq 0$ does not cut or touch x-axis. The curve of parabola remains completely above the x-axis.

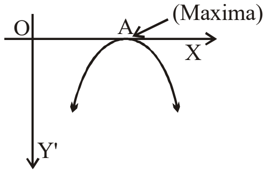
Case-II : If **a < 0**, then graph of the quadratic polynomial looks similar to one of the graphs in Figure (iv), (v) and (vi). In these figures parabola is opening downwards.

(iv) When $b^2 - 4ac > 0$ $\Rightarrow$



The graph $y = ax^2 + bx + c, a \neq 0$ cuts the x-axis at two distinct points A and B. The x-coordinates of these points are the two zeroes of the polynomial $y = ax^2 + bx + c, a \neq 0$.

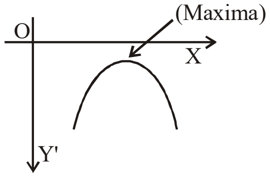
(v) When $b^2 - 4ac = 0$ $\Rightarrow$



In this case, the graph of polynomial $y = ax^2 + bx + c, a \neq 0$, touches the x-axis at exactly one point A and whose coordinates are $\left(\frac{-b}{2a}, 0\right)$. So, in this case the x-coordinates of point A gives two equal zeros of the polynomials $y = ax^2 + bx + c, a \neq 0$.

(vi) When $b^2 - 4ac < 0$ (In this case polynomial $ax^2 + bx + c$ is not factorizable.)

$\Rightarrow$



The graph of polynomial $y = ax^2 + bx + c, a \neq 0$ does not cut or touch x-axis. The curve of parabola remains completely below the x-axis.

Illustrations

Illustration 3 : Draw the graph of quadratic polynomial $x^2 - 2x + 3$ & read off zeros from the graph.

Solution : Let $y = x^2 - 2x + 3$

Put $y = 0$ $\Rightarrow$ $x^2 - 2x + 3 = 0$

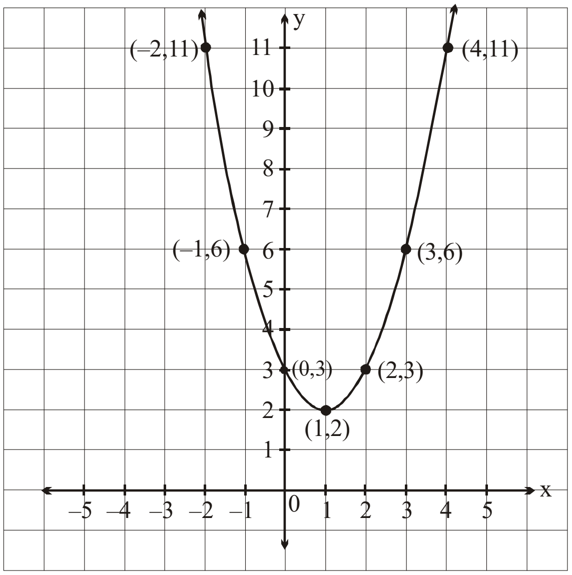
Now, $D = b^2 - 4ac = (-2)^2 - 4 \times 1 \times 3 = 4 - 12 = -8 < 0$. Hence no real zeros :

Now vertex of the parabola $= \left(-\frac{b}{2a}, -\frac{D}{4a}\right) = \left(-\frac{(-2)}{2\times1}, -\frac{(-8)}{4\times1}\right) = (1, 2)$

Required table for $y = x^2 - 2x + 3$

x	$y = x^2 - 2x + 3$			y
	x^2	$-2x$	3	
1	1	– 2	3	2
2	4	– 4	3	3
0	0	0	3	3
3	9	– 6	3	6
4	16	– 8	3	11
– 1	1	2	3	6
– 2	4	4	3	11

Now, we have ordered pairs : (1,2) (2,3) (0,3) (3,6) (–1,6) (–2,11) and (4,11)



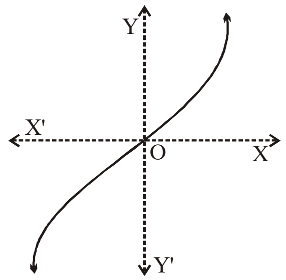
Since the graph of the polynomial does not meet or intersect the x-axis at all, therefore the given polynomial has no real zeros.

- **Graph of a cubic polynomial :**

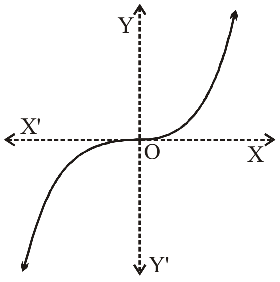
A cubic polynomial is a function of the form $y = ax^3 + bx^2 + cx + d$.

where $a \neq 0$, and a, b, c and d are real constants.

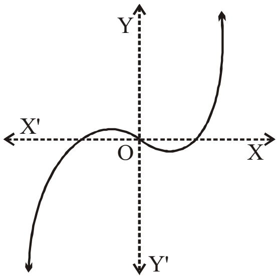
Case-I : If **a > 0**, then graph of a cubic function looks similar to one of the graphs in Figure (i), (ii) and (iii).



(i)

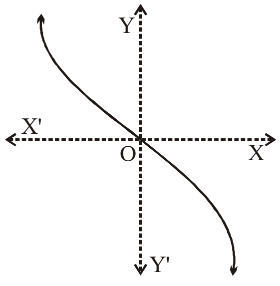


(ii)

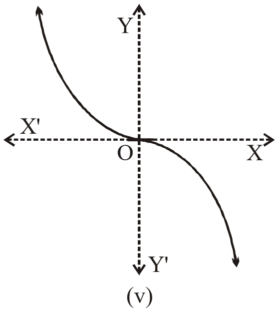


(iii)

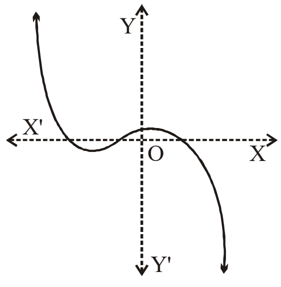
Case-II : If **a < 0**, then graph of the cubic function looks similar to one of the graphs in Figure (iv), (v) and (vi).



(iv)



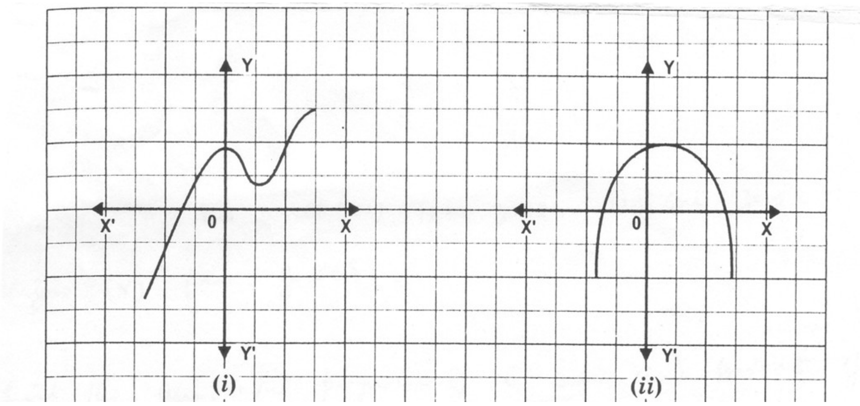
(v)



(vi)

Illustrations

Illustration 4 : Look at the graphs given below. Each is the graph of y = p(x) where p(x) is a polynomial. For each of the following graphs, find the number of zeroes of p(x).



Solution :

(i) Since the graph intersects the x-axis at one point only, the polynomial p(x) has only one zero.

(ii) Since the graph intersects the x-axis at two points, the polynomial p(x) has two zeroes.

KEY CONCEPT

▶ **Relation Between Zero(es) and Coefficient of a polynomial :**

(a) Zero of a linear polynomial ax + b, is $x = -\frac{b}{a}$

(b) If quadratic polynomial $ax^2 + bx + c = k(x - \alpha)(x - \beta)$. where k is any real constant ; then α and β are zeroes of quadratic polynomial $ax^2 + bx + c$
where a, b, c, $\in$ R and a $\neq$ 0

$$\alpha + \beta = -\frac{b}{a}$$

i.e., $\text{sum of zeros} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$

and $\alpha.\beta = \frac{c}{a}$

i.e., $\text{Product of zeros} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

(c) If cubic polynomial $ax^3 + bx^2 + cx + d = k(x-\alpha)(x-\beta)(x-\gamma)$ where k is any real constant, then α, β, and γ are zeroes of cubic polynomial.
$ax^3 + bx^2 + cx + d$ where a, b, c, d $\in R$ and $a \neq 0$

$$\text{sum of zeros} = -\frac{\text{coefficient of } x^2}{\text{coefficient of } x^3}$$

$$\text{i.e. } \alpha + \beta + \gamma = -\frac{\mathbf{b}}{\mathbf{a}};$$

$$\text{sum of the product of the zeros taken two at a time} = \frac{\text{coefficient of } x}{\text{coefficient of } x^3}$$

$$\text{i.e. } \alpha\beta + \beta\gamma + \gamma\alpha = \frac{\mathbf{c}}{\mathbf{a}};$$

$$\text{and product of the zeros} = -\frac{\text{constant term}}{\text{coefficient of } x^3}$$

$$\text{i.e. } \alpha\beta\gamma = \frac{-\mathbf{d}}{\mathbf{a}}.$$

2.3 HCF OF GIVEN POLYNOMIALS :

For two given polynomials, f(x) and g(x), r(x) can be taken as the Highest Common Factor, if

(i) r(x) is a common factor of f(x) and g(x) and

(ii) Every common factor of f(x) and g(x) is also a factor of r(x).
Highest common factor is generally referred to as HCF.

▶ **Method for finding HCF of the given polynomials :**

Step 1 : Express each polynomial as a product of powers of irreducible factors which also requires the numerical factors to be expressed as the product of the powers of primes.

Step 2 : If there is no common factor then HCF is 1 and if there are common irreducible factors, we find the least exponent of these irreducible factors in the factorized form of the given polynomials.

Step 3 : Raise the common irreducible factors to the smallest or the least exponents found in step 2 and take their product to get the HCF.

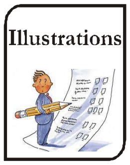


Illustration 5 : Find the HCF of $(x^2 - 6x + 5)$ and $(x^2 + 9x - 10)$.

Solution. : Factorising each expression

$$x^2 - 6x + 5 = (x-1)(x-5)$$

$$x^2 + 9x - 10 = (x-1)(x+10)$$

Since, the common factor of $x^2 - 6x + 5$ and $x^2 + 9x - 10$ is $(x - 1)$
therefore, HCF of $(x^2 - 6x + 5)$ and $(x^2 + 9x - 10)$ is $(x - 1)$.

2.4 LCM OF GIVEN POLYNOMIALS :

Least Common Multiple or the Lowest Common Multiple is the product of all the factors (taken once) of the polynomials given with their highest exponents respectively.

▶ **Method for finding LCM of the given polynomials :**

Step 1 : First express each polynomial as a product of powers of irreducible factors.

Step 2 : Consider all the irreducible factors (only once) occurring in the given polynomials. For each of these factors, consider the greatest exponent in the factorized form of the given polynomials.

Step 3 : Now raise each irreducible factor to the greatest exponent and multiply them to get the LCM.

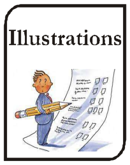


Illustration 6 : Find the LCM of $(x^2 - 6x + 5)$ and $(x^2 + 9x - 10)$.

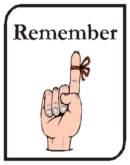
Solution : Factorising each expression

$x^2 - 6x + 5 = (x - 1)(x - 5)$

$x^2 + 9x - 10 = (x - 1)(x + 10)$

Now, LCM of $(x^2 - 6x + 5)$ and $(x^2 + 9x - 10)$ is

$(x - 1)(x - 5)(x + 10)$



Relation between the HCF, the LCM and the product of polynomials :

If f(x) and g(x) are two polynomials then we have the relation,

[HCF of f(x) and g(x)] × [LCM of f(x) and g(x)] = ± [f(x) × g(x)].

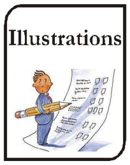


Illustration 7 : Let $f(x) = (x + 3)^2 (x - 1)(x + 2)^3$ and $g(x) = (x + 3)(x - 1)^2 (x + 2)^2$ be two polynomials. Verify that

[f(x) × g(x)] = [HCF of f(x) and g(x)] × [LCM of f(x) and g(x)]

Solution : The common factors with the least exponents are $(x + 3)$, $(x - 1)$, $(x + 2)^2$.

⇒ HCF $= (x - 1)(x + 3)(x + 2)^2$

All the factors (taken only once) with the highest exponents are $(x + 3)^2$, $(x - 1)^2$, $(x + 2)^3$.

⇒ LCM $= (x - 1)^2 (x + 2)^3 (x + 3)^2$

Now $f(x) \times g(x) = \{(x + 3)^2 (x - 1)(x + 2)^3\} \times \{(x + 3)(x - 1)^2 (x + 2)^2\}$

$= (x - 1)^3 (x + 2)^5 (x + 3)^3$

$= [(x - 1)(x + 2)^3 (x + 3)^2] [(x - 1)(x + 2)^2 (x + 3)]$

= [LCM of f(x) and g(x)] × [HCF of f(x) and g(x)]

2.5 RATIONAL EXPRESSIONS :

Rational expression is ‘an algebraic expression which is of the form $\frac{f(x)}{g(x)}$, where f(x) and g(x) are polynomials and g(x) is not a zero polynomial.

For any rational number of the form $\frac{p}{q}$, where $p,q \in z$ and $q \neq 0$. p and q are called numerator and denominator respectively. Eventhough p and q are integers $\frac{p}{q}$ need not to be an integer. Similarly for any rational expression $\frac{f(x)}{g(x)}$, f(x) is called numerator and g(x) is called denominator.

Eventhough f(x) and g(x) are polynomials $\frac{f(x)}{g(x)}$ need not to be a polynomial.

For Example :

1. $\frac{2x-1}{x^2-3x+1}$ is rational expression.
2. $\frac{x^3+5x^2-\sqrt{3}x+\sqrt{5}}{2x^2-\sqrt{5}x+8}$ is a rational expression.
3. $\frac{x^2-5\sqrt{x}-1}{3x-5}$ is not a rational expression as the numerator is not a polynomial.

IMPORTANT

Note : 1. Every polynomial is a rational expression as f(x) can be written as $\frac{f(x)}{1}$.

2. $\frac{f(x)}{g(x)}$ is not a rational expression if either numerator f(x) or denominator g(x) or both f(x) and g(x) are not polynomials.

- **Rational expressions in lowest terms :**

Let f(x) and g(x) have integer coefficients and HCF of f(x) and g(x) is 1, then the rational expression $\frac{f(x)}{g(x)}$ is said to be in its lowest terms.

▶ **Addition/subtraction of rational expressions :**

The sum of any two rational expressions $\frac{f(x)}{g(x)}$ and $\frac{h(x)}{p(x)}$ is written as

$$\frac{f(x)}{g(x)}+\frac{h(x)}{p(x)}=\frac{f(x)p(x)+h(x)g(x)}{g(x)\,p(x)}$$

If the denominators g(x) and p(x) are equal then $\frac{f(x)}{g(x)}+\frac{h(x)}{p(x)}=\frac{f(x)+h(x)}{g(x)}=\frac{f(x)+h(x)}{p(x)}$.

The difference of the above rational expressions can be written as $\frac{f(x)}{g(x)}-\frac{h(x)}{p(x)}=\frac{f(x)p(x)-h(x)g(x)}{g(x)\,p(x)}$

IMPORTANT

Note : 1. Sum or difference of two rational expressions is also a rational expression.

2. For any rational expressions $\frac{f(x)}{g(x)}$, $\frac{-f(x)}{g(x)}$ is called the additive inverse of $\frac{f(x)}{g(x)}$.

i.e., $\frac{f(x)}{g(x)}+\left(\frac{-f(x)}{g(x)}\right)=0$

Illustration 8 : Simplify $\frac{3x-1}{x+2}+\frac{2x-1}{x+1}$

Solution : $\frac{3x-1}{x+2}+\frac{2x-1}{x-1}=\frac{(3x-1)(x-1)+(2x-1)(x+2)}{(x+2)(x-1)}$

$$=\frac{3x^2-3x-x+1+2x^2+4x-x-2}{x^2-x+2x-2}=\frac{5x^2-x-1}{x^2+x-2}$$

▶ **Multiplication of rational expressions :**

The product of two rational expressions $\frac{f(x)}{g(x)}$ and $\frac{h(x)}{p(x)}$ is given by $\frac{f(x)}{g(x)}\times\frac{h(x)}{p(x)}=\frac{f(x).h(x)}{g(x).p(x)}$

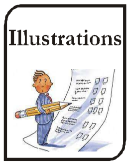
KEY CONCEPT

Note : 1. The process of finding the

(i) product of two rational expressions is similar to the process of finding the product of two rational numbers.

(ii) product of two rational expressions is also a rational expression.

2. After finding the product of two rational expressions the resultant rational expression must be put in its lowest terms.



Example 9 : Find the product of $\frac{x^2-6x+5}{x+10}$ and $\frac{x^2+12x+20}{x^2-1}$

Solution : $\frac{x^2-6x+5}{x+10} \times \frac{x^2+12x+20}{x^2-1} = \frac{(x-1)(x-5)}{(x+10)} \times \frac{(x+2)(x+10)}{(x-1)(x+1)}$

$$= \frac{(x-5)(x+2)}{(x+1)} = \frac{x^2-3x-10}{x+1}$$

▶ **Division of rational expressions :**

Let $\frac{f(x)}{g(x)}$ and $\frac{h(x)}{p(x)}$ be two non-zero rational expressions, then $\frac{f(x)}{g(x)} \div \frac{h(x)}{p(x)} = \frac{f(x)}{g(x)} \times \frac{p(x)}{h(x)}$

i.e., $\frac{f(x)\cdot p(x)}{g(x)\cdot h(x)}$ which is also a rational expression.

Note : The process of dividing two rational expressions is similar to the process of dividing two rational numbers.

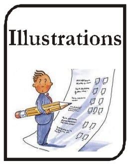


Illustration 10 : Express $\frac{(x^2+6x+5)}{(x^2-4x+3)} \div \frac{(x^2+9x+20)}{(x^2-3x+2)}$ as a rational expression in its lowest terms.

Solution : $\frac{(x^2+6x+5)}{(x^2-4x+3)} \div \frac{(x^2+9x+20)}{(x^2-3x+2)}$

Factorising quadratic expression,

$$= \frac{(x+1)(x+5)}{(x-3)(x-1)} \div \frac{(x+4)(x+5)}{(x-2)(x-1)}$$

$$= \frac{(x+1)(x+5)}{(x-3)(x-1)} \times \frac{(x-2)(x-1)}{(x+4)(x+5)} = \frac{(x+1)(x-2)}{(x-3)(x+4)} = \frac{x^2-x-2}{x^2+x-12}$$

2.6 BASIC OPERATIONS ON POLYNOMIALS :

(i) **Sum and Difference of two polynomials :** The sum and difference of two polynomials can be found by grouping like terms, taking the variable with same index common if any and then taking algebraic sum of the coefficient of like terms.

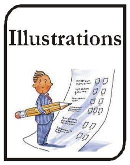


Illustration 11 : Add $p(x) = 4x^3 + 5x^2 + 2x - 1$ and $q(x) = 3x^3 - 6x^2 + 7$

Solution : $p(x) + q(x) = (4x^3 + 5x^2 + 2x - 1) + (3x^3 - 6x^2 + 7)$

$= 4x^3 + 3x^3 + 5x^2 - 6x^2 + 2x - 1 + 7$

$= 7x^3 - x^2 + 2x + 6$

Illustration 12 : Subtract $p(x) = 8x^4 - 3x^2 + x + 9$ from $q(x) = 10x^4 + 5x^3 - 2x^2 - 6x + 4$.

Solution : $q(x) - p(x) = (10x^4 + 5x^3 - 2x^2 - 6x + 4) - (8x^4 - 3x^2 + x + 9)$

$= 10x^4 - 8x^4 + 5x^3 - 2x^2 + 3x^2 - 6x - x + 4 - 9$

$= 2x^4 + 5x^3 + x^2 - 7x - 5$

(ii) Multiplication of Monomials :

Product of monomials = (Product of their numerical coefficients) × (Product of their variable parts)

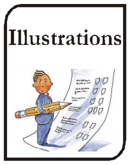


Illustration 13 : Find the product of $4x^2 yz^3$ and $3xy^2$.

Solution : $(4x^2yz^3) \times (3xy^2) = (4 \times 3) \times (x^2yz^3 \times xy^2)$

$= 12 . x^{2+1} . y^{1+2} . z^3 = 12 x^3 y^3 z^3$

(iii) Multiplication of Two Polynomials :

Multiply each term of the multiplicand by each term of the multiplier and take the algebraic sum of theses products.

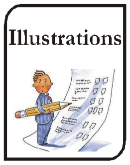


Illustration 14 : Find the product of $2x^2 + 5x + 6$ and $x - 1$.

Solution : $(2x^2 + 5x + 6) (x - 1) = 2x^2 (x -1) + 5x (x - 1) + 6 (x - 1)$

$= 2x^3 - 2x^2 + 5x^2 - 5x + 6x - 6$

$= 2x^3 + 3x^2 + x - 6$

(iv) Division of Polynomials :

On dividing a polynomial p(x) by a polynomial d(x), let the quotient be q(x) and the remainder be r(x), then $p(x) = d(x). q(x) + r(x)$, where either $r(x) = 0$ or deg. r(x) < deg. d (x).

Here, Dividend = p(x), Divisor = d(x), Quotient = q (x) and Remainder = r(x).

Note : Division algorithm helps in determining the other two zeroes of a cubic polynomial when one zero is known.

Division algorithm of a polynomial by a polynomial :

Step 1 : Arrange the terms of the dividend and the divisor in descending order of their degrees.

Step 2 : Divide the first term of the dividend by the first term of the divisor to obtain the first term of the quotient.

Step 3 : Multiply all the terms of the divisor by the first term of the quotient and subtract the result from the dividend.

Step 4 : Consider the remainder as new dividend and proceed as before.

Step 5 : Repeat this process till we obtain a remainder which is either 0 or a polynomial of degree less than that of the division.

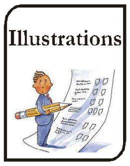


Illustration 15 : Divide p (x) = $x^3 - 3x^2 + 5x - 3$ by the polynomial g (x) = x + 2 , find the quotient and the remainder.

Solution :

$$
\begin{array}{r|l}
 & x^2 - 5x + 15 \\ \hline
x+2 & \cancel{x^3} - 3x^2 + 5x - 3 \\
 & \underset{-}{\cancel{x^3}} + \underset{-}{2x^2} \\ \hline
 & -5\cancel{x^2} + 5x - 3 \\
 & -\cancel{5}x^2 - 10x \\
 & + \qquad + \\ \hline
 & 15\cancel{x} - 3 \\
 & \underset{-}{1\cancel{5}x} \underset{-}{+} 30 \\ \hline
 & -33
\end{array}
$$

Thus quotient = $x^2 - 5x + 15$ and remainder = –33.

Illustration 16 : Find the integral zero of the polynomial : f (y) = $4y^3 - 8y^2 - y + 2$

Solution : Suppose K is an integral zero of the polynomial f(y) = $4y^3 - 8y^2 - y + 2$

Thus K is a factor of constant term 2, then possible values of k are 1, –1 and 2, – 2

Now, $f(1) = 4(1)^3 - 8(1)^2 - 1 + 2$

$= 4 - 8 - 1 + 2 = -3 \neq 0$

$\because$ $f(1) \neq 0$

$\therefore$ 1 is not a zero of f(y).

$f(-1) = 4(-1)^3 - 8(-1)^2 - (-1) + 2$

$= -4 - 8 + 1 + 2 = -9 \neq 0$

Since $f(-1) \neq 0$, therefore –1 is not a zero of f(y)

$f(2) = 4(2)^3 - 8(2)^2 - (2) + 2$

$= 32 - 32 - 2 + 2 = 0$

$\because$ f(2) = 0, therefore 2 is zero of f(y).

$f(-2) = 4(-2)^3 - 8(-2)^2 - (-2) + 2$

$= -32 - 32 + 2 + 2 = -60 \neq 0$

Since $f(-2) \neq 0$, therefore –2 is not a zero of f (y).

The only integral zero of f(y) is 2. If there are other zeroes then they are not integers.

2.7 CONCEPT OF SQUARE ROOTS :

If x is any variable then x^2 is called the square of the variable and for x^2, x is called the square root.

Square root of x^2 can be denoted as $\sqrt{x^2}$.
x and –x can both be considered as the square roots of x^2 because (x) (x) = x^2 and (–x) (–x) = x^2.

In this study we restrict $\sqrt{x^2}$ to x, i.e., + ve value of x.

▶ Methods of finding the square roots of algebraic expression other than monomials :

Here we are discussing two methods to find the square root of an algebraic expression which is not a monomial. They are :
(i) Method of division.
(ii) Method of undetermined coefficients.

(i) Method of division : We discuss the method of division to find the square root of an algebraic expression using the following example :

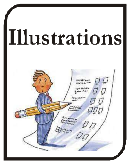


Illustration 17 : Find the square root of $x^2 – 4x + 4$.

Solution :

$$\begin{array}{r|l} & x^2-4x+4 \\ x & x^2 \\ & - \\ \hline 2x-2 & -4x+4 \\ & -4x+4 \\ & + \quad - \\ \hline & 0 \end{array}$$

therefore, $\sqrt{x^2-4x+4} = (x-2)$

Step 1 : Arrange the given expression in the descending powers of the variable x.

Step 2 : Then the square root of the first term in the expression is calculated. In the above problem first term is x^2 whose square root is x. This is now the first term of the square root of the expression.

Step 3 : Then the square of x i.e., x^2 is written below the first term of the expression and subtracted. The difference is zero. Then the next two terms in the expression – 4x + 4 are brought down as the dividend for the next step. Double the first term of the square root and put it down as the first term of the next divisor i.e., 2 (x) = 2x is to be written as the first term of the next divisor. Now the first term – 4x of the dividend – 4x + 4 is to be divided by the first term 2x (of the new divisor). Here we get –2 which is the second term of the square root of the given expression and the second term of the new divisor.

Step 4 : Thus the new divisor becomes 2x – 2. Multiply (2x – 2) by (–2) and the product –4x + 4 is to be brought down under the second dividend –4x + 4 and subtracted where we get 0.

Step 5 : Thus x – 2 is the square root of the given expression $x^2 – 4x + 4$.

Illustration 18 : Find the square root of $4x^6 - 12x^5 + 9x^4 + 8x^3 - 12x^2 + 4$.

Solution : Follow the steps indicated in the previous example.

$$\begin{array}{c|l}
2x^3 & 4x^6 - 12x^5 + 9x^4 + 8x^3 - 12x^2 + 4 \\
 & 4x^6 \\
 & - \\
\hline
4x^3 - 3x^2 & -12x^5 + 9x^4 \\
 & -12x^5 + 9x^4 \\
 & + \qquad - \\
\hline
4x^3 - 6x^2 + 2 & 8x^3 - 12x^2 + 4 \\
 & 8x^3 - 12x^2 + 4 \\
 & + \qquad - \\
\hline
 & 0
\end{array}$$

$$\therefore \sqrt{4x^6 - 12x^5 + 9x^4 + 8x^3 - 12x^2 + 4} = 2x^3 - 3x^2 + 2.$$

(ii) **Method of undetermined coefficients :** The method of undetermined coefficient to find the square root of an algebraic expression is explained in the following example.

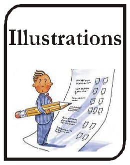


Illustration 19 : **Find the square root of $x^4 + 4x^3 + 10x^2 + 12x + 9$**

Solution : The degree of the given expression is 4, its square root will hence be an expression of degree 2.

Let the expression $ax^2 + bx + c$ to be the square root of $x^4 + 4x^3 + 10x^2 + 12x + 9$

Therefore, $\sqrt{x^4 + 4x^3 + 10x^2 + 12x + 9} = ax^2 + bx + c$

$\Rightarrow$ $x^4 + 4x^3 + 10x^2 + 12x + 9 = (ax^2 + bx + c)^2$

We know that $(p + q + r)^2 = p^2 + q^2 + r^2 + 2pq + 2qr + 2rp$

Here, $p = ax^2$, $q = bx$, $r = c$

$\Rightarrow$ $x^4 + 4x^3 + 10x^2 + 12x + 9$

$= (ax^2)^2 + (bx)^2 + c^2 + 2(ax^2)(bx) + 2(bx)(c) + 2(c)(ax^2)$

$\Rightarrow$ $x^4 + 4x^3 + 10x^2 + 12x + 9 = a^2x^4 + b^2x^2 + 2abx^3 + 2cax^2 + 2bcx + c^2$

Now equating the like terms on either sides of the equality, we have

$x^4 = a^2x^4$

$\Rightarrow$ $a^2 = 1$ $\Rightarrow$ $a = 1$

and $4x^3 = 2abx^3$ $\Rightarrow$ $2ab = 4$

$ab = 2$, but $a = 1$

$\Rightarrow$ $b = 2$

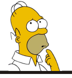
and $b^2 + 2ca = 10$ $\Rightarrow$ $2^2 + 2c(1) = 10$ $(\because a = 1, b = 2)$

$2c = 6$

$\Rightarrow$ $c = 3$

$\therefore$ The square root of the given expression is

$ax^2 + bx + c$ i.e., $x^2 + 2x + 3$.



IMPORTANT FORMULAE

Algebraic Identities :

(i) $(x + y)^2 = x^2 + y^2 + 2xy$

(ii) $(x - y)^2 = x^2 + y^2 - 2xy$

(iii) $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(iv) $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$

(v) $(x - y)^3 = x^3 - y^3 - 3xy (x - y)$

(vi) $(x^2 - y^2) = (x + y) (x - y)$

(vii) $x^3 + y^3 = (x + y) (x^2 - xy + y^2)$

(viii) $x^3 - y^3 = (x - y) (x^2 + xy + y^2)$

(ix) $(x^3 + y^3 + z^3 - 3xyz) = (x + y + z) (x^2 + y^2 + z^2 - xy - yz - zx)$

(x) Conditional Identity : if $(x + y + z) = 0$, then $x^3 + y^3 + z^3 = 3xyz$

SOLVED EXAMPLES

Example 1

Factorise :

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$ **(ii)** $a^4 + \frac{1}{a^4} - 3$

(iii) $4x^2 + \frac{1}{4x^2} + 2 - 9y^2$ **(iv)** **Factorise :** $y^4 + y^2 - 2ay + 1 - a^2$

Solution

(i)
$$\begin{aligned} 8a^3 + b^3 + 12a^2b + 6ab^2 &= (2a)^3 + b^3 + 6ab(2a + b) \\ &= (2a + b)(4a^2 + b^2 - 2ab) + 6ab(2a + b) \\ &= (2a + b)(4a^2 + b^2 + 4ab) \\ &= (2a + b)(2a + b)^2 \\ &= (2a + b)^3 \end{aligned}$$

(ii)
$$(a^2)^2 + \left(\frac{1}{a^2}\right)^2 - 2.(a^2)\left(\frac{1}{a^2}\right) - 1 = \left(a^2 - \frac{1}{a^2}\right)^2 - (1)^2 = \left(a^2 - \frac{1}{a^2} + 1\right)\left(a^2 - \frac{1}{a^2} - 1\right)$$

(iii)
$$\begin{aligned} 4x^2 + \frac{1}{4x^2} + 2 - 9y^2 &= (2x)^2 + 2.(2x).\left(\frac{1}{2x}\right) + \left(\frac{1}{2x}\right)^2 - (3y)^2 \\ &= \left(2x + \frac{1}{2x}\right)^2 - (3y)^2 = \left(2x + \frac{1}{2x} + 3y\right)\left(2x + \frac{1}{2x} - 3y\right) \end{aligned}$$

(iv) We arrange the expression in powers of a.

We have the given expression.

$$\begin{aligned} &= -a^2 - 2ay + 1 + y^2 + y^4 \\ &= -[a^2 + 2ay - y^2 - y^4 - 1] \\ &= -[a^2 + 2ay + y^2 - 2y^2 - y^4 - 1] \\ &= -[(a + y)^2 - (y^2 + 1)^2] \\ &= -[y^2 + 1 + a + y][-y^2 - 1 + a + y] \\ &= [y^2 + 1 + a + y][y^2 + 1 - a - y] \end{aligned}$$

Example 2

If $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ is a polynomial such that when it is divided by $(x - 1)$ and $(x + 1)$ the remainders are respectively 5 and 19. Determine the remainder when $f(x)$ is divided by $(x - 2)$.

Solution

When $f(x)$ is divided by $(x - 1)$ and $(x + 1)$ the remainders are 5 and 19 respectively.

$\therefore$ $f(1) = 5$ and $f(-1) = 19$

$\Rightarrow$ $1^4 - 2(1)^3 + 3(1)^2 - a(1) + b = 5$

and $(-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + b = 19$

$\Rightarrow$ $1 - 2 + 3 - a + b = 5$

and $1 + 2 + 3 + a + b = 19$

$\Rightarrow$ $-a + b = 3$ ...(i)

and $a + b = 13$ ...(ii)

From equation (i) and (ii)

We have $a = 5$ and $b = 8$

So, $f(x) = x^4 - 2x^3 + 3x^2 - 5x + 8$

The remainder when $f(x)$ is dividing by $(x - 2)$ is equal to

$$= f(2) = 2^4 - 2(2^3) + 3(2)^2 - 5(2) + 8 = 16 - 16 + 12 - 10 + 8 = 10$$

Example 3

If both $x - 2$ and $x - \frac{1}{2}$ are factors of $px^2 + 5x + r$, show that $p = r$.

Solution

Let $f(x) = px^2 + 5x + r$ be the given polynomial. Since $x - 2$ and $x - \frac{1}{2}$ are factors of f(x).

$\therefore \quad f(2) = 0$ and $f\left(\frac{1}{2}\right) = 0 \qquad [\because x - 2 = 0 \Rightarrow x = 2$ and $x - \frac{1}{2} = 0 \Rightarrow x = \frac{1}{2}]$

$\Rightarrow \quad p \times 2^2 + 5 \times 2 + r = 0$ and $p\left(\frac{1}{2}\right)^2 + 5 \times \frac{1}{2} + r = 0$

$\Rightarrow \quad 4p \times 10 + r = 0$ and $\frac{p}{4} + \frac{5}{2} + r = 0$

$\Rightarrow \quad 4p + r = -10$ and $\frac{p + 4r + 10}{4} = 0$

$\Rightarrow \quad 4p + r = -10$ and $p + 4r + 10 = 0$

$\Rightarrow \quad 4p + r = -10$ and $p + 4r = -10$

$\Rightarrow \quad 4p + r = p + 4r \qquad [\because$ RHS of the two equations are equal]

$\Rightarrow \quad 3p = 3r$

$\Rightarrow \quad p = r$

Example 4

What must be subtracted from $8x^4 + 14x^3 - 2x^2 + 7x - 8$ so that the resulting polynomial is exactly divisible by $4x^2 + 3x - 2$.

Solution

We know that

Dividend = Quotient × Divisor + Remainder

$\Rightarrow$ Dividend – Remainder = Quotient × Divisor

Clearly, RHS of the above result is divisible by the divisor.

Therefore, LHS is also divisible by the divisor. Thus, if we subtract remainder from the dividend, then it will be exactly divisible by the divisor.

Dividing $8x^4 + 14x^3 - 2x^2 + 7x - 8$ by $4x^2 + 3x - 2$, we get

$$\begin{array}{r|l}
 & 2x^2 + 2x - 1 \\
\hline
4x^2 + 3x - 2 & 8x^4 + 14x^3 - 2x^2 + 7x - 8 \\
 & 8x^4 + 6x^3 - 4x^2 \\
 & - \quad - \quad + \\
\hline
 & 8x^3 + 2x^2 - 7x - 8 \\
 & 8x^3 + 6x^2 - 4x \\
 & - \quad - \quad + \\
\hline
 & -4x^2 + 11x - 8 \\
 & -4x^2 - 3x + 2 \\
 & + \quad + \quad - \\
\hline
 & 14x - 10
\end{array}$$

$\therefore$ Quotient $= 2x^2 + 2x - 1$ and Remainder $= 14x - 10$

Thus, if we subtract the remainder $14x - 10$ from $8x^4 + 14x^3 - 2x^2 + 7x - 8$, it will be divisible by $4x^2 + 3x - 2$.

Example 5

Find the zeros of the quadratic polynomial $f(x) = abx^2 + (b^2 - ac)x - bc$, and verify the relationship between the zeros and its coefficients

Solution

We have,

$$f(x) = abx^2 + (b^2 - ac)x - bc$$

$$\Rightarrow \quad f(x) = abx^2 + b^2x - acx - bc$$

$$\Rightarrow \quad f(x)\ bx(ax + b) - c\,(ax + b)$$

$$\Rightarrow \quad f(x) = (ax + b)(bx - c)$$

The zeros of f(x) are given by

$$f(x) = 0$$

$$\Rightarrow \quad (ax + b)\,(bx - c) = 0$$

$$\Rightarrow \quad ax + b = 0 \text{ or, } bx - c = 0$$

$$\Rightarrow \quad x = -\frac{b}{a} \text{ or, } x = \frac{c}{b}$$

Thus, the zeros of (x) are : $\alpha = -\frac{b}{a}$ and $\beta = \frac{c}{b}$

Now, $\alpha + \beta = -\frac{b}{a} + \frac{c}{b} = \frac{ac - b^2}{ab}$ and $\alpha\beta = -\frac{b}{a} \times \frac{c}{b} = -\frac{c}{a}$

Also, $-\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\left(\frac{b^2 - ac}{ab}\right) = \frac{ac - b^2}{ab}$

And, $\frac{\text{Constan t trem}}{\text{Coefficient of } x^2} = -\frac{bc}{ab} = -\frac{c}{a}$

Hence, Sum of the zeros $= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

And, Product of the zeros $= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Example 6

Draw the graph of the polynomial $f(x) = -4x^2 + 4x - 1$. Also, find the vertex of this parabola.

Solution

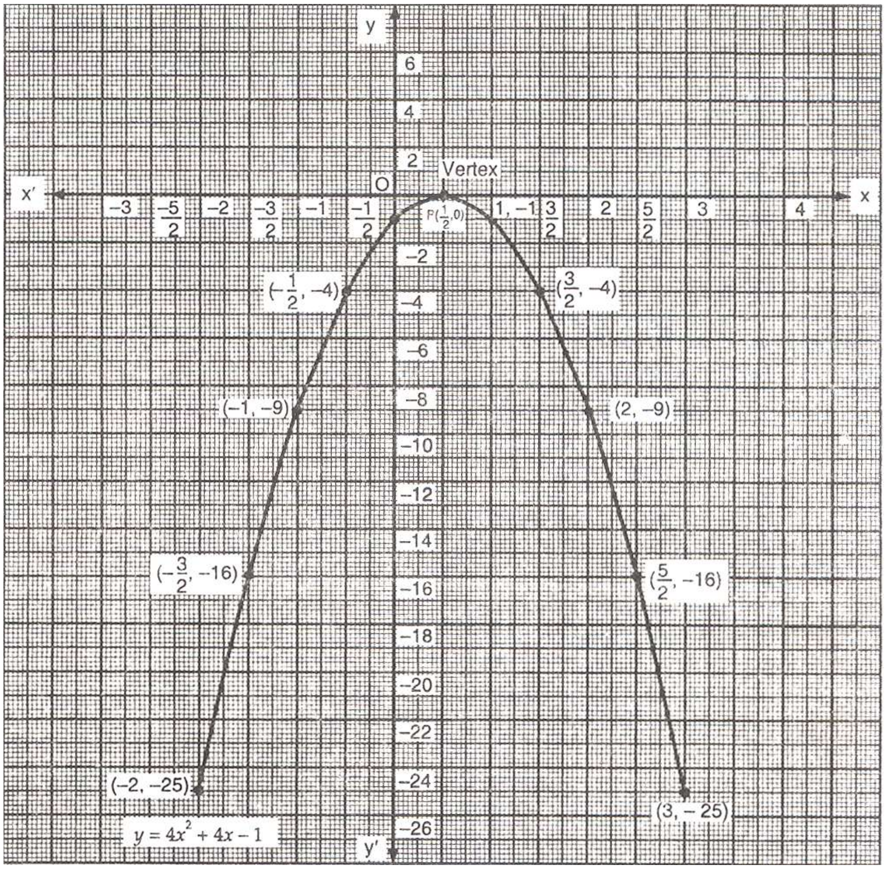
Let $y = f(x)$ or, $y = -4x^2 + 4x - 1$

The following table gives the values of y for various values of x.

x	– 2	–3/2	–1	–1/2	0	1/2	1	3/2	2	5/2	3
$y = -4x^2 + 4x - 1$	– 25	–16	–9	–4	–1	0	–1	–4	–9	–16	–25

Thus the following points lie on the graph of $y = -4x^2 + 4x - 1$:

(– 2, 25), (–3/2, – 16), (– 1, – 9), (– 1/2, – 4), (0, – 1), (1/2, 0), (1, – 1), (3/2, – 4), (2, – 9), (5/2, –16), (3, – 25) etc.



Observations : From the graph of the polynomial $f(x) = -4x^2 + 4x - 1$, we make the following observations

(i) The coefficient of x^2 in $f(x) = -4x^2 + 4x - 1$ is $-4m$ a negative real number and so the parabola opens downwards.

(ii) The polynomial $f(x) = -4x^2 + 4x - 1 = -(2x - 1)^2$ is factorizable into two equal factors equal to $2x - 1$. So, the parabola cuts X-axis at two coincident points having coordinates (1/2, 0).

(iii) The polynomial $f(x) = -4x^2 + 4x - 1$ has two equal roots each equal to 1/2. So, the parabola touches X-axis at one point (1/2, 0) only i.e. it cuts X-axis at coincident points.

(iv) On comparing the polynomial $-4x^2 + 4x - 1$ with $ax^2 + bx + c$, we get $a = -4$, $b = 4$ and $c = 1$.
The vertex of the parabola has the coordinates (1/2, 0) i.e. $\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$,
where $D = b^2 - 4ac$.

(v) $d = b^2 - 4ac = 4 - 4 = 0$. So, the parabola touches X-axis.

Example 7

Obtain all the zeros of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeros are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Solution

$f(x) = 3x^4 + 6x^2 - 2x^2 - 10x - 5$

Given : $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are the zeros of f (x).

$\therefore$ $\left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right)$ are the factors of f (x).

$\left(x^2 - \frac{5}{3}\right)$ are the factor of f (x) or $(3x^2 - 5)$ are the factor of f (x).

$$
\begin{array}{r|l}
 & \quad x^2 + 2x + 1 \\
\hline
3x^2 - 5 & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \;(\\
 & \underset{-}{3x^4} \quad \underset{+}{-} 5x^2 \\
\hline
 & \quad 6x^3 + 3x^2 - 10x - 5 \\
 & \quad \underset{-}{6x^3} \quad \underset{+}{-} 10x \\
\hline
 & \qquad 3x^2 - 5 \\
 & \qquad \underset{-}{3x^2} \underset{+}{-} 5 \\
\hline
 & \qquad\quad 0
\end{array}
$$

So, $3x^4 + 6x^3 - 2x^2 - 10x - 5 = (3x^2 - 5)(x^2 + 2x + 1)$

$$= 3\left(x^2 - \frac{5}{3}\right)(x + 1)^2 = 3\left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right)(x + 1)^2$$

$\therefore$ Zeros are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -1$ and -1.

Example 8

If α, β are zeros of quadratic polynomial $kx^2 + 4x + 4$, find the value of k such that $(\alpha + \beta)^2 - 2\alpha\beta = 24$.

Solution

$f(x) = kx^2 + 4x + 4$

$\alpha + \beta = -\frac{4}{k}, \alpha\beta = \frac{4}{k}$

Now, $(\alpha + \beta)^2 - 2\alpha\beta = 24$

$\Rightarrow \left(-\frac{4}{k}\right)^2 - 2\left(\frac{4}{k}\right) = 24 \qquad \Rightarrow \frac{16}{k^2} - \frac{8}{k} = 24$

$\Rightarrow 24k^2 + 8k - 16 = 0 \qquad \Rightarrow 3k^2 + k - 2 = 0$

$\Rightarrow 3k^2 + 3k - 2k - 2 = 0 \qquad \Rightarrow 3k(k + 1) - 2(k + 1) = 0$

$\Rightarrow (k + 1)(3k - 2) = 0 \qquad \Rightarrow k = -1$ or $k = \frac{2}{3}$.

Example 9

The polynomials $ax^3 + 3x^2 - 3$ and $2x^3 - 5x + a$ when divided by $(x - 4)$ leaves remainders R_1 & R_2 respectively then value of 'a' if $2R_1 - R_2 = 0$.

Solution

$R_1 = a(4)^3 + 3(4)^2 - 3$

$R_1 = 64a + 45$

$R_2 = 2(4)^3 - 5(4) + a = 128 - 20 + a = 108 + a$

Given: $2R_1 - R_2 = 0$

$2(64a + 45) - (108 + a) = 0$

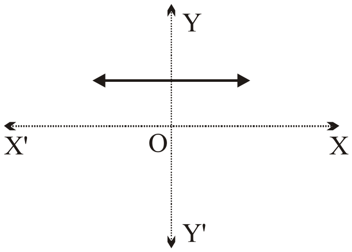
$128a + 90 - 108 - a = 0$

$127a = 18 \qquad \Rightarrow \qquad a = \frac{18}{127}$

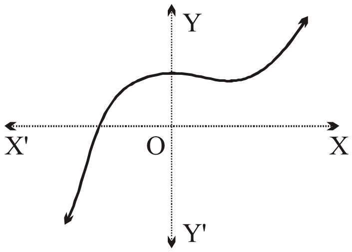
CONCEPT APPLICATION LEVEL - I [NCERT Questions]

Q.1 The graphs of y = p(x) are given in figure for some polynomials p(x). Find the number of zeroes of p(x), in each case.

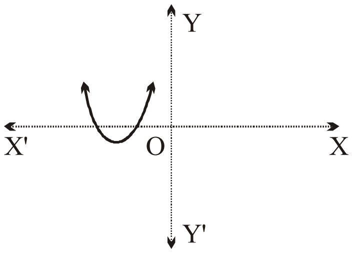
(i)



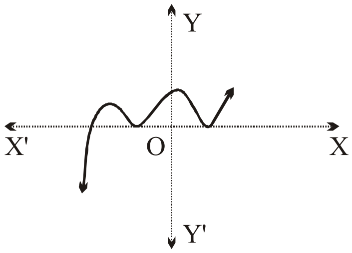
(ii)



(iii)



(iv)



Sol. (i) no zero (ii) one zeroe (iii) two zeroes (iv) three zeroes

Q.2 Find the Zeroes of the quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeroes and the coefficients.

Sol. $6x^2 - 7x - 3 = 6x^2 - 9x + 2x - 3$

$= 3x(2x - 3) + 1(2x - 3)$

$= (2x - 3)(3x + 1)$

$\because$ $(2x - 3)$ and $(3x + 1)$ are the factors of $6x^2 - 7x - 3$ therefore, $\frac{3}{2}$ and $-\frac{1}{3}$ are the zeroes of $6x^2 - 7x - 3$,

Now, relationship between the zeroes and the coefficients :

(i) Sum of zeroes $= \frac{3}{2} + \left(-\frac{1}{3}\right) = \frac{3}{2} - \frac{1}{3} = \frac{9-2}{6} = \frac{7}{6} = \frac{-(-7)}{6}$

Sum of zeroes $= \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$

(ii) product of zeroes $= \left(\frac{3}{2}\right)\left(-\frac{1}{3}\right) = -\frac{1}{2} = \frac{-3}{6}$

product of zeroes $= \frac{\text{constant term}}{\text{coefficient of } x^2}$

Q.3 Find a quadratic polynomial with $\frac{1}{4}$ and –1 as the sum and product of its zeroes respectively.

Sol. sum of zeroes $= \frac{1}{4}$

product of zeroes $= -1$

quadratic polynomial $= x^2 - (\text{sum of zeroes})\, x + \text{product of zeroes}$

$$= x^2 - \left(\frac{1}{4}\right)x + (-1) = x^2 - \frac{1}{4}x - 1$$

$$= \frac{4x^2 - x - 4}{4} = \frac{1}{4}\,(4x^2 - x - 4)$$

or quadratic polynomial $= 4x^2 - x - 4$

Q.4 Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in the following :
$p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$

Sol.

$$\begin{array}{r|l|l}
x^2 - x + 1 & x^4 - 3x^2 + 4x + 5 & x^2 + x - 3 \\
 & x^4 + x^2 \qquad\quad - x^3 & \\
 & - \quad - \qquad\qquad + & \\
\hline
 & \quad x^3 - 4x^2 + 4x + 5 & \\
 & \quad x^3 - \ \ x^2 + \ x & \\
 & \quad - \quad + \qquad - & \\
\hline
 & \qquad - 3x^2 + 3x + \ 5 & \\
 & \qquad - 3x^2 + 3x - \ 3 & \\
 & \qquad + \qquad - \quad + & \\
\hline
 & \qquad\qquad 8 & \\
\hline
\end{array}$$

quotient $= x^2 + x - 3$

remainder $= 8$

Q.5 Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial : $x^3 - 3x + 1$, $x^5 - 4x^3 + x^2 + 3x - 1$

Sol.

$$\begin{array}{r|l|l}
x^3 - 3x + 1 & x^5 - 4x^3 + x^2 + 3x - 1 & x^2 - 1 \\
 & x^5 - 3x^3 + x^2 & \\
 & - \quad + \quad - & \\
\hline
 & \quad - x^3 + 3x - 1 & \\
 & \quad - x^3 + 3x - 1 & \\
 & \quad + \quad - \quad + & \\
\hline
 & \qquad\quad 0 & \\
\hline
\end{array}$$

∵ remainder is 0 therefore, $x^3 - 3x + 1$ is a factor of $x^5 - 4x^3 + x^2 + 3x - 1$

Q.6 Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Sol. Two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ therefore, $\left(x - \sqrt{\frac{5}{3}}\right)$ and $\left(x + \sqrt{\frac{5}{3}}\right)$ are two factors of

$3x^4 + 6x^3 - 2x^2 - 10x - 5$.

now $\left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = x^2 - \left(\sqrt{\frac{5}{3}}\right)^2 = x^2 - \frac{5}{3} = \frac{1}{3}(3x^2 - 5)$

Now dividing $3x^4 + 6x^3 - 2x^2 - 10x - 5$ by $(3x^2 - 5)$,

$$
\begin{array}{r|l|l}
3x^2 - 5 & 3x^4 + 6x^3 - 2x^2 - 10x - 5 & x^2 + 2x + 1 \\
 & 3x^4 \qquad - 5x^2 & \\
 & - \qquad\quad + & \\
\hline
 & \quad + 6x^3 + 3x^2 - 10x - 5 & \\
 & \quad + 6x^3 \qquad - 10x & \\
 & \quad - \qquad\quad + & \\
\hline
 & \qquad 3x^2 - 5 & \\
 & \qquad 3x^2 - 5 & \\
 & \qquad - \quad + & \\
\hline
 & \qquad\quad 0 & \\
\hline
\end{array}
$$

Quotient $= x^2 + 2x + 1 = (x + 1)^2 = (x + 1)(x + 1)$
therefore, other two zeroes are $-1, -1$.

Q.7 On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

Sol. Division algorithm :

dividend = quotient × divisor + remainder

$x^3 - 3x^2 + x + 2 = (x - 2) \times g(x) + (-2x + 4)$

or $g(x) = \dfrac{x^3 - 3x^2 + x + 2 + 2x - 4}{x - 2}$

or $g(x) = \dfrac{x^3 - 3x^2 + 3x - 2}{x - 2} = \dfrac{(x^2 - x + 1)(x - 2)}{(x - 2)} = x^2 - x + 1$

Q.8 Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, –7, – 14 respectively.

Sol. Sum of zeroes = 2

Sum of the product of the zeroes taken two at a time = – 7

product of zeroes = –14

Cubic polynomial

$= x^3 -$ (sum of zeroes) $x^2 +$ (sum of the product of the zeroes taken two at a time) $x -$ product of zeroes

$= x^3 - 2x^2 + (-7)x - (-14)$

$= x^3 - 2x^2 - 7x + 14$

Q.9 If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are $a - b, a, a + b$, find a and b.

Sol. $(a - b)$, a and $(a + b)$ are zeroes of $x^3 - 3x^2 + x + 1$

or $x^3 - 3x^2 + x + 1 = x^3 - [(a - b) + (a) + (a + b)]\, x^2 + [(a - b)(a) + (a)(a + b) + (a + b)(a - b)]x - (a-b)(a)(a + b)$

$= x^3 - (3a)\, x^2 + (a^2 - ab + a^2 + ab + a^2 - b^2)\, x - a(a^2 - b^2)$

or $x^3 - 3x^2 + x + 1 = x^3 - 3ax^2 + (3a^2 - b^2)x - a(a^2 - b^2)$

Comparing both sides, we get.

$-3a = -3$

$\Rightarrow$ $a = 1$ [on comparing coefficients of x^2]

and $3a^2 - b^2 = 1$ [on comparing coefficients of x]

$\Rightarrow$ $3(1)^2 - b^2 = 1 \Rightarrow 3 - b^2 = 1 \Rightarrow b = \pm\sqrt{2}$

$a = 1, b = \pm\sqrt{2}$ Ans.

Q.10 If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + K$, the remainder comes out to be $x + a$, find K and a.

Sol.

$$\begin{array}{r|l|l}
x^2 - 2x + K & x^4 - 6x^3 + 16x^2 - 25x + 10 & x^2 - 4x + (8 - K) \\
 & x^4 - 2x^3 + Kx^2 & \\
 & - \quad + \quad - & \\
\hline
 & -4x^3 + (16 - K)\, x^2 - 25x + 10 & \\
 & -4x^3 + 8x^2 \qquad -4Kx & \\
 & + \quad - \qquad + & \\
\hline
 & (8-K)\, x^2 + (4K - 25)\, x + 10 & \\
 & (8-K)\, x^2 - 2(8 - K)\, x + K(8 - K) & \\
 & - \quad + \quad - & \\
\hline
 & (-25 + 16 + 2K)\, x + 10 - 8K + K^2 & \\
\text{Remainder} = & (-9 + 2K)\, x + (K^2 - 8K + 10) &
\end{array}$$

or $(-9 + 2K)x + (K^2 - 8K + 10) = x + a$ (given)

Comparing both sides,

$-9 + 2K = 1$ (on comparing coefficients of x)

$2K = 10$

or $K = 5$

and $K^2 - 8K + 10 = a$ (On comparing constant terms)

$(5)^2 - 8 \times 5 + 10 = a$

or $a = 25 - 40 + 10$

$a = 35 - 40$ or $a = -5$

CONCEPT APPLICATION LEVEL - II [Previous Year Questions]

Multiple choice questions with one correct answer.(From Q.1 to Q. 13)

Q.1 Which polynomial represents $(3x^2 + x - 4)(2x - 5)$?

(A) $6x^3 - 13x^2 - 13x - 20$ (B) $6x^3 - 13x^2 - 13x + 20$

(C) $6x^3 - 13x^2 + 13x - 20$ (D) $6x^3 + 13x^2 + 3x + 20$ **(IMO, 2011)**

Q.2 $2x + 7\overline{\big)\,2x^4 + 21x^3 + 35x^2 - 37x + 46}\ =$

(A) $x^3 - 7x^2 - 7x + 6 - \dfrac{4}{2x+7}$ (B) $2x^3 + 14x^2 - 14x + 12 - \dfrac{4}{2x+7}$

(C) $x^3 - 7x^2 + 7x - 6 + \dfrac{4}{2x+7}$ (D) $x^3 + 7x^2 - 7x + 6 + \dfrac{4}{2x+7}$ **(IMO, 2011)**

Q.3 If α and β are the zeros of the polynomial $f(x) = x^2 - 5x + K$ such that $\alpha - \beta = 1$, then find the value of K.

(A) 5 (B) 6 (C) 7 (D) 2 **(IMO, 2012)**

Q.4 When $6x^3 + 11x^2 - 39x - 65$ is divided by $(x^2 - 1 + x)$ then the remainder is :

(A) $-38x - 60$ (B) $-40x + 5$ (C) $-21x - 20$ (D) $31x + 60$ **(IMO, 2012)**

Q.5 Find a quadratic polynomial whose zeroes are $(2\alpha + 1)$ and $(2\beta + 1)$ if α and β are the zeroes of the polynomial $f(t) = 2t^2 - 7t + 6$. **(IMO, 2012)**

(A) $2t^2 - 9t + 10$ (B) $t^2 - 9t + 20$ (C) $t^2 - 7t + 10$ (D) $2t^2 - 7t + 10$

Q.6 Simplify : $\dfrac{3}{(x-1)} - \dfrac{2}{x} + \dfrac{(x-3)}{(x-1)(x-1)}$ **(NIMO, Level-I)**

(A) $\dfrac{2x^2+6x+2}{x^3-x}$ (B) $\dfrac{2x^2+6x+2}{x-1}$ (C) $\dfrac{2x^2+6x}{x-1}$ (D) None of these

Q.7 The zeroes of the polynomial $f(x) = 3x^2 - x - 4$ are : **(NIMO 2011)**

(A) $-\dfrac{4}{3}, \dfrac{-4}{3}$ (B) $\dfrac{4}{3}, -1$ (C) $\dfrac{-4}{3}, \dfrac{4}{3}$ (D) $\dfrac{-4}{3}, -1$

Q.8 The cubic polynomial whose three roots are 4, –2 and –4 is : **(NIMO 2011)**

(A) $x^3 + 2x^2 - 16x - 32$ (B) $x^3 + 2x^2 + 16x + 32$

(C) $x^3 + 2x^2 - 16x + 32$ (D) $x^3 + 2x^2 + 16x - 32$

Q.9 Divide the polynomial $g(x) = x^3 - 3x^2 + 3x - 5$ by the polynomial $h(x) = x^2 + x + 1$ and the quotient and remainder are : **(NIMO 2011)**

(A) $(x - 4, 6x - 1)$ (B) $(x + 4, 6x + 1)$ (C) $(x^2 + 1, 3x + 2)$ (D) $(x^2 + 1, x - 2)$

Q.10 If a quadratic polynomial of the form $x^2 + ax + b$ has no linear term and the constant term is negative, then: **(5th NIMO Level-I)**

(A) one of the zeroes is reciprocal of the other (B) one of the zeroes is negative of the other.

(C) one of the zeroes is twice of the other (D) one of the zeroes is half of the other

Q.11 If $x = \sqrt{7} - \sqrt{5}$, $y = \sqrt{5} - \sqrt{3}$ and $z = \sqrt{3} - \sqrt{7}$, then the value of $x^3 + y^3 + z^3 - 2xyz$ is :

(A) $-4\sqrt{5} - 12\sqrt{3} + \sqrt{7}$ (B) $-4\sqrt{5} + 2\sqrt{3} + 2\sqrt{7}$

(C) $4\sqrt{5} + 12\sqrt{3} + 2\sqrt{7}$ (D) $4\sqrt{5} - 12\sqrt{3} + \sqrt{7}$ **(5th NIMO Level-I)**

Q.12 If x, y and z are real numbers such that $x + y + z = 5$ and $xy + yz + zx = 3$, what is the largest value that x can have ? **(5th NIMO Level-I)**

(A) $\frac{5}{3}$ (B) $\sqrt{19}$ (C) $\frac{13}{3}$ (D) None of these

Q.13 Graph drawn from the equation $y = x^2 - 3x - 4$ will be : **(NTSE 2013)**

(A) Circle (B) Parabola (C) Straight Line (D) Hyperbola

Q.14 Let $p_1(x) = ax^2 - bx - c$, $p_2(x) = bx^2 - cx - a$, $p_3(x) = cx^2 - ax - b$ be three quadratic polynomials where a, b, c are non-zero real numbers. Suppose there exists a real number such that $p_1(\alpha) = p_2(\alpha) = p_3(\alpha)$. Prove that $a = b = c$. **(RMO 2010)**

Q.15 If the zero of the polynomial $f(x) = k^2x^2 - 17x + k + 2 (k > 0)$ are reciprocal of each other, then the value of k is **[Delhi NTSE Stage-1_2013]**

(A) 2 (B) –1 (C) –2 (D) 1

Q.16 If x, y, z are positive real numbers and a, b, c are rational numbers, then the value of

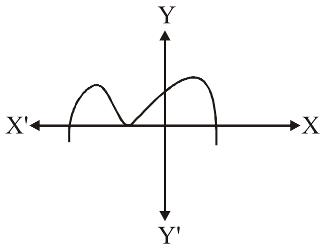
$$\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{b-c}+x^{a-c}}$$ **[Raj.NTSE Stage-1_2014]**

(A) –1 (B) 0 (C) 1 (D) None of these

Q.17 If $x^2 - x - 1 = 0$, then the value of $x^3 - 2x + 1$ is **[Harayana NTSE Stage-1_2014]**

(A) 0 (B) 2 (C) $\frac{1+\sqrt{5}}{2}$ (D) $\frac{1-\sqrt{5}}{2}$

Q.18 The graph of $y = p(x)$ is given below. The number of zeroes of polynomial p(x), is **[Raj. NTSE Stage-1_2015]**



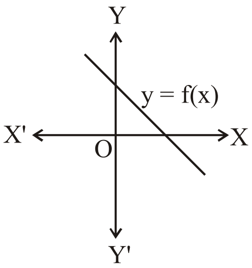
(A) 3 (B) 2 (C) 1 (D) 0

Q.19 On dividing $6x^3 + 8x^2 - 3x + 8$ by a polynomial $g(x)$, the quotient and remainder were $3x + 4$ and $6x + 20$, respectively. Find $g(x)$ **[IMO - 2016]**

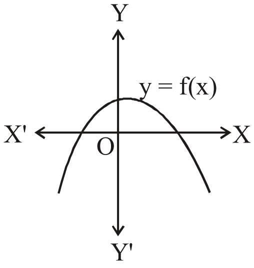
(A) $2x - 3$ (B) $2x^2 + 4$ (C) $3x^2 - 4$ (D) $2x^2 - 3$

Q.20 Find the number of zeroes of $f(x)$, in each case **[IMO - 2016]**

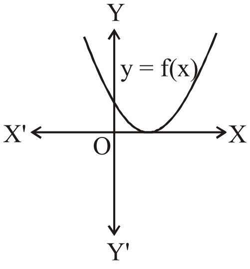
(i)



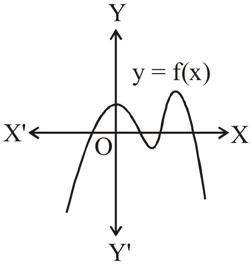
(ii)



(iii)



(iv)



	(i)	(ii)	(iii)	(iv)
(A)	1	2	1	4
(B)	1	2	4	1
(C)	4	2	1	4
(D)	1	2	3	4

Q.21 The cube root of $x + y + 3x^{\frac{1}{3}}y^{\frac{1}{3}}\left(x^{\frac{1}{3}} + y^{\frac{1}{3}}\right)$ is **[NTSE - 2016]**

(A) $x + y$ (B) $x^{\frac{1}{3}} + y^{\frac{1}{3}}$ (C) $(x + y)^{\frac{1}{3}}$ (D) $(x + y)^3$

Q.22 If $\left(x + \sqrt{2}\right)$ is a factor $kx^2 - \sqrt{2}\,x + 1$, then the value of k is **[NTSE - 2016]**

(A) $\frac{-3}{2}$ (B) $\frac{-2}{3}$ (C) $\frac{3}{2}$ (D) $\frac{2}{3}$

Q.23 If $a = x - y$, $b = y - z$ and $c = z - x$ then the value of $a^3 + b^3 + c^3$ is **[NTSE - 2016]**

(A) $3\,(x - y)\,(y - z)\,(z - x)$ (B) $(x - y)^3\,(y - z)^3\,(z - x)^3$

(C) $(x + y + z)^3$ (D) $x^3 + y^3 + z^3$

CONCEPT APPLICATION LEVEL - III

SECTION-A

- **Fill in the blanks**

1. A polynomial of degree 1 is called _________.
2. A quadratic polynomial can have _________ zeros.
3. If α, β, γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d = 0$ then $\alpha\beta + \beta\gamma + \alpha\gamma =$ _________
4. Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a _________ open. If $a < 0$.
5. If discriminant $D < 0$, a quadratic polynomial $ax^2 + bx + c$ has ________ real roots.

SECTION-B

- **Multiple choice questions with one correct answer**

Q.1 Zero of $p(x) = x^2 - 2x - 3$ is :

(A) 0 (B) 1 (C) –1 (D) –3

Q.2 If one root of the polynomial $5x^2 + 13x + K$ is reciprocal of the other, then the value of k is :

(A) 0 (B) 5 (C) 6 (D) 1/6

Q.3 If α, β are the zeroes of the polynomial $f(x) = x^2 - p(x + 1) - c$ then $(\alpha + 1)(\beta + 1)$ is equal to :

(A) $1 + c$ (B) $1 - c$ (C) $c - 1$ (D) c

Q.4 Quadratic polynomial having zeroes 1 and –2 is :

(A) $x^2 - x + 2$ (B) $x^2 - x - 2$ (C) $x^2 + x - 2$ (D) $x^2 + x + 2$

Q.5 If a, b, c are three natural numbers such that c is a factor of ab and c is coprime to a then :

(A) b is a factor of c (B) c is a factor of b (C) a is a factor of b (D) b is a factor of a

Q.6 If deg p(x) = m and deg q(x) = n, then deg [p(x) – q(x)] equal to:

(A) max {m, n} (B) min {m, n} (C) m + n (D) m – n

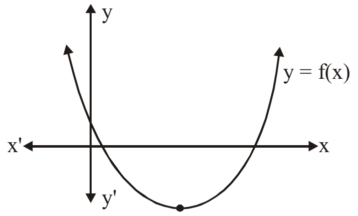
Q.7 p(x) and q(x) are two reducible (factorisable) unequal polynomial with real coefficient and neither of them is a factor of the other. If ℓ and h are their LCM and HCF respectively, then ℓ and h must satisfy the equality:

(A) $\ell p(x) = hq(x)$ (B) $hp(x) = \ell q(x)$ (C) $p(x)q(x) = \ell h$ (D) $\ell h = 1$

Q.8 The G.C.D. of $(x + a)^2$ and $x^3 + a^3$ is :

(A) $(x + a)^2$ (B) $x^2 - a^2$ (C) $x + a$ (D) $x^2 + a^2$

Q.9 The graph of the polynomial $f(x) = ax^2 + bx + c$ is as shown below, then which of the following is true :



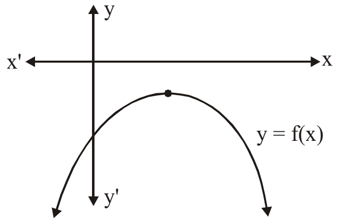
(A) $a > 0, b < 0, c > 0, D > 0$

(B) $a > 0, b > 0, c > 0, D > 0$

(C) $a < 0, b < 0, c > 0, D > 0$

(D) $a < 0, b > 0, c < 0, D < 0$

Q.10 The graph of the polynomial $f(x) = ax^2 + bx + c$ is as shown in fig, then which of the following is true :



(A) $a > 0, b > 0, c < 0, D < 0$

(B) $a < 0, b > 0, c > 0, D < 0$

(C) $a < 0, b > 0, c < 0, D < 0$

(D) $a < 0, b < 0, c < 0, D > 0$

Q.11 If α and β are the zeroes of the polynomials $f(x) = x^2 - 5x + k$ such that $\alpha - \beta = 1$, then value of k is

(A) 4 (B) –6 (C) 6 (D) 12

Q.12 If zeroes of the polynomial $f(x) = x^3 - 3px^2 + qx - r$ are in A.P, then :

(A) $2p^3 = pq - r$ (B) $2p^3 = pq + r$ (C) $p^3 = pq - r$ (D) None of these

Q.13 If the sum of two roots of the polynomial $f(x) = x^3 - px^2 + qx - r$ is zero, then which of the following condition holds good :

(A) $p^3 = r^3$ (B) $pq = r$ (C) $p^3 - pr + r = 0$ (D) $p^2q^2 = r^3$

Q.14 The coordinates of the vertex of the parabola $4u^2 + 8u$ are :

(A) (–1, 0) (B) (–1, –4) (C) (1, 4) (D) (–1, 4)

Q.15 The product of the zeroes of the polynomial $bx(ax + b) - c(ax + b)$ is :

(A) c/b (B) $\frac{ac - b^2}{ab}$ (C) $-c/a$ (D) $-b/a$

SECTION-C

- **Multiple choice questions with one or more than one correct answers**

Q.1 If the zeroes of a quadratic polynomial $ax^2 + bx + c$ are equal but opposite in sign then :

(A) $a = b - c$ (B) $c = 0$ (C) $b = a$ (D) $b = 0$

Q.2 If the zeroes of a quadratic polynomial $\alpha x^2 + \beta x + \gamma$ are reciprocals of each other then :

(A) $\beta = \alpha$ (B) $\gamma = \alpha$ (C) $\gamma - \alpha = 0$ (D) $\gamma = \alpha + \beta$

Q.3 The polynomials of degree 2 is/are :

(A) $xy + 1$ (B) $x^2 + ax + c$ (C) $x^2 + 5x + y^3$ (D) $xy + yz + xz$

Q.4 If $(x + a)$ is the factor of the polynomials $x^2 + px + q$ and $x^2 + mx + n$, then which of the following is/are true :

(A) $am - pa - n + q = 0$ (B) $am - pm = an + q$

(C) $a = \dfrac{n - q}{m - p}$ (D) $a = \dfrac{m - q}{n - p}$

Q.5 If $(x^2 - 1)$ is a factor of $ax^4 + bx^3 + cx^2 + dx + e$, then which of the following is/are correct :

(A) $a + c + e = b + d$ (B) $a + b + c + d + e = 0$

(C) $a + b + e = c + d$ (D) $a + c + e = 0$

Q.6 What are the value of a for which $3x^5 + 9x^4 - 7x^3 - 5x^2 - 3ax + 3a^2$ is divisible by $x - 1$?

(A) 0 (B) 2 (C) –1 (D) 1

Q.7 What is the remainder when $3x^3 - 2x^2y - 13xy^2 + 10y^3$ is divided by $x - 2y$?

(A) $x + y$ (B) 0 (C) $0x^9$ (D) $x + 2y$

Q.8 If $x - y = 1$ and $x^2 + y^2 = 41$, then the value of $x + y$ will be :

(A) 5 (B) 9 (C) 4 (D) –9

Q.9 If $(x + a)$ is the HCF of $x^2 + px + q$ and $x^2 + \ell x + m$, then the value of 'a' is given by :

(A) $\dfrac{q - m}{p - \ell}$ (B) $\dfrac{p - \ell}{q - m}$ (C) $\dfrac{q + m}{p + \ell}$ (D) $\dfrac{-(m - q)}{p - \ell}$

Q.10 If the polynomial $f(x) = ax^3 + bx - c$ is divisible by the polynomial $g(x) = x^2 + bx + c$, then :

(A) $ab = 1$ (B) $ac - 2b^2 = 0$ (C) $ac = 2b^2$ (D) $ab + 1 = 0$

SECTION-D

- **Assertion & Reason (From Q. 1 to Q. 3)**

Instructions: In the following questions an Assertion (A) is given followed by a Reason (R). Mark your responses from the following options.

(A) Both Assertion and Reason are true and Reason is the correct explanation of 'Assertion'

(B) Both Assertion and Reason are true and Reason is not the correct explanation of 'Assertion'

(C) Assertion is true but Reason is false

(D) Assertion is false but Reason is true

Q.1 **Assertion:** If α, β, γ are the zeroes of $x^3 - 2x^2 + qx - r$ and $\alpha + \beta = 0$, then $2q = r$.

Reason: If α, β, γ are the zeroes of $ax^3 + bx^2 + cx + d$, then $\alpha + \beta + \gamma = \frac{-b}{a}$, $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$, $\alpha\beta\gamma = \frac{-d}{a}$.

Q.2 **Assertion:** If one zero of polynomial $p(x) = (k^2 + 4)x^2 + 13x + 4k$ is reciprocal of other, then $k = 2$.

Reason: If $x - \alpha$ is a factor of $p(x)$, then $p(\alpha) = 0$, i.e. α is a zero of $p(x)$.

Q.3 **Assertion:** Degree of a zero polynomial is not defined.

Reason: Degree of a non-zero constant polynomial is '0'.

SECTION-E

- **Match the following (one to many)**

Column-I and **column-II** contains **four** entries each. Entries of column-I are to be matched with some entries of column-II. One or more than one entries of column-I may have the matching with the some entries of column-II and one entry of column-II may have one or more than one matching with entries of column-I

Q.1 **Match the polynomials given in column I with their zeros in column II**

	Column I		**Column II**
(i)	$4 - x^2$	(p)	7
(ii)	$x^3 - 2x^2$	(q)	-2
(iii)	$6x^2 - 3 - 7x$	(r)	2
(iv)	$-x + 7$	(s)	3/2
		(t)	0
		(u)	$-1/3$

ANSWER KEY

CONCEPT APPLICATION LEVEL - II

Q.1	B	Q.2	D	Q.3	B	Q.4	A	Q.5	B	Q.6	D	Q.7	B
Q.8	A	Q.9	A	Q.10	B	Q.11	B	Q.12	B	Q.13	B	Q.15	A
Q.16	C	Q.17	B	Q.18	A	Q.19	D	Q.20	A	Q.21	B	Q.22	A
Q.23	A												

CONCEPT APPLICATION LEVEL - III

SECTION-A

Q.1	Linear polynomial	Q.2	At most 2	Q.3	c/a
Q.4	parabola, down wards.	Q.5	no		

SECTION-B

Q.1	C	Q.2	B	Q.3	B	Q.4	C	Q.5	B	Q.6	A	Q.7	C
Q.8	C	Q.9	A	Q.10	C	Q.11	C	Q.12	A	Q.13	B	Q.14	B
Q.15	C												

SECTION-C

Q.1	D	Q.2	BC	Q.3	ABD	Q.4	AC	Q.5	ABD	Q.6	AD	Q.7	BC
Q.8	BD	Q.9	AD	Q.10	AD								

SECTION-D

Q.1	A	Q.2	B	Q.3	B

SECTION-E

Q.1 (i) r, q; (ii) r, t; (iii) s, u; (iv) p