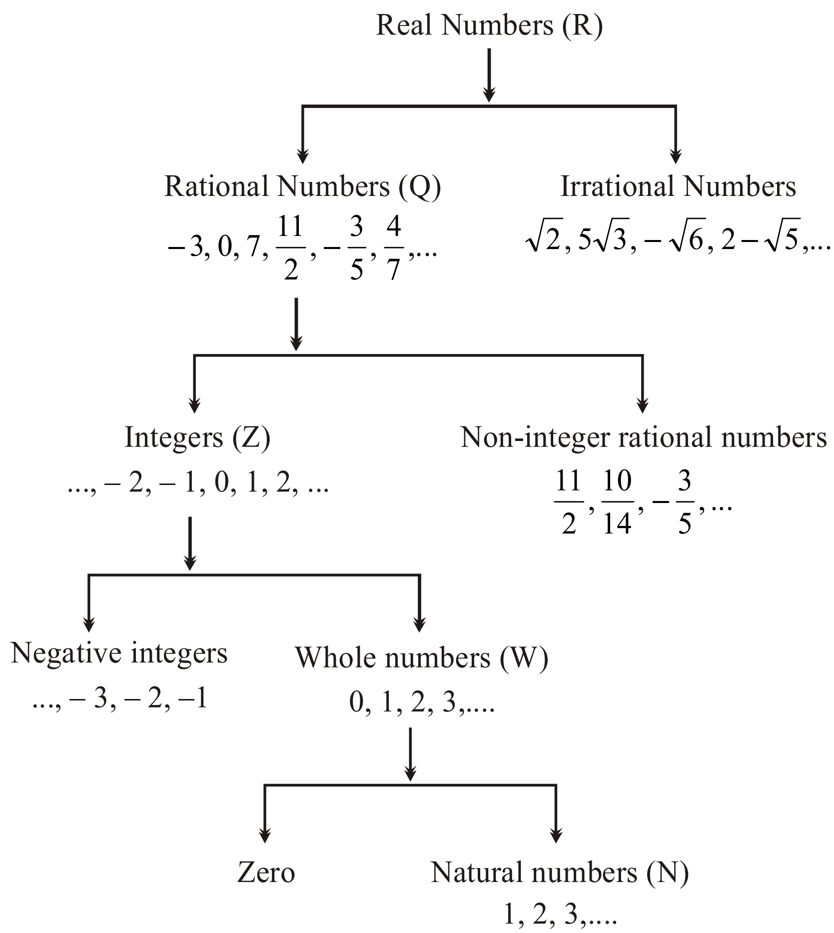
1 REAL NUMBERS

THEORY

1.1 INTRODUCTION :

KEY CONCEPT



- **Natural numbers :** The counting numbers 1,2,3..... are called natural numbers. It is denoted by N.

  N = {1,2.3..........}

- **Whole numbers :** In the set of natural number if we include the number 0, the resulting set is known as the set of whole numbers.

  It is represented by W.

  W = {0,1,2.......}

- **Integers :** Natural numbers along with 0 and their negatives are called integers and the set of integers is denoted by I

$$I = \{.........-4, -3, -2, -1, 0, 1, 2, 3..........\}$$

- **Rational numbers :** A rational number is a number which can be expressed in the form of p/q, where p and q are integers and q is not zero.

- **Irrational numbers :** A number is called irrational if it can not be written in the form of p/q, where p and q are integers and $q \neq 0$

The system R of real numbers includes rational as well irrational numbers.

In this chapter we will begin with a brief recall of divisibility of integers as well state some important properties of integers.

1.2 RATIONAL NUMBERS

Decimal Representation of Rational Numbers :

KEY CONCEPT

Real Numbers in decimal form

→ Rational numbers
All terminating or non-terminating repeating decimals

→ Irrational numbers
All non-terminating and non-repeating decimals

Rational numbers →

→ Terminating decimals
$x = \frac{p}{q}$, q can be expressed in the form $2^m 5^n$, where m, n are non-negative integers

→ Non-terminating decimals
$x = \frac{p}{q}$, q can not be expressed in the form $2^m 5^n$, where m, n are non-negative integers

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(i) Finite or Terminating Decimal : Every fraction p/q can be expressed as a decimal, if the decimal expression of p/q terminates, i.e. comes to an end, then the decimal so obtained is called a terminating decimal.

e.g,. $1/4 = 0.25$, $5/8 = 0.625$, $2\frac{3}{5} = \frac{13}{5} = 2.6$

Thus, each of the numbers $\frac{1}{4}$, $\frac{5}{8}$ and $2\frac{3}{5}$ can be expressed in the form of a terminating decimal.

Important : A fraction p/q is a terminating decimal only, when prime factors of q are 2 and 5 only.

e.g. Each one of the fractions $\frac{1}{2}, \frac{3}{4}, \frac{7}{20}, \frac{13}{25}$ is a terminating decimal, since the denominator of each has no prime factor other than 2 and 5.

(ii) Repeating (or Recurring) Decimals: A decimal in which a digit or a set of digits repeats periodically, is called a repeating or a recurring decimal.

In a recurring decimal, we place a bar over the first block of the repeating part and omit the other repeating blocks.

e.g. (i) $\frac{2}{3} = 0.666 \ldots\ldots = 0.\overline{6}$

(ii) $\frac{15}{7} = 2.142857142857 \ldots\ldots = 2.\overline{142857}$

REMEMBER

Special Characteristics of Rational Numbers :

(i) Every rational number is expressible either as a terminating decimal or as a repeating decimal.

(ii) Every terminating decimal is a rational number.

(iii) Every repeating decimal is a rational number.

Fractions :

(a) Common fraction : Fractions whose denominator is not 10.

(b) Decimal fraction : Fractions whose denominator is 10 or any power of 10.

(c) Proper fraction : Numerator $<$ Denominator i.e. $\frac{2}{7}$

(d) Improper fraction : Numerator $>$ Denominator i.e. $\frac{7}{2}$

(e) Mixed fraction : Consists of integral as well as fractional part i.e. $5\frac{2}{9}$

(f) Compound fraction : Fraction whose numerator and denominator themselves are fractions. i.e. $\frac{4/5}{3/7}$.

(g) Continued fraction : Fraction consists of the fractional denominators.

i.e., $1+\cfrac{1}{2-\cfrac{3}{5+\cfrac{4}{7}}}$

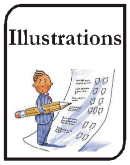


Illustration 1 : Simplify: $3+\cfrac{2}{1+\cfrac{1}{4-\cfrac{2}{3}}}$

Sol. $3+\cfrac{2}{1+\cfrac{1}{\left(\frac{4}{1}-\frac{2}{3}\right)}} = 3+\cfrac{2}{1+\cfrac{1}{\left(\frac{10}{3}\right)}} = 3+\cfrac{2}{\left(1+\frac{3}{10}\right)} = 3+\cfrac{2}{\left(\frac{13}{10}\right)}$

$= 3 + 2\times\frac{10}{13} = 3+\frac{20}{13} = \frac{39+20}{13} = \frac{59}{13}.$

- **Prime numbers :** All natural numbers that have one and itself only as their factors are called prime numbers i.e. prime numbers are exactly divisible by 1 and themselves.
  Example : 2, 3, 5, 7, 11, 13, 17, 19, 23 ....etc.

- **Twin Primes :** The term twin primes is used for a pair of odd prime numbers that differ by two.
  Example : 3 and 5 are twin primes.

- **Co-prime numbers :** If the H.C.F. of the given numbers (not necessarily prime) is 1 then they are known as co-prime numbers.
  Example : 5, 6, are co-prime as H.C.F. of (5, 6) = 1.

IMPORTANT

Note : (i) 1 is neither prime nor composite number.
(ii) 2 is the only prime number which is even.
(iii) Any two consecutive numbers will always be co-prime.

- **Composite numbers :** All natural numbers that have more than two different factors are called composite numbers. If C is the set of composite numbers then C = {4,6,8,9,10,12,.....}.

- **Perfect Number :** If the sum of all factors of a number is twice the number then this number is called perfect number.
  If $2^k - 1$ = Prime number, then $(2^k - 1)(2^{k-1})$ is a perfect number.
  Example : 6, 28, etc.

- **Imaginary Numbers:** All the numbers whose square is negative are called imaginary numbers.
  Example : 2i, – 7i, i, ... where $i = \sqrt{-1}$ $(i^2 = -1)$.

- **Complex Numbers :** The combined form of real and imaginary numbers is known as complex numbers. It is denoted by Z = a + ib where a is real part and b is imaginary part of Z and a, b $\in$ R.
  The set of complex numbers is the super set of all the sets of numbers.

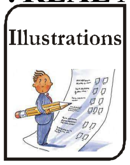


Illustration 2 : Express $\frac{2157}{625}$ in the decimal form.

Sol. We have,

```
625)2157.0000(3.4512
    1875
    ----
     2820
     2500
     ----
      3200
      3125
      ----
       750
       625
       ----
       1250
       1250
       ----
          0
```

$\therefore \quad \frac{2157}{625} = 3.4512$ **Ans.**

Illustration 3 : Find the decimal representation of $\frac{-16}{45}$.

Sol. By long division, we have

```
45)160   (0.3555
   135
   ---
    250
    225
    ---
     250
     225
     ---
      250
      225
      ---
       25
```

$\therefore \quad \frac{16}{45} = 0.355 \ldots = 0.3\overline{5}$

Hence, $\frac{-16}{45} = -\,0.3\overline{5}$. **Ans.**

Conversion of Decimal Numbers into Rational Numbers of the form p/q :

(i) Procedure for terminating decimal :

Step. 1 : Count the number of numerals to the right of the decimal point. Let it be m.

Step. 2 : Drop the decimal point and in the denominator write 1 followed by m zeros.

Step. 3 : Simplify the fraction.

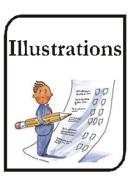


Illustration 4 : Convert 6.225 to the form p/q.

Sol. 1. Number of numerals to the right of decimal is 3 i.e. m = 3.

2. Write $6.225 = \frac{6225}{1000}$

3. Simplify (divide the numerator and denominator by 25) = $6.225 = \frac{249}{40}$

(ii) Conversion of Pure Recurring Decimal to the form p/q.

Step.1 : Obtain the repeating decimal and put it equal to x.

Step 2 : Write the number in decimal form by removing bar from the top of repeating digits and listing repeating digits at least twice :

e.g. write $x = 0.\overline{8}$ as $x = 0.888$ ....

Step 3 : Determine the no. of digits having bar on their heads.

Step 4 : If the repeating decimal has 1 place repetition, multiply by 10, a two place repetition, multiply by 100, a three place repetition, multiply by 1000 and so on.

Step 5 : Subtract the number in step II from the numbers obtained in step IV.

Step 6 : Divide both sides of the equation by the coefficient of x.

Step 7 : Write the rational number in its simplest form.

Illustrations

Illustration 5 : Express $0.\overline{585}$ in the form p/q.

Sol. Let $x = 0.\overline{585}$

$x = 0.585585585$ ....(i)

Here, we have 3 repeating digits after the decimal point. So, we multiply both sides of (i) by $10^3 = 1000$ to get

$1000\,x = 585.585585$ ....(ii)

Subtracting (i) from (ii), we get

$1000\,x - x = (585.585585 \text{ ....}) - (0.585585\ldots)$

$999\,x = 585 \quad \Rightarrow \quad x = \dfrac{585}{999}$

(iii) Conversion of a Mixed Recurring Decimal to the form p/q.

Step 1 : Obtain the mixed recurring decimal and write it equal to x.

Step 2 : Determine the number of digits after the decimal point which do not have bar on them. Let there be n digits without bar just after the decimal point.

Step 3 : Multiply both sides of x by 10^n, so that only the repeating decimal is on the right side of the decimal point.

Step 4 : Use the method of converting pure recurring decimal to the form p/q and obtain the value of x.

Illustrations

Illustration 6 : Express $0.22\overline{5}$ in the form p/q.

Sol. Let $x = 0.22\overline{5}$ ....(i)

The no of digits after the decimal point which do not have bar on them is 2.

$\therefore$ Multiply both sides of x by 10^2.

$100\,x = 22.\overline{5}$ ....(ii)

Here, we have 1 repeating digit after the decimal point. So, multiply both sides of (ii) by 10 to get.

$1000\,x = 225.55$ ....(iii)

Subtracting (ii) from (iii)

$1000\,x - 100\,x = (225.55\text{......}) - (22.55\text{......})$

$900\,x = 203 \Rightarrow x = \dfrac{203}{900}$.

1.3 IRRATIONAL NUMBERS :

A number is an irrational number, if it has a non terminating and non-repeating decimal representations. A number that cannot be put in the form p/q where p, q are integers and $q \neq 0$ is called irrational number.

e.g. $\sqrt{2}$, $\sqrt{3}$, $\sqrt{11}$, π etc.

Illustration 7 : Prove that $\sqrt{5}$ is an irrational number.

Sol. Let us assume on the contrary that $\sqrt{5}$ is a rational number. Then, there exist co-prime positive integers a and b such that

$\sqrt{5} = \frac{a}{b} \Rightarrow 5b^2 = a^2$

$\Rightarrow$ $5 \mid a^2$ $[\because 5 \mid 5b^2]$

$\Rightarrow$ $5 \mid a$ ......(i)

$\Rightarrow$ a = 5c for some positive integer c

$\Rightarrow$ $a^2 = 25c^2$

$\Rightarrow$ $5b^2 = 25c^2$ $[\because a^2 = 5c^2]$

$\Rightarrow$ $b^2 = 5c^2$

$\Rightarrow$ $5 \mid b^2$ $[\because 5 \mid 5c^2]$

$\Rightarrow$ $5 \mid b$ ....(ii)

From (i) and (ii), we find that a and b have at least 5 as a common factor. This contradicts the fact that a and b are co-prime.

Hence, $\sqrt{5}$ is an irrational number.

Illustration 8 : Prove that $\sqrt{3} - \sqrt{2}$ is an irrational number.

Sol. If possible, let $\sqrt{3} - \sqrt{2}$ be a rational number equal to x.

Then, $x = \sqrt{3} - \sqrt{2}$

$\Rightarrow$ $x^2 = \left(\sqrt{3} - \sqrt{2}\right)^2$

$\Rightarrow$ $x^2 = 3 + 2 - 2\sqrt{3}\sqrt{2}$

$\Rightarrow$ $x^2 = 5 - 2\sqrt{6}$

$\Rightarrow$ $x^2 - 5 = -2\sqrt{6}$

$\Rightarrow$ $\frac{5 - x^2}{2} = \sqrt{6}$

Now, x is rational

$\Rightarrow$ x^2 is rational

$\Rightarrow$ $\frac{5 - x^2}{2}$ is rational

$\Rightarrow$ $\sqrt{6}$ is rational.

But, $\sqrt{6}$ is irrational.

Thus, we arrive at a contradiction. So, our supposition that $\sqrt{3} - \sqrt{2}$ is rational, is wrong.

Hence, $\sqrt{3} - \sqrt{2}$ is an irrational number. **Ans.**

REMEMBER

Some Properties of irrational numbers :

(a) The –ve of an irrational number is an irrational number.
(b) The sum of a rational and an irrational number is an irrational number.
(c) The product of a non–zero rational number with an irrational number is always an irrational number.

1.4 REAL NUMBERS :

The collection of real numbers consists of all the rational and irrational numbers and is denoted by R. Every real number corresponds to a point on the line and conversely, every point on the number line represents a real number.

REMEMBER

Properties of all real numbers :

(a) Closure property of addition :
The sum of two real numbers is always a real number.

(b) Commutative law for addition :
$a + b = b + a$, $\forall$ real numbers 'a' and 'b'.

(c) Associative law for addition :
$(a + b) + c = a + (b + c)$,
$\forall$ real numbers a, b and c.

(d) Existence of additive identity :
Zero is the additive identity.
$a + 0 = 0 + a = a$, $\forall$ real numbers a.

(e) Existence of addition inverse :
For each real number 'a', there exists a real number '–a' such that $a + (-a) = (-a) + a = 0$.

(f) Closure property for multiplication :
The product of two real numbers is a real number.

(g) Commutative law of multiplication :
$ab = ba$, $\forall$ real numbers a and b.

(h) Associative law of multiplication :
$(ab)c = a(bc)$, $\forall$ real numbers a, b and c.

(i) Existence of multiplicative identity :
1 is called the multiplicative identity.
$1.a = a.1 = a$, $\forall$ real numbers a.

(j) Existence of multiplicative inverse :
Every non–zero real number 'a' has its multiplicative inverse $\frac{1}{a}$.

(k) Distributive law of multiplication over addition :
$a(b + c) = ab + ac$, $\forall$ real numbers a, b and c.

Important

"Zero is a real number which has no multiplicative inverse".

KEY CONCEPT

1.5 TEST OF DIVISIBILITY :

Divisibility by	Divisibility Rule
2	The unit digit of the number must be even
3	The sum of digits of the number must be divisible by 3
4	The number formed by last two digits of the given number must be divisible by 4
5	The unit digit of the number must be 0 or 5.
6	The given number must be divisible by 2 and 3.
7	Subtract two times of the unit digit from the number formed by excluding the unit digit from the given number, then the result obtained must be 0 or divisible by 7.
8	The number formed by last three digits of given number must be divisible by 8
9	The sum of digits of the given number must be divisible by 9
11	The difference between the sums of the digits at even and odd places must be zero or multiple of 11
12	The number must be divisible by 3 & 4.
13	Add four times of the unit digit to the given number formed by excluding unit digit, then the result obtained must be divisible by 13.
17	Subtract five times of the unit digit from the number formed by excluding the unit digit from the given number then the result must be 0 or divisible by 17.
19	Add two times of the unit digit to the given number formed by excluding unit digit, then the result obtained must be divisible by 19.

1.6 RADICALS :

An expression written under a radical sign is called a radical expression. The **radicand** is the number under the radical.

A surd is the simplest type of irrational number, one whose radicand is a rational number.

e.g. $\sqrt{5}$, $3\sqrt{7}$ and $\frac{1}{\sqrt{3}}$ are surds whereas $3\sqrt{5-\sqrt{2}}$ and $\sqrt{\sqrt{3}}$ are not surds.

The order of a surd is indicated by its index.

The order of a radical is the denominator of its fractional exponent.

e.g., $\sqrt[n]{a}$ is the surd of n^{th} order.

(i) Pure surd : A surd in which the whole of the rational number is under the radical sign. & makes the **radicand**, is called pure surd.

e.g. $\sqrt{8}$, $\sqrt[3]{15}$, $\sqrt[4]{11}$ etc.

(ii) Mixed surd : If some part of the quantity under the radical sign is taken out of it then it makes the surd mixed.

e.g. $3\sqrt[4]{2}$, $4\sqrt{2}$ etc.

(iii) Like surds (Similar surds) : The surds having the same irrational factor are called similar surds.

e.g. $\sqrt{3}, 5\sqrt{3}, \frac{2}{5}\sqrt{3}$ are like surds.

(iv) Unlike surds : The surds having different irrational factor are called dissimilar surds.

e.g. $\sqrt{2}, 2\sqrt{3}, 2\sqrt{5}$ are unlike surds.

KEY CONCEPT

Laws of Radicals :

If a, b are positive rational numbers and m, n, p are positive integers, then

(i) $a^0 = 1$

(ii) $(\sqrt[n]{a})^n = a$, $\sqrt[n]{a^n} = a = (\sqrt[n]{a})^n$

(iii) $(\sqrt[n]{a})(\sqrt[n]{b}) = \sqrt[n]{ab}$

(iv) $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

(v) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$

(vi) $\sqrt[n]{a^p} = (a^p)^{1/n} = \sqrt[mn]{a^{pm}}$

(vi) $a^{-n} = \frac{1}{a^n}$

Illustrations

Illustration 9 : Convert :

(i) $\sqrt[4]{1875}$ into mixed surd. (ii) $3\sqrt[5]{7}$ into pure surd.

Sol. (i) We have $1875 = 5^4 \times 3$

$$\sqrt[4]{1875} = \sqrt[4]{5^4 \times 3} = 5\sqrt[4]{3}$$

(ii) $$3\sqrt[5]{7} = \sqrt[5]{3^5 \times 7} = \sqrt[5]{243 \times 7} = \sqrt[5]{1701}$$

Illustration 10 : Simplify : $\left(\frac{81}{16}\right)^{-3/4} \times \left[\left(\frac{25}{9}\right)^{-3/2} \div \left(\frac{5}{2}\right)^{-3}\right]$

Sol. We have,

$$\left(\frac{81}{16}\right)^{-3/4} \times \left[\left(\frac{25}{9}\right)^{-3/2} \div \left(\frac{5}{2}\right)^{-3}\right] = \left(\frac{3^4}{2^4}\right)^{-3/4} \times \left[\left(\frac{5^2}{3^2}\right)^{-3/2} \div \left(\frac{5}{2}\right)^{-3}\right]$$

$$= \left[\left(\frac{3}{2}\right)^4\right]^{-3/4} \times \left[\left(\frac{5}{3}\right)^2\right]^{-3/2} \div \left[\left(\frac{5}{2}\right)^{-3}\right] = \left(\frac{3}{2}\right)^{4\times -3/4} \times \left[\left(\frac{5}{3}\right)^{2\times -3/2} \div \left(\frac{5}{2}\right)^{-3}\right]$$

$$= \left(\frac{3}{2}\right)^{-3} \times \left[\left(\frac{5}{3}\right)^{-3} \div \left(\frac{5}{2}\right)^{-3}\right] = \left(\frac{2}{3}\right)^{3} \times \left[\left(\frac{3}{5}\right)^{3} \div \left(\frac{2}{5}\right)^{3}\right] = \frac{2^3}{3^3} \times \left[\frac{3^3}{5^3} \div \frac{2^3}{5^3}\right]$$

$$= \frac{2^3}{3^3} \times \left[\frac{3^3}{5^3} \times \frac{5^3}{2^3}\right] = 1. \textbf{ Ans.}$$

Illustration 11 : If x, y, z are positive real numbers show that :

$$\sqrt{x^{-1}y} \cdot \sqrt{y^{-1}z} \cdot \sqrt{z^{-1}x} = 1$$

Sol. We have,

$$\sqrt{x^{-1}y} \cdot \sqrt{y^{-1}z} \cdot \sqrt{z^{-1}x}$$

$$= \sqrt{\frac{y}{x}} \cdot \sqrt{\frac{z}{y}} \cdot \sqrt{\frac{x}{z}} = \left(\frac{y}{x}\right)^{1/2} \left(\frac{z}{y}\right)^{1/2} \left(\frac{x}{z}\right)^{1/2} = \frac{y^{1/2}}{x^{1/2}} \cdot \frac{z^{1/2}}{y^{1/2}} \cdot \frac{x^{1/2}}{z^{1/2}} = 1. \textbf{ Ans.}$$

1.7 RATIONALISATION OF DENOMINATOR :

Sometimes we come across expressions containing square root in their denominators. Addition, subtraction, multiplication and division of such expressions is convenient if their denominators are free from square roots. To make the denominators free from square roots, we multiply the numerator and denominator by an irrational number. Such a number is called rationlisation factor.

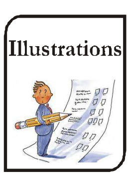


Illustration 12 : Rationalise the denominator of $\frac{1}{3+\sqrt{2}}$

Sol. We have,

$$\frac{1}{3+\sqrt{2}} = \frac{1}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}} = \frac{3-\sqrt{2}}{9-2} = \frac{3-\sqrt{2}}{7}$$

Illustration 13 : If both a and b are rational numbers, find the values of a and b.

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = a + b\sqrt{15}$$

Sol. Rationalising the denominator, we get

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{\left(\sqrt{5}+\sqrt{3}\right)^2}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$$\Rightarrow \quad \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{\left(\sqrt{5}\right)^2+\left(\sqrt{3}\right)^2+2\sqrt{5}\times\sqrt{3}}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$$\Rightarrow \quad \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{5+3+2\sqrt{5\times3}}{5-3} = \frac{8+2\sqrt{15}}{5-3} = 4+\sqrt{15}$$

$$\therefore \quad \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = a+b\sqrt{15} \Rightarrow 4+\sqrt{15} = a+b\sqrt{15} \Rightarrow a = 4 \text{ and } b = 1.$$ **Ans.**

Illustration 14 : If $x = 3 - 2\sqrt{2}$, find $x^2 + \frac{1}{x^2}$.

Sol. We have, $x = 3 - 2\sqrt{2}$

$$\therefore \quad \frac{1}{x} = \frac{1}{3-2\sqrt{2}} = \frac{1}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}} = \frac{3+2\sqrt{2}}{3^2-\left(2\sqrt{2}\right)^2} = \frac{3+2\sqrt{2}}{9-8} = 3+2\sqrt{2}.$$

Thus, $$x^2+\frac{1}{x^2} = \left(3-2\sqrt{2}\right)^2+\left(3+2\sqrt{2}\right)^2$$

$$\Rightarrow \quad x^2+\frac{1}{x^2} = 3^2+\left(2\sqrt{2}\right)^2 - 2\times3\times2\sqrt{2} + 3^2 + \left(2\sqrt{2}\right)^2 + 2\times3\times2\sqrt{2}$$

$$\Rightarrow \quad x^2+\frac{1}{x^2} = 9+8-12\sqrt{2}+9+8+12\sqrt{2} = 34.$$ **Ans.**

Illustration 15 : Show that : $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}} = 1$

Sol. We have,

$$= \frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}}$$

$$= \frac{x^a}{x^a+x^{b-a+a}+x^{c-a+a}} + \frac{x^b}{x^b+x^{a-b+b}+x^{c-b+b}} + \frac{x^c}{x^c+x^{b-c+c}+x^{a-c+c}}$$

[Multiplying N^r and D^r of three terms by x^a, x^b and x^c respectively]

$$= \frac{x^a}{x^a+x^b+x^c} + \frac{x^b}{x^b+x^a+x^c} + \frac{x^c}{x^c+x^b+x^a}$$

$$= \frac{x^a+x^b+x^c}{x^a+x^b+x^c} = 1$$

Illustration 16 : Simplify : $\frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}} + \frac{2\sqrt{3}}{\sqrt{6}+2}$

Sol. $\frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}} + \frac{2\sqrt{3}}{\sqrt{6}+2} = T_1 - T_2 + T_3$ (say)

$$T_1 = \frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}} \times \frac{\sqrt{6}+\sqrt{3}}{\sqrt{6}+\sqrt{3}} = \frac{3\sqrt{2}(\sqrt{6}+\sqrt{3})}{6-3}$$

$$= \sqrt{2}(\sqrt{6}+\sqrt{3})$$

$$= 2\sqrt{3}+\sqrt{6}$$

$$T_2 = \frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}} \times \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}} = \frac{4\sqrt{18}+4\sqrt{6}}{6-2}$$

$$= \frac{4(3\sqrt{2}+\sqrt{6})}{4} = 3\sqrt{2}+\sqrt{6}$$

$$T_3 = \frac{2\sqrt{3}}{\sqrt{6}+2} \times \frac{\sqrt{6}-2}{\sqrt{6}-2} = \frac{2\sqrt{3}\left(\sqrt{6}-2\right)}{6-4} = 3\sqrt{2}-2\sqrt{3}$$

$\therefore$ Given expression $= T_1 - T_2 + T_3$

$$= 2\sqrt{3} + \sqrt{6} - 3\sqrt{2} - \sqrt{6} + 3\sqrt{2} - 2\sqrt{3} = 0$$

Illustration 17 : If $x = \frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}}$, then show that $bx^2 - ax + b = 0$

Sol. $x = \frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}} \text{ x } \frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}+\sqrt{a-2b}}$

$$= \frac{\left(\sqrt{a+2b}+\sqrt{a-2b}\right)^2}{(a+2b)-(a-2b)} = \frac{a+2b+a-2b+2\sqrt{(a+2b)(a-2b)}}{4b}$$

$$x = \frac{2(a+\sqrt{(a^2-4b^2)}}{2\times 2b}$$

$2\,bx = a + \sqrt{(a^2-4b^2)}$ $\Rightarrow$ $2bx - a = \sqrt{(a^2-4b^2)}$

On squaring both sides, we get :

$\Rightarrow 4b^2x^2 + a^2 - 4abx = a^2 - 4b^2$

or $4b^2x^2 - 4abx + 4b^2 = 0$

Dividing by 4b , we get, $bx^2 - ax + b = 0$

Hence the result.

Illustration 18 : Find the square root of $2+\sqrt{3}$.

Sol. Let $\sqrt{2+\sqrt{3}} = \sqrt{x}+\sqrt{y}$

$2+\sqrt{3} = x + y + 2\sqrt{xy}$ (By squaring both sides.)

Now comparing the parts of both sides,

$x + y = 2$ .... (1)

$2\sqrt{xy} = \sqrt{3}$ .... (2)

or $4xy = 3$ .... (3) (By squaring both sides of equation 2.)

we know that

$(x-y)^2 = (x+y)^2 - 4xy$

$(x-y)^2 = 4-3$

$(x-y)^2 = 1$

$x - y = 1$ .... (4)

On solving equations (1) and (4), we get

$$x = \frac{3}{2},\ y = \frac{1}{2}$$

Since $\sqrt{2+\sqrt{3}} = \sqrt{x}+\sqrt{y}$

$$= \pm\left[\sqrt{\frac{3}{2}}+\sqrt{\frac{1}{2}}\right]$$

$$= \pm\left[\frac{1}{\sqrt{2}}\left(\sqrt{3}+1\right)\right]$$

$$\sqrt{2+\sqrt{3}} = \pm\frac{1}{\sqrt{2}}\left(1+\sqrt{3}\right)$$

1.8 TO FIND THE NUMBER OF DIVISORS OF A COMPOSITE NUMBER :

IMPORTANT FORMULAE

Let N denotes a natural number such that

$N = a^p\, b^q\, c^r$ ...., where a, b, c, ... are different prime numbers and p, q, r, ... are positive integers, then

(i) n = The number of divisors of N = (p + 1) (q + 1) (r + 1) ....

Note : This includes as divisors, both unity and the number itself.

(ii) The sum of divisors of $N = \left(\frac{a^{p+1}-1}{a-1}\right)\left(\frac{b^{q+1}-1}{b-1}\right)\left(\frac{c^{r+1}-1}{c-1}\right)$......

(iii) The product of divisors of N = $(N)^{\frac{n}{2}}$, where n is the total number of divisors of N.

Illustrations

Illustration 19 : If a number N = 30 then, find

(i) The number of divisors of N

(ii) The sum of divisors of N

(iii) The product of divisors of N

Sol. Prime factorisation of $30 = 2^1 \times 3^1 \times 5^1$

(i) Let n = the number of divisors of 30 = $(1 + 1)(1 + 1)(1 + 1) = 2 \times 2 \times 2 = 8$.

and these divisors are : 1, 2, 3, 5, 6, 10, 15, 30

(ii) The sum of divisors of 30 $= \left(\dfrac{2^{1+1}-1}{2-1}\right)\left(\dfrac{3^{1+1}-1}{3-1}\right)\left(\dfrac{5^{1+1}-1}{5-1}\right)$

$$= \left(\frac{2^2-1}{1}\right)\left(\frac{3^2-1}{2}\right)\left(\frac{5^2-1}{4}\right)$$

$$= 3 \times \frac{8}{2} \times \frac{24}{4}$$

$$= 9 \times 8 = 72.$$

(iii) Product of divisors of 30 $= (N)^{n/2} = (30)^{8/2} = (30)^4 = 810000$.

1.9 ABSOLUTE VALUE OR MODULUS OF A RATIONAL NUMBER :

1. The absolute value of a rational number is the number without any regard to its sign. Thus, for any rational number x,

   The absolute value of x $= |x| = \begin{cases} x, \text{if } x > 0 \\ 0, \text{if } x = 0 \\ -x, \text{if } x < 0 \end{cases}$

2. $|x + y| \le |x| + |y|$, where x and y are rational numbers.
3. $|x \times y| = |x| \times |y|$, where x and y are rational numbers.
4. $|x - y| \ge |x| - |y|$, where x and y are rational numbers.

REMEMBER

5. Modulus of a real number given by $|x|$.

(i)	If	$\|x\| = a$	$\Rightarrow$	$x = \pm a$
(ii)	If	$\|x\| < a$	$\Rightarrow$	$-a < x < a$
(iii)	If	$\|x\| > a$	$\Rightarrow$	$x > a$ or $x < -a$
(iv)	If	$\|x - a\| < l$	$\Rightarrow$	$a - l < x < a + l$
(v)	If	$\|x - a\| > l$	$\Rightarrow$	$x > a + l$ or $x < a - l$

Illustration 20 : Find the value of x, when $x - 3|x| = -4$ is simplified

Sol.

(i) If $x \geq 0$, then $|x| = x$ and given equation becomes $x - 3x = -4$ or $-2x = -4$.

$x = +2.$

(ii) If $x < 0$, then $|x| = -x$ and given equation becomes,

$$x - 3(-x) = -4$$
$$x + 3x = -4$$
$$4x = -4$$
$$x = -1.$$

Therefore, $x = 2$ or -1. **Ans.**

1.10 HCF AND LCM OF FRACTIONS :

Note : Make sure that the fractions are in the most reducible form.

(1) HCF of fractions $= \dfrac{\text{HCF of their numerators}}{\text{LCM of their denominators}}$.

Illustration 21 : Find HCF of $\frac{9}{2}, \frac{3}{4}$ and $\frac{6}{7}$.

Sol. $\text{HCF}\left(\frac{9}{2}, \frac{3}{4}, \frac{6}{7}\right) = \dfrac{\text{HCF }(9, 3, 6)}{\text{LCM }(2, 4, 7)} = \dfrac{3}{28}$.

---

(2) LCM of fractions $= \dfrac{\text{LCM of their numerators}}{\text{HCF of their denominators}}$.

Illustration 22 : Find LCM of $\frac{9}{2}, \frac{3}{4}$ and $\frac{6}{7}$.

Sol. $\text{LCM}\left(\frac{9}{2}, \frac{3}{4}, \frac{6}{7}\right) = \dfrac{\text{LCM }(9, 3, 6)}{\text{HCF }(2, 4, 7)} = \dfrac{18}{1} = 18.$

1.11 TO FIND UNIT DIGIT IN EXPONENTIAL EXPRESSION :

Cyclicity : In number system, every number repeats its unit digit after some definite number of powers.

For example

1 repeat its unit digit after every consecutive power. So the cyclicity of 1 is 1.

2 repeat its unit digit after every four powers. So the cylicity of of 2 is 4.

REMEMBER

Cyclicity chart :

Unit digit in the number x	Possible unit digit in the number x^n (where n is positive integer)	Cyclicity of unit digit in the number x
0	0	1
1	1	1
2	2, 4, 8, 6	4
3	3, 9, 7, 1	4
4	4, 6	2
5	5	1
6	6	1
7	7, 9, 3, 1	4
8	8, 4, 2, 6	4
9	9, 1	2

Illustrations

Illustration 23 : Find the unit digit in 3^{47}.

Sol. Cyclicity of 3 is 4,

Remainder of $\frac{47}{4}$ is 3.

In cyclicity chart, the possible unit digit in 3^n at 3^{rd} place is 7, so the unit digit in 3^{47} is 7.

1.12 DIVISIBILITY :

A non zero integer 'a' is said to divide an integer 'b' if there exists an integer 'c' such that b = ac

The integer 'b' is called the dividend, integer 'a' is known as the divisor and integer 'c' is known as the quotient. For example, 3 divides 36 because there is an integer 12 such that 36 = 3 × 12. However, 3 does not divide 35 because there do not exist an integer 'c' such that 35 = 3 × c. In other words, 35 = 3 × c is not true for any integer 'c'.

Note: If a non zero integer 'a' divides an integer 'b', then we write a | b. This is read as "a divides b". When a | b, we say that 'b is divisible by a' or 'a is a factor of b' or 'b is a multiple of a' or 'a is divisor of b'.

REMEMBER

Some Properties of Divisibility

(i) ±1 divides every non-zero integer.

(ii) 0 does not divide any integer.

(iii) If a is a non zero integer and b is any integer, then $a \mid b \Rightarrow a \mid -b, -a \mid b$ and $-a \mid -b$.

(iv) If a and b are non-zero integers, then

$a \mid b$ and $b \mid a \Rightarrow a = \pm b$

(v) If a is a non-zero integer and b,c are any two integers, then

$$a \mid b \text{ and } a \mid c \Rightarrow \begin{cases} a \mid b \pm c \\ a \mid bc \\ a \mid bx \text{ for any integer } x \end{cases}$$

(vi) If a and c are non zero integers and b,d are any two integers, then

(a) $a \mid b$ and $c \mid d \Rightarrow ac \mid bd$

(b) $ac \mid bc \Rightarrow a \mid b$

IMPORTANT

1.13 EUCLID'S DIVISION LEMMA

(a) **Euclid's division Lemma:** Let *a* and *b* be any two positive integers. Then, there exist unique *q* and *r* such that :

$a = bq + r, 0 \le r < b$

If $b \mid a$, then r = 0, otherwise, r satisfies the strict inequality.

$0 < r < b$

Highest Common Factor (HCF) : HCF of two or more numbers is the largest number that divides all the given numbers completely.
It is also called the Greatest Common Divisor (GCD).

Lowest or least Common Multiple (LCM) : The LCM of two or more numbers is the smallest number which is multiple of each of the numbers or in other words the LCM of two or more numbers is the smallest number which is divisible by all the given numbers.

Illustrations

Illustration 24 : Show that any positive odd integer is of the form 6q + 1 or, 6q + 3 or, 6q + 5, where q is some integer.

Sol. Let 'a' be any positive integer and b = 6. Then, by Euclid's division lemma there exists integers 'a' and 'r' such that

$a = 6q + r$, where $0 \le r < 6$.

$\Rightarrow$ a = 6q or, a = 6q + 1 or, a = 6q + 2 or, a = 6a + 3 or, a = 6q + 4 or, a = 6q + 5.

[$\therefore 0 \le r < 6 \Rightarrow r = 0, 1,2,3,4,5$]

$\Rightarrow$ a = 6q + 1 or, a = 6q + 3 or, a = 6q + 5.

[$\therefore$ a is an odd integer, $\therefore$ 6q, $a \ne 6q + 2$, $a \ne 6q + 4$]

Hence, any odd integer is of the form 6q + 1 or, 6q + 3 or, 6q + 5.

Illustration 25 : Use Euclid's division algorithm to find the HFC of 4052 and 12576.

Sol. Given integers are 4052 and 12576 such that 12576 > 4052. Applying Euclid's division lemma to 12576 and 4052.

$\Rightarrow \quad 12576 = 4052 \times 3 + 420$

$\Rightarrow \quad 4052 = 420 \times 9 + 272$

$\Rightarrow \quad 420 = 272 \times 1 + 148$

$\Rightarrow \quad 272 = 148 \times 1 + 124$

$\Rightarrow \quad 148 = 124 \times 1 + 24$

$\Rightarrow \quad 124 = 24 \times 5 + 4$

$\Rightarrow \quad 24 = 4 \times 6 + 0$

We observe that the remainder at this stage is zero. Therefore, the divisor at this stage i.e. 4(or the remainder at the earlier stage) is the HCF of 4052 and 12576.

REMEMBER

Properties of HCF and LCM of given numbers

(i) The HCF of given numbers is *not greater than* any of two numbers.

(ii) The LCM of given numbers is *not less than* any of given numbers.

(iii) The HCF of given numbers is always factor of their LCM

(iv) The HCF of two coprime numbers is 1.

(v) The LCM of two or more coprime numbers is equal to their product.

IMPORTANT FORMULAE

(i) If a and b are two positive integers then
$a \times b = \text{HCF}(a, b) \times \text{LCM}(a, b)$

(ii) If a, b,c are positive integers

$$\text{LCM}(a, b, c) = \frac{a.b.c.\,\text{HCF}(a,b,c)}{\text{HCF}(a,b).\text{HCF}(b,c).\text{HCF}(c,a)}$$

$$\text{HCF}(a, b, c) = \frac{a.b.c.\,\text{LCM}(a,b,c)}{\text{LCM}(a,b).\text{LCM}(b,c).\text{LCM}(c,a)}$$

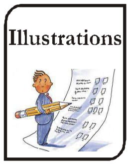


Illustration 26 : If the HCF of 210 and 55 is expressible in the form $210 \times 5 + 55y$, find y.

Sol. Applying Euclid's division lemma on 210 and 55, we get

$\Rightarrow \quad 210 = 55 \times 3 + 45$

$\Rightarrow \quad 55 = 45 \times 1 + 10$

$\Rightarrow \quad 45 = 4 \times 10 + 5$

$\Rightarrow \quad 10 = 5 \times 2 + 0$

We observe that the remainder at this stage is zero. So, the last divisor i.e. 5 is the HCF of 210 and 55.

$\therefore \quad 5 = 210 \times 5 + 55y$

$\Rightarrow \quad 55y = 5 - 210 \times 5 = 5 - 1050$

$\Rightarrow \quad 55y = -1045$

$\Rightarrow \quad y = \frac{1045}{55} = -19.$

Illustration 27 : Find the largest number which divides 245 and 1029 leaving remainder 5 in each case.

Sol. Given that the required number which divides 245 and 1029, the remainder is 5 in each case. This means 245 – 5 = 240 and 1029 – 5 = 1024 are completely divisible by the required number. Now is common factor of 240 and 1024,

Let us now find the HCF of 240 and 1024 by Euclid's algorithm.

3	240 192	1024 960	4
3	48 48	64 48	1
	0 (Remainder)	(16) (HCF)	

Clearly, HCF of 240 and 1024 is the last divisor i.e., 16.

Hence, required number = 16.

- **Prime factorization method to find HCF**

  Step–1 : Find prime factorisation of each of the given number.

  Step–2 : Identify common prime factors.

  Step–3 : Find the product of all the common prime factors, using each common prime factor the least number of times it appears in the prime factorisation of any of the given numbers. The product so obtained is the required HCF.

- **Prime factorization method to find LCM**

  Step–1 : Write the prime factorisation of each of the given numbers.

  Step–2 : Find the product of all different prime factors of the numbers using each common prime factor the highest number of times it appears in the prime factorisation of any of the numbers. The product so obtained is the required LCM of the given number.

KEY CONCEPT

Euclid's Division Alogrithm :

Algorithm: An algorithm is a series of well defined steps which provide a procedure of calculation repeated successively on the results of earlier steps till the desired result is obtained.
Euclid's division algorithm is an algorithm to compute the highest common factor (HCF) of two given positive integers.

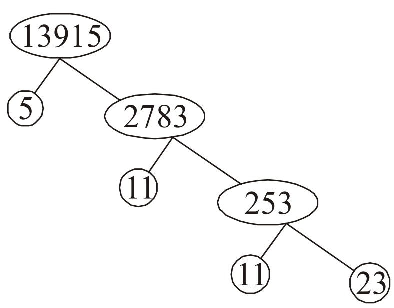
IMPORTANT

Some Important Results

(i) If b is a factor of a, then HCF of (a,b) = b which is simply written as (a,b) = b.
(ii) If a = q.b+r, r < b then HCF of (a,b) = HCF of (b, r) or (a,b) = (b,r)
(iii) If (a,b) = 1 and bc is divisible by a, then c is divisible by a. This is known as Gauss's Theorem.
(iv) If a and b are primes and a | bc $\Rightarrow$ a | c
(v) The HCF(d) of two positive integers a and b can be expresed as a linear combination of a and b i.e.d = xa + yb for some integers x and y. Also this representation is not unique.

Illustrations

Illustration 28 : Determine the prime factorization of the number 13915.
Sol. Using the prime factorization tree, we have



$\therefore$ $13915 = 5 \times 11 \times 11 \times 23 = 5 \times 11^1 \times 23$. Ans.

Illustration 29 : Prove that there is no natural number for which 4^n ends with the digit zero.
Sol. We know that any positive integer ending with the digit zero is divisible by 5 and so its prime factorization must contain the prime 5.
We have,
$4^n = (2^2)^n = 2^{2n}$
$\Rightarrow$ The only prime in the factorization of 4^n is 2.
$\Rightarrow$ There is no other primes in the factorization of $4^n = 2^{2n}$
$\Rightarrow$ 5 does not occur in the prime factorization of 4^n for any.
$\Rightarrow$ 4^n does not end with the digit zero for any natural number.

1.14 THE FUNDAMENTAL THEOREM OF ARITHMETIC :

REMEMBER

Theorem (Fundamental Theorem of Arithmetic) :

Every composite number can be expressed (factorised) as a product of primes and this factorisation is unique, apart from the order in which the prime factors occurs.

The prime factorisation of a natural number is unique except the order of its factors.

Let x be a composite number we factorise it as $x = p_1 \times p_2 \times p_3....p_n$, where $p_1, p_2...p_n$ are primes and written in ascending order i.e. $p_1 \le p_2 \le p_3...p_n$. If we combine we will get powers of primes.

Theorem :

Let p be a prime number and a be positive integer. If p divides a^2 then p also divides a

1.15 DETERMINING THE NATURE OF THE DECIMAL EXPANSION OF RATIONAL NUMBERS :

We have studied that the decimal expansion of a rational number is either terminating or non terminating repeating (or recurring) without knowing when it is terminating and when it is non-terminating repeating. Here in this section we will explore exactly when the decimal expansion of a rational number is terminating and when it is non terminating repeating.

REMEMBER

Theorem :

Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form p/q where p and q are coprimes and the prime factorisation of q is of the form $2^m \times 5^n$ where m and n are non negative integers.

Theorem :

Let $x = p/q$ be a rational number such that the prime factorization of q is the form of $2^m \times 5^n$ where m and n are non-negative integers. Then x has a decimal expansion which terminates.

Theorem :

Let $x = p/q$ be a rational number such that prime factorization of q ***is not*** of the form $2^m \times 5^n$ where m and n are non negative integers. Then x has a decimal expansion which is non-terminating repeating.

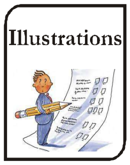


Illustration 30 : Find the HCF and LCM of 144, 180 and 192 by prime factorization method.

Sol. Using the factor tree for the prime factorization of 144, 180 and 192, we have

$144 = 2^4 \times 3^2$, $180 = 2^2 \times 3^2 \times 5$ and $192 = 2^6 \times 3$

Common prime factors and their smallest exponents in 144, 180 and 192 as follows **:**

Common prime factors	Least exponents
2	2
3	1

$\therefore$ HCF $= 2^2 \times 3^1 = 12$

Prime factors of 144, 180, 192 and their greatest exponents as follows :

Prime factors of 144, 180 and 192	Greatest exponents
2	6
3	2
5	1

$\therefore$ LCM $= 2^6 \times 3^2 \times 5^1 = 64 \times 9 \times 5 = 2880$.

Illustration 31 : In a seminar, the number of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively. Find the minimum number of rooms required if in each rooms required if in each room the same number of participlants are to be seated and all of them being in the same subject.

Sol. The number of participants in each room must be the HCF of 60, 84 and 108

Prime factorizations of 60, 84 and 108

$60 = 2^2 \times 3 \times 5$, $84 = 2^2 \times 3 \times 7$ and $108 = 2^2 \times 3^3$

$\therefore$ HCF of 60, 80 and 108 is $2^2 \times 3 = 12$

Therefore, in each room 12 participants can be seated.

$\therefore$ Number of rooms required $= \dfrac{\text{Total number of participants}}{12} = \dfrac{60+84+108}{12} = \dfrac{252}{12} = 21.$

Illustration 32 : Without actually performing the long division, state whether the following rational numbers will have terminating decimal expansion or a non-terminating repeating decimal expansion. Also, find the number of places of decimals after which the decimal expansion terminates.

(i) $\dfrac{13}{3125}$ (ii) $\dfrac{23}{2^3 5^2}$

Sol.

(i) We have, $\dfrac{13}{3125} = \dfrac{13}{2^0 \times 5^5}$

This shows that the prime factorization of the denominator of $\dfrac{13}{3125}$ is of the form $2^m \times 5^n$.

Hence, it has terminating decimal expansion which terminates after 5 places of decimals.

(ii) Prime factorization of the denominator of $\dfrac{23}{2^3 \times 5^2}$ is of the form $2^m \times 5^n$. So, it has terminating decimal expansion which terminates after 3 places of decimals.

SOLVED EXAMPLES

Example 1

Find two irrational number between 0.12 and 0.13

Solution

0.1201001000100001............., 0.12101001000100001.............

Example 2

If $\frac{9^n \times 3^2 \times 3^n - (27)^n}{(3^3)^5 \times 2^3} = \frac{1}{27}$. Find the value of 'n'.

Solution

$\frac{3^{2n+2+n} - 3^{3n}}{3^{15} \times 2^3} = \frac{1}{27} \Rightarrow \frac{3^{3n}(9-1)}{3^{15} \times 2^3} = \frac{1}{27} \Rightarrow 3^{3n-15} = 3^{-3}$

$\Rightarrow 3n - 15 = -3 \Rightarrow 3n = 12 \Rightarrow n = 4$

Example 3

A-5 digit number abcde is such that the 6-digit number abcde1 is the product of the 6-digit number 1abcde and 3. The sum of the digits of the number abcde is -

Solution

A.T.Q.

abcde1 = 1abcde × 3

i.e.

$a \times 10^5 + b \times 10^4 + c \times 10^3 + d \times 10^2 + e \times 10 + 1 = 3 \times [1 \times 10^5 + a \times 10^4 + b \times 10^3 + c \times 10^2 + d \times 10 + e]$

$a \times 10^4 (10-3) + b \times 10^3(10-3) + c \times 10^2(10-3) + d \times 10(10-3) + e(10-3) + 1 - 3 \times 10^5 = 0$

$a \times 10^4 + b \times 10^3 + c \times 10^2 + d \times 10^1 + e = \frac{3 \times 10^5 - 1}{7}$

abcde = 42857

$\therefore$ a + b + c + d + e = 4 + 2 + 8 + 5 + 7 = 26

Example 4

Seven Oranges weigh the same as four Apples and five Apples weigh the same as six Guava. Which of the following gives the description of the fruits in increasing order of weights ?

Solution

7Or = 4A = k

5A = 6G

$Or = \frac{k}{7}$

$A = \frac{k}{4}$

$G = \frac{5}{6}A = \frac{5}{6} \times \frac{k}{4} = \frac{5k}{24}$

LCM of 7, 4, 24 is 168

Or : A : G

$\frac{k}{7} : \frac{k}{4} : \frac{5k}{24}$

= 24k : 42k : 35k

= 24 : 42 : 35

$\therefore$ O < G < A

Example 5

What least number must be subtracted from 16160 to get a perfect square ? Also find the square root of this perfect square.

Solution

Let us try to find the square root of 16160.

```
          127
      ___________
  1  | 1  61  60
     | 1
     |----
 22  |   61
     |   44
     |-------
247  |   1760
     |   1729
     |   ----
     |     31
```

This shows that $(127)^2$ is less than 16160 by 31. So in order to get a perfect square, 31 must be subtracted from the given number.

$\therefore$ Required perfect square number = (16160 – 31) = 16129

Also, $\sqrt{16129} = 127$

Example 6

Find the square root of $3 + \sqrt{2}$.

Solution

Let $\sqrt{3+\sqrt{2}} = \sqrt{p} + \sqrt{q}$

$3 + \sqrt{2} = p + q + 2\sqrt{pq}$ [By squaring both sides]

by equating the parts

$p + q = 3$ ...(i)

$\Rightarrow 2\sqrt{pq} = \sqrt{2}$ ...(ii)

$\Rightarrow 4pq = 2$ ...(iii) [By squaring both sides of (ii)]

$\Rightarrow (p - q)^2 = (p + q)^2 - 4pq$

$\therefore (p - q)^2 = 9 - 2$

$\Rightarrow (p - q)^2 = 7$

$\Rightarrow p - q = \sqrt{7}$ ...(iv)

$\because p + q = 3$ [By eq^n (i)]

$\Rightarrow p = \frac{1}{2}(3 + \sqrt{7})$ [On adding (i) & (iv)]

$\Rightarrow q = \frac{1}{2}(3 - \sqrt{7})$ [On subtracting (i) & (iv)]

$\therefore \sqrt{3+\sqrt{2}} = \pm\frac{1}{\sqrt{2}}\left(\sqrt{3+\sqrt{7}} + \sqrt{3-\sqrt{7}}\right)$

Example 7

Find the least number which must be subtracted from 2509 to make it a perfect square.

Solution

Let us find the square root of 2509.

	50
5	25 09
	– 25
100	00 09
	– 00 00
100	09

So 2509 is 9 more than the square of 50 so 09 must be substracted to make it. a perfect square.

Example 8

If the number 357 * 25 * is divisible by both 3 and 5, then the missing digit in the unit's place and the thousandth place respectively are :

Solution

Let the required number be 357y25x.

Then, for divisibility by 5, we must have x = 0 or x = 5.

Case I When x = 0.

Then, sum of digits = (22 + y). For divisibility by 3, (22 + y) must be divisible by 3.

$\therefore$ y = 2 or 5 or 8.

$\therefore$ Number are (0, 2) or (0, 5) or (0, 8)

Case II When x = 5.

Then, sum of digits = (27 + y). For divisibility by 3, we must have y = 0 or 3 or 6 or 9.

$\therefore$ Numbers are (5, 0) or (5, 3) or (5, 6) or (5, 9).

Example 9

Which of the following numbers $\sqrt{\pi^2}, \sqrt[3]{0.8}, \sqrt[4]{0.00016}, \sqrt[3]{-1}, \sqrt{0.001}$ is/are rational ?

Solution

(A) $\sqrt{\pi^2} = \pi$ (irrational no.)

(B) $\sqrt[3]{0.8}$

(C) $\sqrt[4]{0.00016}$

(D) $\sqrt[3]{-1} = -1$ (rational no.)

(E) $\sqrt{0.001} = \sqrt{1/1000} = 0.1/\sqrt{10}$ (irrational no.)

Example 10

If $\dfrac{5+\sqrt{3}}{7-4\sqrt{3}} = 47a + \sqrt{3}\,b$ then find the value of 'a' and 'b'.

Solution

Taking L.H.S

$$= \frac{5+\sqrt{3}}{7-4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}} = \frac{(5+\sqrt{3})(7+4\sqrt{3})}{49-48} = 35 + 12 + 20\sqrt{3} + 7\sqrt{3}$$

$$= 47 + 27\sqrt{3} = 47a + 27\sqrt{3} \quad \text{[Given]}$$

Hence a = 1 and b = 27

Example 11

Find the value of :

$$\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}.$$

Solution

$$\left(\frac{1}{3-\sqrt{8}}\times\frac{3+\sqrt{8}}{3+\sqrt{8}}\right)-\left(\frac{1}{\sqrt{8}-\sqrt{7}}\times\frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}}\right)+\left(\frac{1}{\sqrt{7}-\sqrt{6}}\times\frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}\right)-\left(\frac{1}{\sqrt{6}-\sqrt{5}}\times\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}\right)+\left(\frac{1}{\sqrt{5}-2}\times\frac{\sqrt{5}+2}{\sqrt{5}+2}\right)$$

$$=\left(\frac{3+\sqrt{8}}{9-8}\right)-\left(\frac{\sqrt{8}+\sqrt{7}}{8-7}\right)+\left(\frac{\sqrt{7}+\sqrt{6}}{7-6}\right)-\left(\frac{\sqrt{6}+\sqrt{5}}{6-5}\right)-\left(\frac{\sqrt{5}+2}{5-4}\right)$$

$$=3+\sqrt{8}-\sqrt{8}-\sqrt{7}+\sqrt{7}+\sqrt{6}-\sqrt{6}-\sqrt{5}+\sqrt{5}+2$$

$= 5.$

Example 12

Three pieces of cakes of weights $4\frac{1}{2}$ Ibs, $6\frac{3}{4}$ Ibs and $7\frac{1}{5}$ Ibs respectively are to be divided into parts of equal weights. Further, each must be as heavy as possible. If one such part is served to each guest, then what is the maximum number of guests that could be entertained ?

Solution

$$\text{HCF}\left(\frac{9}{2},\frac{27}{4},\frac{36}{5}\right)=\frac{\text{HCF}(9,27,36)}{\text{LCM}(2,4,5)}=\frac{9}{20}\text{ Ibs}\qquad=\text{weight of each piece.}$$

$$\text{Total weight}=\frac{9}{2}+\frac{27}{4}+\frac{36}{5}=18.45$$

$$\text{Maximum no. of guests}=\frac{18.45\times 20}{9}=41.$$

CONCEPT APPLICATION LEVEL - I [NCERT Questions]

Q.1 Use Euclid's division algorithm to find the HCF of 196 and 38220.

Sol. $38220 = 196 \times 195 + 0$

Therefore, HCF (196, 38220) = 196. **Ans.**

Q.2 Find the HCF and LCM of 6, 72 and 120, using the prime factorization method.

Sol.

2	6
3	3
	1

2	72
3	36
2	18
3	9
3	3
	1

2	120
2	60
2	30
3	15
5	5
	1

$6 = 2 \times 3$

$72 = 2 \times 2 \times 2 \times 3 \times 3$

$120 = 2 \times 2 \times 2 \times 3 \times 5$

HCF (6, 72, 120) = $2 \times 3 = 6$.

LCM (6, 72, 120) = $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$. **Ans.**

Q.3 Express each number as a product of its prime factors :
(i) 3825 (ii) 5005

Sol. Prime factorization of 3825 & 5005 are :

(i)

3	3825
3	1275
5	425
5	85
17	17
	1

(ii)

5	5005
7	1001
11	143
13	13
	1

$3825 = 3 \times 3 \times 5 \times 5 \times 17$ **Ans.**

$5005 = 5 \times 7 \times 11 \times 13$. **Ans.**

Q.4 Find the LCM and HCF of the following integers by applying the prime factorization method 17, 23 and 29

Sol. LCM = $17 \times 23 \times 29 = 11339$

HCF = 1. **Ans.**

Q.5 Given that HCF (306, 657) = 9, find LCM (306, 657).

Sol. HCF = (306, 657) × LCM (306, 657) = 306 × 657

or $9 \times$ LCM (306, 657) = 306×657

LCM (306, 657) = $\dfrac{306 \times 657}{9} = 22338$. **Ans.**

Q.6 Check whether 6^n can end with the digit 0 for any natural number n.

Sol. Let for any natural number n. Then number 6^n ends with 0, then 6^n will be divisible by 5.

But $6^n = (2 \times 3)^n$

The prime factors of 6 are 2 and 3.

In prime factorisation of 6^n, there is no factor 5. Therefore 6^n is not divisible by 5 and there does not exist any natural number n for which 6^n ends with zero. **Ans.**

Q.7 Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.

Sol. $7 \times 11 \times 13 + 13 = 13(7 \times 11 \times 1 + 1)$
$= 13 \times (77 + 1)$
$= 13 \times 78$
= It is a composite number.
$7\ \ 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5\ (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$
$= 5 \times (1008 + 1)$
$= 5 \times 1009$ = It is a composite number. **Ans.**

Q.8 Prove that $3+2\sqrt{5}$ is irrational.

Sol. Let us assume, to the contrary, that $3+2\sqrt{5}$ is a rational number.

Now, $3+2\sqrt{5} = \frac{a}{b}$, (where a and b are integers, $b \neq 0$ and a and b are coprime.)

$\Rightarrow \quad 2\sqrt{5} = \frac{a}{b} - 3$

$\sqrt{5} = \frac{a}{2b} - \frac{3}{2}$

$\because$ a and b are integers

$\therefore \quad \frac{a}{2b} - \frac{3}{2}$ is a rational number.

$\Rightarrow \quad \sqrt{5}$ is rational number.

But $\sqrt{5}$ is an irrational number. This shows that our assumption is incorrect. Therefore, $3+2\sqrt{5}$ is irrational number. **Ans.**

Q.9 Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i) $\frac{13}{3125}$ **(ii)** $\frac{23}{2^3\ 5^2}$ **(iii)** $\frac{129}{2^2\ 5^7 7^5}$ **(iv)** $\frac{77}{210}$

Sol. (i) $\frac{13}{3125}$

we have, denominator $= 3125 = 5 \times 5 \times 5 \times 5 \times 5 = 5^5 = 2^0 \times 5^5 = 2^m \times 5^n$.

Therefore, $\frac{13}{3125}$ must have a terminating decimal expansion.

(ii) $\frac{23}{2^3\ 5^2}$ = denometor $= 2^3\ \ 5^2 = 2^m \times 5^n$

Therefore, $\frac{23}{2^3 5^2}$ must have a terminating.

(iii) $\frac{129}{2^2\ 5^7 7^5}$ denominator $= 2^2\ 5^7\ 7^5 \neq 2^m \times 5^n$. (Non-terminating).

(iv) $\frac{77}{210} = \frac{11}{30}$ denominator $= 30 = 2 \times 3 \times 5 \times 7 \neq 2^m \times 5^n$ (Non-terminating). **Ans.**

CONCEPT APPLICATION LEVEL - II [Previous Year Questions]

Multiple Choice Questions with one correct answer :

Q.1 The product of the additive inverse and multiplication inverse of 6 is :

(A) –6 (B) 1 (C) –1 (D) $\frac{1}{6}$

Q.2 The value of $\sqrt{3\sqrt{3\sqrt{3\sqrt{3\sqrt{3.......}}}}}$ is :

(A) 0 (B) 3 (C) Both 0 and 3 (D) Can't be determined

Q.3 Which of the following statements is correct?
(A) 0 is called the additive identity for rational numbers.
(B) 1 is called the multiplicative identity for rational numbers.
(C) The additive inverse of 0 is zero itself.
(D) All the above.

Q.4 The value of $\sqrt{80}-\sqrt{45}$ is :

(A) $\sqrt{35}$ (B) $\sqrt{5}$ (C) $4\sqrt{5}$ (D) $4\left(\sqrt{5}-\sqrt{3}\right)$

Q.5 The number $3^{11}-3^{8}$ is divisible by prime numbers:
(A) 2 and 3 only (B) 2, 3 and 11 only (C) 3^{11} only (D) 2, 3 and 13 only

Q.6 Find the unit digit in $132^{74}-(3498)^{49}$:
(A) 6 (B) 0 (C) 4 (D) 2

Q.7 Which of the following alternatives is wrong? Given that :
(i) Difference of two rational numbers is a rational number.
(ii) Subtraction is commutative on rational numbers
(iii) Addition is not commutative on rational numbers.
(A) (ii) & (iii) (B) (i) only (C) (i) & (iii) (D) All the above

Q.8 The average of the middle two rational numbers if $\frac{2}{5},\frac{1}{2},\frac{3}{4},\frac{5}{7}$ are arranged in ascending order is :

(A) $\frac{5}{4}$ (B) $\frac{9}{10}$ (C) $\frac{39}{35}$ (D) $\frac{17}{28}$

Q.9 Which is smallest among :
$10-3\sqrt{11}$, $18-5\sqrt{13}$ and $51-10\sqrt{26}$
(A) $10-3\sqrt{11}$ (B) $18-5\sqrt{13}$ (C) $51-10\sqrt{26}$ (D) All are equal

Q.10 The greater between $\sqrt{19}-\sqrt{14}$ and $\sqrt{12}-\sqrt{7}$
(A) $\sqrt{19}-\sqrt{14}$ (B) $\sqrt{12}-\sqrt{7}$ (C) Both are equal (D) Can't say

Q.11 The smallest among $\sqrt[2]{2}$, $\sqrt[3]{4}$ and $\sqrt[4]{3}$ is :
(A) $\sqrt[2]{2}$ (B) $\sqrt[4]{3}$ (C) $\sqrt[3]{4}$ (D) Can't compare

Q.12 A number that has to be added to 9247653140 in order to make it divisible by 8 is :
(A) 2 (B) 8 (C) 6 (D) 4

Q.13 In order that the six digit number 1x0x3x be divisible by 11, the digit x should be :
(A) 2 (B) 1 (C) 4 (D) 5

Q.14 What are the values of x and y if 15x 0468913y is divisible by 8 and 11, where x and y are single digit integers?
(A) x = 3, y = 6 (B) x = 6, y = 9 (C) x = 9, y = 12 (D) x = 0, y = 3

Q.15 What is the complete solution to the equation :
$|3 - 4x| = 13$?
(A) $x = \frac{5}{2}, x = 4$ (B) $x = \frac{5}{2}, x = -4$ (C) $x = -\frac{5}{2}, x = 4$ (D) $x = -\frac{5}{2}, x = -4$

Q.16 If $x^{p^q} = (x^p)^q$, then p = **[Harayana NTSE Stage-1_2013]**
(A) $q^{\frac{1}{q}}$ (B) 1 (C) q^q (D) $q^{\frac{1}{q-1}}$

Q.17 The rationalizing factor of $\sqrt[n]{\frac{a}{b}}$ is **[Karnataka NTSE Stage-1_2014]**
(A) $ab\sqrt[n]{\frac{a}{b}}$ (B) $\sqrt[n]{\frac{a}{b}}$ (C) $\sqrt[n]{\frac{a^{n-1}}{b^{n-1}}}$ (D) $\sqrt[n]{\frac{a^{n+1}}{b^{n+1}}}$

Q.18 The value of $\left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \cdot \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} \cdot \left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}}$ on simplifying is **[Rajasthan NTSE Stage-1_2015]**
(A) x (B) $\frac{1}{x}$ (C) 1 (D) –1

Q.19 Which real number lies between 2 and 2.5 **[Chandigarh NTSE Stage-1_2014]**
(A) $\sqrt{11}$ (B) $\sqrt{8}$ (C) $\sqrt[3]{7}$ (D) $\sqrt[3]{9}$

Q.20 The HCF of any two prime numbers a and b is **[Rajasthan NTSE Stage-1_2015]**
(A) a (B) ab (C) b (D) 1

Q.21 The traffic lights at three different singals change after 48 seconds, 72 seconds and 108. If they change at 7 a.m. simultaneously. How many times they will change between 7 a.m. to 7.30 a.m. simultaneously? **[Harayana NTSE Stage-1_2015]**
(A) 3 (B) 4 (C) 5 (D) 2

Q.22 Which of the following is rational number? **[IMO-2016]**
(A) Sum of $(2+\sqrt{3})$ and its reciprocal (B) Square root of 18
(C) Square root of $7+4\sqrt{3}$ (D) None of these

Q.23 Expressing $0.\overline{23} + 0.2\overline{3}$ as a single decimal, we get **[NTSE-2016]**
(A) $0.46\overline{5}$ (B) $0.4\overline{65}$ (C) $0.\overline{465}$ (D) $0.465\overline{4}$

CONCEPT APPLICATION LEVEL - III

SECTION-A

Subjective Type Questions :

Q.1 Find the largest number that will divide 398, 436 and 542 leaving remainders 7, 11 and 15 respectively.

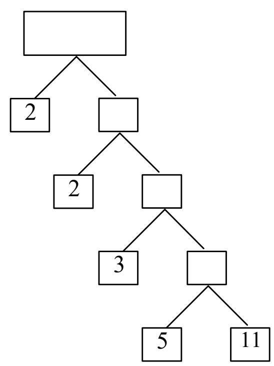
Q.2 Find the largest number that divides 2053 and 967 and leaves a remainder of 5 and 7 respectively.

Q.3 In a seminar, the number of participants in Hindi, English and Mathematics are 60, 84 and 108, respectively. Find the minimum number of rooms required if in each room the same number of participants are to be seated and all of them being in the same subject.

Q.4 Three sets of English, Hindi and Mathematics books have to be stacked in such a way that all the books are stored topic wise and the height of each stack is the same. The number of English books is 96, the number of Hindi books is 240 and the number of Mathematics books is 336. Assuming that the books are of the same thickness, determine the number of stacks of English, Hindi and Mathematics books.

Q.5 Prove that $\sqrt{2}+\sqrt{5}$ is irrational.

Q.6 Find the missing number in the following factorisation sequence.



Q.7 Find the largest number that divides 2053 and 967 leaves remainder of 5 and 7 respectively.

Q.8 Two tankers contains 850 litres and 680 litres of petrol respectively. Find the maximum capacity of a container which can measure the petrol of either tank in exact number of times.

Q.9 The length, breadth and height of a room are 8 m 25 cm, 6 m 75 cm and 4m 50 cm respectively. Determine the longest rod which can measure the three dimensions of the room exactly.

Q.10 If a number N= 126, then

Find : (i) The number of divisors of N. (ii) The sum of divisors of N.
(iii) The product of divisors of N. (iv) The number of distinct prime divisors.
(v) The number of proper divisors of N.

Q.11 Find the HCF of $\frac{2}{3},\frac{3}{4}$ and $\frac{6}{7}$.

Q.12 Find the LCM of $\frac{5}{6},\frac{10}{17}$ and $\frac{15}{16}$.

SECTION-B

Multiple choice questions with one correct answer :

Q.1 If the LCM of a and 18 is 36 and the HCF of a and 18 is 2, then a =
(A) 2 (B) 3 (C) 4 (D) 1

Q.2 The number of decimal places after which the decimal expansion of the rational number $\frac{23}{2^2 \times 5}$ will terminate, is
(A) 1 (B) 2 (C) 3 (D) 4

Q.3 If n is a natural number, then $9^{2n} - 4^{2n}$ is always divisible by
(A) 5 (B) 13 (C) both 5 and 13 (D) None of these

Q.4 If $x^{1/8} = m$ and $x^{1/4} = n$ and $n = 4$ m, then find the value of $\sqrt{x}$.
(A) 512 (B) 216 (C) 324 (D) 256

Q.5 The LCM of two numbers is 567 and their HCF is 9. If the difference between the two numbers is 18, find the two numbers :
(A) 36 and 18 (B) 78 and 60 (C) 63 and 81 (D) 52 and 34

Q.6 If n is any natural number, then $6^n - 5^n$ always ends with
(A) 1 (B) 3 (C) 5 (D) 7

Q.7 The smallest number by which $\sqrt{27}$ should be multiplied so as to get a rational number is
(A) $\sqrt{27}$ (B) $3\sqrt{3}$ (C) $\sqrt{3}$ (D) 3

Q.8 If $a = 2^3 \times 3$, $b = 2 \times 3 \times 5$ and LCM (a, b, c) = $2^3 \times 3^2 \times 5$, then c =
(A) 1 (B) 9 (C) 3 (D) 4

Q.9 Which of the following is a pair of coprimes?
(A) (14, 35) (B) (18,25) (C) {31,93} (D) (32,62)

Q.10 HCF of $2^3 \times 3^2 \times 5$, $2^3 \times 3^3 \times 5^2$ and $2^2 \times 3 \times 5^3 \times 7$ is
(A) 30 (B) 48 (C) 60 (D) 105

Q.11 The product of two numbers is 2160 and their GCD is 12. The numbers are
(A) 72, 30 (B) 36, 60 (C) 96, 25 (D) None

Q.12 LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one number is 280, then the other is
(A) 40 (B) 60 (C) 80 (D) 100

Q.13 The ratio of two numbers is 3 : 4 and their HCF is 4. Then LCM is
(A) 12 (B) 16 (C) 24 (D) 48

Q.14 The HCF of $(4a^2b^3 - 9b)$ and $(2a^2b^2 - ab - 3)$ is
(A) (2a – 3) (B) (2ab – 3) (C) (2b – 3a) (D) None of these

Q.15 What is the greatest number which shall divide 305 and 629 and leave a remainder 8 in each case?
(A) 24 (B) 36 (C) 27 (D) 35

SECTION-C

Multiple choice Questions with one or more than one correct answers :

Q.1 If n is a natural number, then $\sqrt{n}$ can be

(A) a natural number (B) always an rational number

(C) an irrational number (D) always a natural number

Q.2 Which of the following statements for natural numbers a, b and c is/are true

(A) If a is divisible by b and b is divisible by c, then a must be divisible by c.

(B) If a is a factor of both b and c, then a must be a factor of b+c

(C) If a is a factor of both b and c then a must be a factor of b–c.

(D) If a is a factor of b and b anc c are coprime, then a,c must also be coprimers.

Q.3 Which of the following rational numbers have terminating decimal expansion

(A) 64/455 (B) 29/343 (C) 13/325 (D) 1/308

Q.4 The number $\sqrt{14+6\sqrt{5}}+\sqrt{14-6\sqrt{5}}$ is,

(A) is a rational numbers (B) is not a rational number

(C) simplifies to 5 (D) simplifies to 6

Q.5 The number $\sqrt{3+2\sqrt{2}}-\sqrt{3-2\sqrt{2}}$ can be

(A) rational number (B) natural number (C) prime number (D) irrational number

Q.6 Simplify: $1+\cfrac{6}{5+\cfrac{4}{3-\cfrac{1}{2}}}$

(A) $\frac{14}{45}$ (B) $\frac{63}{33}$ (C) $\frac{13}{19}$ (D) $\frac{21}{11}$

SECTION-D

Match the following (one to one) :

Column-I and **column-II** contain **four** entries each. One entry of column-I is to be matched with one entry of column-II.

Q.1

Column I		Column II
(i)	–10	(P) Natural number
(ii)	π	(Q) Integer but not a natural number
(iii)	3	(R) Rational number but not an integer
(iv)	5/2	(S) Irrational

Q.2

Column I		Column II	
(i)	Every odd integer is of the form 2m – 1 where m is	(P)	Rational
(ii)	$3.\overline{27}$ is	(Q)	Equal
(iii)	The LCM and HCF of two rational numbers are equal then the numbers must be	(R)	Integer
(iv)	$\sqrt{2}+\sqrt{5}$ is	(S)	Irrational

Q.3

Column I		Column II	
(i)	$\frac{-10}{15}$	(P)	1
(ii)	$3+2\sqrt{5}$	(Q)	Prime number
(iii)	$\left(\frac{2p^2q}{xy}\right)^0$	(R)	Irrational
(iv)	2	(S)	Fractional but not natural number

Match the following (one to many) :

Column-I and **column-II** contains **four** entries each. Entries of column-I are to be matched with some entries of column-II. One or more than one entries of column-I may have the matching with the some entries of column-II and one entry of column-II may have one or more than one matching with entries of column-I

Q.4

Column I		Column II	
(i)	If two irrational numbers are added then sum may be	(P)	Irrational number
(ii)	Division of a natural number by another natural number gives	(Q)	Rational number
(iii)	Natural number	(R)	Neither rational nor irrational
(iv)	Every real number	(S)	None of these

Q.5

Column I		Column II	
(i)	The sum of two prime numbers	(P)	Natural number
(ii)	The product of three consecutive natural numbers is	(Q)	Even number
(iii)	Sum of two even numbers	(R)	Odd number
(iv)	Sum of two odd numbers	(S)	None of these

ANSWER KEY

CONCEPT APPLICATION LEVEL - II

Multiple Choice Questions with one correct answer :

Q.1	C	Q.2	B	Q.3	D	Q.4	B	Q.5	D
Q.6	A	Q.7	A	Q.8	D	Q.9	B	Q.10	B
Q.11	B	Q.12	D	Q.13	D	Q.14	A	Q.15	C
Q.16	D	Q.17	C	Q.18	C	Q.19	D	Q.20	D
Q.21	B	Q.22	A	Q.23	B				

CONCEPT APPLICATION LEVEL - III

SECTION-A

Q.1	17	Q.2	64	Q.3	12	Q.4	14

Q.6 Going upwards 55, 165, 330, 660

Q.7 64

Q.8 170 litres

Q.9 75 cm

Q.10 (i) 12 (ii) 312 (iii) 126^6 (iv) 3 (v) 10

Q.11 $\frac{1}{84}$

Q.12 30

SECTION-B

Q.1	C	Q.2	B	Q.3	C	Q.4	D	Q.5	C
Q.6	A	Q.7	C	Q.8	B	Q.9	B	Q.10	C
Q.11	B	Q.12	C	Q.13	D	Q.14	B	Q.15	C

SECTION-C

Q.1	A,C	Q.2	A,B,C,D	Q.3	C	Q.4	A,D
Q.5	A, B, C	Q.6	B,D				

SECTION-D

Q.1 (i)-(Q), (ii)-(S), (iii)-(P), (iv)-(R)

Q.2 (i)-(R), (ii)-(P), (iii)-(Q), (iv)-(S)

Q.3 (i)-(S), (ii)-(R), (iii)-(P), (iv)-(Q)

Q.4 (i)-(P,Q), (ii)-(Q), (iii)-(Q), (iv)-(P,Q)

Q.5 (i)-(PQR), (ii)-(PQ), (iii)-(Q), (iv)-(Q)