Class Xth – <u>Real Numbers</u>

Q.1   A number when divided by 53 gives 34 as quotient and 21 as remainder. Find the number.

1. 1823

Q.2   Show that every positive even integer is of the form 2q and that every positive odd integer is of the form 2q + 1 for some integer q.

Q.3   Show that every positive odd integer is of the form (4q + 1) or (4q + 3) for some integer q.

Q.4   If n is an odd positive integer, show that $(n^2 - 1)$ is divisible by 8.

Q.5   Show that every positive odd integer is of the form (6q + 1) or (6q + 3) or (6q + 5) for some integer q.

Q.6   Show that every positive integer is of the form 2q and that every positive odd integer is of the form (2q + 1) for some positive integer q.

Q.7   Show that one and only one out of n, (n + 1) and (n + 2) is divisible by 3, where n is any positive integer.

Q.8   Show that one and only one out of n, n + 2, n + 4 is divisible by 3, where n is any positive integer.
                                                                            [CBSE 2008C]

Q.9   Use Euclid's Division Lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

Q.10   Using Euclid's Division Lemma, show that the cube of any positive integer is of the form 9q or (9q + 1) or (9q + 8) for some integer q.   **[CBSE 2009C]**

Q.11   Show that any number of the form 4^n, n ∈ N can never end with the digit 0.

Q.12   Show that any number of the form 6^n, where n ∈ N can never end with the digit 0.

Q.13   Use Euclid's algorithm to find the HCF of 408 and 1032.

13. 24

Q.14   Find the HCF and LCM of 612 and 1314 using prime factorization method.

14. 44676

Q.15   The HCF of two number is 27 and their LCM is 162. If one of the number is 54, find the other.

15. 81

Q.16   Find the HCF and LCM of 60, 84 and 108 using prime factorization method.

16. 3780

Q.17   Find the largest number which divides 248 and 1032 leaving remainder 8 in each case.

17. 24

Q.18   Find the largest number which divides 546 and 764, leaving remainders 6 and 8 respectively.

18. 108

Q.19   Find the simplest form of $\dfrac{148}{185}$.

19. $\dfrac{4}{5}$

Q.20   Two contain 504 and 735 litres of milk respectively. Find the maximum capacity of a container which can measure the milk of either tank an exact number of times.

20. 21

Q.21 Ravi and Sikha drive around a circular sports field. Ravi takes 16 minutes to take one round, while Sikha completes the round in 20 minutes. If both start at the same point, at the same time and go in the same direction, after much time will they meet at the stating point?

21. 80

EXERCISE 1A

Q 1. What do you mean by Euclid's division algorithm.

Q 2. A number when divided by 61 gives 27 as quotient and 32 as remainder. Find the number.

Q 3. By what number should 1365 be divided to get 31 as quotient and 32 as remainder?

Q 4. Using Euclid's algorithm, find the HCF of
(i) 405 and 2520 (ii) 504 and 1188 (iii) 960 and 1575

Q 5. Using prime factorisation, find the HCF and LCM of
(i) 144, 198 (ii) 396, 1080 (iii) 1152, 1664

Q 6. Using prime factorisation, find the HCF and LCM of
(i) 24,36,40 (ii) 30,72,432 (iii) 21,28,36,45

Q 7. The HCF of two numbers is 23 and their LCM is 1449. If one of the numbers is 161, find the other.

Q 8. The HCF of two numbers is 11 and their LCM is 7700. If one of the numbers is 275, find the other.

Q 9. Find the missing numbers in the following factorisation:
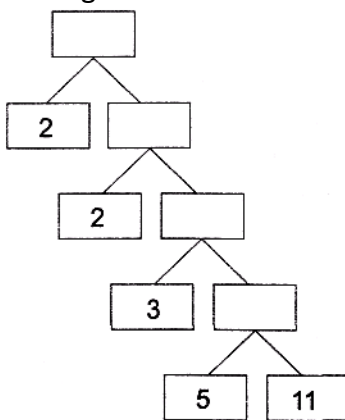
Q 10. Find the largest number which divides 378 and 510 leaving remainder 6 in each case.

Q 11. Find the largest number which divides 320 and 457 leaving remainders 5 and 7 respectively.

Q 12. Find the simplest form of

(i) $\dfrac{69}{92}$ (ii) $\dfrac{561}{748}$ (iii) $\dfrac{1095}{1168}$

Q 13. Three pieces of timber 42 m, 49 m and 63 m long have to be divided into planks of the same length. What is the greatest possible length of each plank?

Q 14. Find the greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm and 12 m 95 cm.

Q 15. Find the maximum number of students among whom 1001 pens and 910 pencils can be distributed in such a way that each student gets the same number of pens and the same number of pencils.

Q 16. Three sets of English, Mathematics and Science books containing 336, 240 and 96 books respectively have to be stacked in such a way that all the books are stored subjectwise and the height of each stack is the same. How many stacks will be there?

Q 17. Find the least number of square tiles reqired to pave the ceiling of a room 15 m 17 cm long and 9 m 2 cm broad.

Q 18. Three measuring rods are 64 cm, 80 cm and 96 cm in length. Find the least length of cloth that can be measured an exact number of times, using any of the rods.

Q 19. The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they all change simultaneously at 8 hours, then at what time will they again change simultaneously?

Q 20. An electronic device makes a beep after every 60 seconds. Another device makes a beep after every 62 seconds. They beeped together at 10 a.m. At what time will they beep together at the earliest?

Q 21. Six bells commence tolling together and toll at intervals of 2,4,6,8,10, 12 minutes respectively. In 30 hours, how many times do they toll together?

ANSWERS (EXERCISE 1A)

2.1679 3. 43 4. (i) 45 (ii) 36 (iii) 15 5. (i) HCF = 18, LCM = 1584 (ii) HCF = 36, LCM = 11880 (iii) HCF = 128, LCM = 14976 6. (i) HCF = 4, LCM = 360 (ii) HCF = 6, LCM = 2160 (iii) HCF = 1, LCM = 1260 7.207 8.308 9.

From last number to first (55,165, 330, 660) 10.12 11.45 12.(i) $\dfrac{3}{4}$ (ii) $\dfrac{3}{4}$ (iii) $\dfrac{15}{16}$ 13.7m 14.35cm 15.91

16.14 17.814 18.9.6 m 19. 8 : 7 :12 hrs 20.10 : 31 hrs 21.16 times

Solved Examples

Q.1 Without actual division show that each of the following rational numbers is a non-terminating repeating decimal:

(i) $\dfrac{121}{(2^3 \times 3^2 \times 7^5)}$ (ii) $\dfrac{17}{90}$ (iii) $\dfrac{53}{343}$ Iv) $\dfrac{66}{180}$

Q.2 Without actual division, show that each of the following rational numbers is a terminating decimal. Express each in decimal form:

(i) $\dfrac{31}{(2^2 \times 5^3)}$ (ii) $\dfrac{33}{50}$ (iii) $\dfrac{17}{625}$ (v) $\dfrac{41}{1000}$

Q.3 The decimal expansion of the rational number $\dfrac{43}{2^4 . 5^3}$, will terminate after how places of decimals?

[CBSE 2009]

3. 0.0215

Q.4 Express each of the following as a rational number in simplest form:

(a) $0.\overline{6}$ (b) $1.\overline{8}$ (c) $0.1\overline{6}$

4. (a) $\dfrac{2}{3}$ (b) $\dfrac{8}{9}$ (c) $\dfrac{1}{6}$

Q.5 Express $0.\overline{32}$ as a fraction in simplest form.

5. $\dfrac{32}{99}$

Q.6 Express $0.2\overline{54}$ as a fraction in simplest form.

6. $\dfrac{14}{55}$

EXERCISE 1B

Q 1. Without actual division, show that each of the following rational numbers is a non-terminating repeating decimal:

(i) $\dfrac{11}{(2^3 \times 3)}$ (ii) $\dfrac{73}{(2^3 \times 3^3 \times 5)}$ (iii) $\dfrac{9}{35}$ (iv) $\dfrac{32}{147}$

(v) $\dfrac{64}{455}$ (vi) $\dfrac{77}{210}$ (vii) $\dfrac{29}{343}$ (viii) $\dfrac{1269}{(2^2 \times 5^7 \times 7^5)}$

Q 2. Without actual division, show that each of the following rational numbers is a terminating decimal. Express each in decimal form:

(i) $\dfrac{23}{(2^3 \times 5^2)}$ (ii) $\dfrac{24}{125}$ (iii) $\dfrac{17}{320}$ (iv) $\dfrac{171}{800}$

(v) $\dfrac{15}{1600}$ (vi) $\dfrac{19}{3125}$

Q 3. Express each of the following as a fraction in simplest form:
(i) $0.\overline{8}$ (ii) $2.\overline{4}$ (iii) $0.\overline{24}$ (iv) $0.1\overline{2}$ (v) $2.2\overline{4}$ 4 (vi) $2.3\overline{65}$

Q 4. Decide whether the given number is rational or not:
(1) 53.123456789 (ii) $31.\overline{123456789}$ (iii) 0.12012001200012 … Give reason to support your answer.

ANSWERS (EXERCISE 1B)

2. (i) 0.115 (ii) 0.192 (iii) 0.053125 (iv) 0.21375 (v) 0.009375 (vi) 0.00608

3. (i) $\dfrac{8}{9}$ (ii) $\dfrac{22}{9}$ (iii) $\dfrac{8}{33}$ (iv) $\dfrac{11}{90}$ (v) $\dfrac{101}{45}$ (vi) $\dfrac{181}{495}$

4. (i) rational, since it is a terminating decimal (ii) rational, since it is a repeating decimal (iii) not rational, since it is a non-terminating and non-repeating decimal

Solved Examples

Q.1 Show that $2\sqrt{3}$ is irrational

Q.2 Show that $3\sqrt{5}$ is irrational.

Q.3 If a is a nonzero rational and $\sqrt{b}$ is irrational, then show that $a\sqrt{b}$ is

Q.4 Show that $(2 + \sqrt{3})$ is an irrational number. **[CBSE 2008C, 09]**

Q.5 Show that $(4 - \sqrt{3})$ is irrational **[CBSE 2008]**

Q.6 If a is rational and $\sqrt{b}$ be irrational, then prove that $(a + \sqrt{b})$ is irrational.

Q.7 Prove that $(3 + 5\sqrt{2})$ is irrational. **[CBSE 2009]**

Q.8 Show that $(5 - 2\sqrt{3})$ is irrational. **[CBSE 2008]**

Q.9 Show that $\dfrac{1}{\sqrt{2}}$ is irrational

Q.10. Write a rational number between $\sqrt{2}$ and $\sqrt{3}$. **[CBSE 2008]**

10. $\dfrac{3}{2}$

Q.11 Prove that $\sqrt{5}$ is irrational **[CBSE 2008, '09]**

Q.12 Prove that $(\sqrt{2} + \sqrt{3})$ is irrational.

EXERCISE 1C

Q 1. Define (i) rational numbers (ii) irrational numbers (iii) real numbers.

Q 2. Classify the following numbers as rational or irrational:

(i) $\dfrac{22}{7}$ (ii) 3.1416 (iii) π (iv) $3.\overline{142857}$

(v) 5.636363... (vi) 2.040040004... (vii) 1.535335333... (viii) 3.121221222... (ix) $\sqrt{21}$ (x) $\sqrt[3]{3}$

Q 3. Prove that each of the following numbers is irrational:

(i) $\sqrt{6}$ (ii) $(2 - \sqrt{3})$ **[CBSE 2008]**

(iii) $(3 + \sqrt{2})$ **[CBSE 2009]** (iv) $(2 + \sqrt{5})$ **[CBSE 2008C]**

(v) $(5 + 3\sqrt{2})$ **[CBSE 2008]** (vi) $3\sqrt{7}$ (vii) $\dfrac{3}{\sqrt{5}}$ (viii) $(2 - 3\sqrt{5})$ **[CBSE 2010]**

(ix)$(\sqrt{3} + \sqrt{5})$

Q 4. Prove that $\dfrac{1}{\sqrt{3}}$ is irrational.

Q 5. (i) Give an example of two irrationals whose sum is rational.
(ii) Give an examples of two irrationals whose product is rational.
HINT (i) Take $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$. (ii) Take $(3 + \sqrt{2})$ and $(3 - \sqrt{2})$.

Q 6. State whether the given statement is true or false:
(i) The sum of two rationals is always rational.
(ii) The product of two rationals is always rational.
(iii) The sum of two irrationals is an irrational.
(iv) The product of two irrationals is an irrational.

(v) The sum of a rational and an irrational is irrational.

(vi) The product of a rational and an irrational is irrational.

ANSWERS (EXERCISE 1C)

2. (i) rational (ii) rational (iii) irrational (iv) rational (v) rational (vi) irrational (vii) irrational (viii) irrational (ix) irrational (x) irrational

6. (i) True (ii) True (iii) False (iv) False (v) True (vi) True

CCE QUESTIONS

Objective Questions MCQ (2 marks)

Q 1. Euclid's division lemma states that for any positive integers a and b, there exist unique integers q and r such that a = bq + r, where r must satisfy

(a) 1 < r < b   (b) 0 < r ≤ b   (c) 0 ≤ r < b   (d) 0 < r < b

Q 2. What is the largest number that divides each one of 1152 and 1664 exactly?

(a) 32   (b) 64   (c) 128   (d) 256

Q 3. What is the largest number that divides 70 and 125, leaving remainders 5 and 8 respectively?

(a) 13   (b) 9   (c) 3   (d) 585

Q 4. What is the largest number that divides 245 and 1029, leaving remainder 5 in each case?

(a) 15   (b) 16   (c) 9   (d) 5

Q 5. The simplest form of $\dfrac{1095}{1168}$ is

(a) $\dfrac{17}{26}$ (b) $\dfrac{25}{26}$ (c) $\dfrac{13}{16}$ (d) $\dfrac{15}{16}$

Q 6. If the HCF of 65 and 117 is of the form (65m - 117), then m = ?

(a) 1   (b) 2   (c) 3   (d) 4

Q 7. A number when divided by 143 leaves 31 as remainder. What will be the remainder when the same number is divided by 13?

(a) 0   (b) 1   (c) 3   (d) 5

Q 8. For some positive integer m, every positive even integer is of the form

(a) m - l   (b) m + l   (c) 2m   (d) 2m + l

Q 9. For some positive integer n, every positive odd integer is of the form

(a) n   (b) n + l   (c) 2n   (d) 2n + l

Q 10. $(n^2 -1)$ is divisible by 8, if n is

(a) any natural number   (b) any integer

(c) any odd positive integer   (d) any even positive integer

Q 11. If a and b are positive integers such that $a = x^3y^2$ and $b = xy^3$, where x, y are prime numbers, then HCF (a, b) = ?

(a) xy   (b) xy^2 (c) x^2y^2 (d) x^3y^3

Q 12. If p and q are positive integers such that $p = ab^2$ and $q = a^3b$, where a, b are prime numbers, then LCM (p, q) = ?

(a) ab   (b) a^2b^2 (c) a^3b^2 (d) a^3b^3

Q 13. If $a = (2^2 \times 3^3 \times 5^4)$ and $b = (2^3 \times 3^2 \times 5)$, then HCF (a, b) = 1

(a) 90   (b) 180   (c) 360   (d) 540

Q 14. HCF of $(2^3 \times 3^2 \times 5)$, $(2^2 \times 3^3 \times 5^2)$ and $(2^4 \times 3 \times 5^3 \times 7)$ is

(a) 30   (b) 48   (c) 60   (d) 105

Q 15. Which of the following is a pair of co-primes?

(a) (14,35)   (b) (18,25)   (c) (31,93)   (d) (32,62)

HINT (a, b) are co-primes, if HCF (a, b) - 1

Q 16. LCM of $(2^3 \times 3 \times 5)$ and $(2^4 \times 5 \times 7)$ is

(a) 40   (b) 560   (c) 1680   (d) 1120

Q 17. What is the least number that is divisible by all the natural numbers from 1 to 10 (both inclusive)?

(a) 100   (b) 1260   (c) 2520   (d) 5040

Q 18. The HCF of two numbers is 27 and their LCM is 162. If one of the numbers is 54, what is the other number?

(a) 36   (b) 45   (c) 9   (d) 81

Q 19. The product of two numbers is 1600 and their HCF is 5. The LCM of the numbers is

(a) 8000   (b) 1600   (c) 320   (d) 1605

Q 20. A positive integer n when divided by 9, gives 7 as remainder. What will be the remainder when (3n -1) is divided by 9?

(a) l   (b) 2   (c) 3   (d) 4

Q 21. a and b are two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5. Then, the least prime factor of (a + b) is

(a) 2   (b) 3   (c) 5   (d) 8

Q 22. The decimal expansion of the rational number $\dfrac{37}{2^2 \times 5}$ will terminate after

(a) one decimal place (b) two decimal places (c) three decimal places (d) four decimal places

Q 23. The decimal expansion of the number $\dfrac{14753}{1250}$ will terminate after

(a) one decimal place (b) two decimal places (c) three decimal places (d) four decimal places

Q 24. Which of the following rational numbers is expressible as a terminating decimal?

(a) $\dfrac{124}{165}$ (b) $\dfrac{131}{30}$ (c) $\dfrac{2027}{625}$ (d) $\dfrac{1625}{462}$

Q 25. Which of the following rational numbers is expressible as a non-terminatine repeating decimal?

(a) $\dfrac{1351}{1250}$ (b) $\dfrac{2017}{250}$ (c) $\dfrac{3219}{1800}$ (d) $\dfrac{1723}{625}$

Q 26. The product of a non-zero rational and an irrational number is

(a) always rational   (b) always irrational   (c) rational or irrational (d) 1

Q 27. 3.24636363... is
(a) an integer   (b) a rational number (c) an irrational number (d) none of these

Q 28. 2.13113111311113... is
(a) an integer   (b) a rational number (c) an irrational number (d) none of these

Q 29. Which of the following is an irrational number?
(a) $\dfrac{22}{7}$ (b) 3.1416   (c) $3.\overline{1416}$ (d) 3.141141114...

Q 30. π is
(a) an integer   (b) a rational number (c) an irrational number (d) none of these

Q 31. $2.\overline{35}$ is
(a) an integer   (b) a rational number (c) an irrational number (d) none of these

Q 32. $1.23\overline{48}$ is
(a) an integer   (b) an irrational number (c) a rational number (d) none of these

Q 33. $\sqrt{3}$ is
(a) an integer   (b) a rational number (c) an irrational number (d) none of these

Q 34. $2\sqrt{5}$ is
(a) an integer   (b) an irrational number   (c) a rational number (d) none of these

Q 35. $(2 + \sqrt{2})$ is
(a) an integer   (b) a rational number (c) an irrational number (d) none of these

Q 36. $\sqrt{12}$ is
(a) an integer   (b) a rational number (c) an irrational number (d) none of these

Q 37. $\dfrac{1}{\sqrt{2}}$ is
(a) a fraction   (b) a rational number (c) an irrational number (d) none of these

MCQ Assertion-and-Reason Type (2 marks)

Each question consists of two statements, namely. Assertion (A) and Reason (R). For selecting the correct answer, use the following code:
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).
(c) Assertion (A) is true and Reason (R) is false.
(d) Assertion (A) is false and Reason (R) is true.

Assertion (A)	Reason (R)
$\dfrac{123}{6250}$ is a terminating decimal.	The rational number $\dfrac{p}{q}$ is a terminating decimal, if q = ($2^m \times 5^n$) for some whole numbers m and n.

The correct answer is: (a)/(b)/(c)/(d).

Assertion (A)	Reason (R)
$\dfrac{1242}{49}$ is a non-terminating, repeating decimal.	The rational number $\dfrac{p}{q}$ is a terminating decimal, if q = ($2^m \times 5^n$) for some whole numbers m and n.

The correct answer is: (a)/(b)/(c)/(d).

Assertion (A)	Reason (R)
$\sqrt{8}$ is an irrational number.	If m is a natural number which is not a perfect square, then $\sqrt{m}$ is irrational.

The correct answer is: (a)/(b)/(c)/(d).

Assertion (A)	Reason (R)
The HCF of two numbers is 9 and their LCM is 2016. If one of the numbers is 54, then the other is 306.	tor any two positive integers a and b, we have: HCF (a, b) × LCM (a,b) = a × b.

The correct answer is: (a)/(b)/(c)/(d).

Matching of columns (2 marks)

Q 42. Match the following columns:

Colum I	Column II
(a) $\sqrt[3]{6}$.	(p) terminates after 4 places of decimal
(b) 0.3465	(q) is irrational
(c) $\dfrac{1234}{625}$	(r) terminates after 3 places of decimal
(d) $\dfrac{3416}{125}$	(s) is a non-terminating and repeating decimal

The correct answer is :

(a)-.......   (b)-......,   (c)-......,   (d)-......

ANSWERS

l.(c) 2.(c) 3. (a) 4.(b) 5.(d) 6. (b) 7. (d) 8. (c) 9.(d) 10. (c) 11. (b) 12. (c) 13. (b) 14. (c) 15. (b) 16, (c)
17. (c) 18. (d) 19. (c) 20. (b) 21. (a) 22. (b) 23. (d) 24. (c) 25. (c) 26. (b) 27. (b) 28. (c) 29. (d) 30. (c) 31. (b) 32. (c)
33. (c) 34. (b) 35. (c) 36. (c) 37. (c) 38. (a) 39. (a) 40. (a) 41. (d)
42. (a)-(q), (b)-(s), (c)-(p), (d)-(r)