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MATHEMATICS

CLASS-XII-CBSE

DIFFERENTIABILITY
EXRCISE 5.2

Prepared By:
PARDEEP KUMAR

DNG TUTORIAL
Crack the Concepts, Not just the Exams

1. $\sin(x^2+5)$

$\therefore \frac{dy}{dx} = \frac{d}{dx}(\sin(x^2+5)) = \cos(x^2+5)\frac{d}{dx}(x^2+5)$

$\because \frac{d}{dx}\sin x = \cos x$

$= \cos(x^2+5)(2x) = 2x\cdot\sin(x^2+5)$

$\therefore \frac{dy}{dx} = 2x\cos(x^2+5)$ Ans.

2 $\cos(\sin x)$

$\therefore \frac{dy}{dx} = \frac{d}{dx}\cos(\sin x) = -\sin(\sin x)\frac{d}{dx}(\sin x)$

$= -\sin(\sin x)\cdot\cos x$ $\left[\text{using } \frac{d}{dx}\cos x = -\sin x\right]$

$\therefore \frac{dy}{dx} = -\cos x\sin(\sin x)$ Ans.

3 $\sin(ax+b)$

$\frac{dy}{dx} = \frac{d}{dx}\sin(ax+b) = \cos(ax+b)\frac{d}{dx}(ax+b)$

$= \cos(ax+b)(a)$

$\frac{dy}{dx} = a\cos(ax+b)$ Ans.

4 $\sec(\tan\sqrt{x})$

$\frac{dy}{dx} = \frac{d}{dx}\sec(\tan\sqrt{x}) = \sec(\tan\sqrt{x})\tan(\tan\sqrt{x})\frac{d}{dx}\tan\sqrt{x}\cdot\frac{d}{dx}\sqrt{x}$

$= \sec(\tan\sqrt{x})\tan(\tan\sqrt{x})\cdot\sec^2\sqrt{x}\cdot\frac{1}{2\sqrt{x}}$

$\left\{\begin{array}{l}\frac{d}{dx}\tan x = \sec^2 x \\ \frac{d}{dx}\cdot\sqrt{x} = \frac{1}{2\sqrt{x}}\end{array}\right.$

5 $\frac{\sin(ax+b)}{\cos(cx+d)}$

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\sin(ax+b)}{\cos(cx+d)}\right) = ?$

$$\frac{dy}{dx} = \frac{d}{dx}\frac{\sin(ax+b)}{\cos(cx+d)} = \frac{\cos(cx+d)\frac{d}{dx}\sin(ax+b) - \sin(ax+b)\frac{d}{dx}\cos(cx+d)}{(\cos(cx+d))^2}$$

$$\left[\text{using } \left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}\right]$$

$$= \frac{\cos(cx+d)\cos(ax+b) - \sin(ax+b)\left[-\sin(cx+d)\right]}{(\cos(cx+d))^2}$$

$$= \frac{a\cos(cx+d)\cos(ax+b) + c\sin(ax+b)\sin(cx+d)}{(\cos(cx+d))^2}$$

$$= \frac{a\cos(cx+d)\cos(ax+b) + c\sin(ax+b)\cancel{\sin}\sin(cx+d)}{\cos^2(cx+d)}$$

6 $\cancel{\cos^3 x \sin^2(x^5)}$

$\cos x^3 \sin^2(x^5)$

Let $y = \cos x^3 \sin^2(x^5) = \cos x^3 (\sin(x^5))^2$

$\therefore \quad \frac{dy}{dx} = \cos x^3 \frac{d}{dx}(\sin x^5)^2 + (\sin x^5)^2 \frac{d}{dx}(\cos x^3)$

(using product rule)

$\frac{dy}{dx} = \cos x^3 \cdot 2\sin x^5 \frac{d}{dx}\sin x^5 + (\sin x^5)^2(-\sin x^3)\frac{d}{dx}x^3$

$= 2\cos x^3 \sin x^5 \cos x^5 (5x^4) + \sin^2 x^5(-\sin x^3)\cdot(3x^2)$

$= 10x^4 \cos^3 x \sin x^5 \cos x^5 - 3x^2 \sin^2 x^5 \sin x^3$

OR

$= x^2 \sin x^5 (10x^2 \cos^3 x \cos x^5 - 3\sin x^5 \sin x^3)$

Q.No.7 $2\sqrt{\cot(x^2)}$

Let $y = 2\sqrt{\cot(x^2)} = 2(\cot(x^2))^{\frac{1}{2}}$

$$\frac{dy}{dx} = 2\,\frac{1}{2}(\cot x^2)^{1/2-1}\frac{d}{dx}\cot(x^2)$$

$$= (\cot x^2)^{-1/2}(-\operatorname{cosec}^2 x^2)\frac{d}{dx}x^2$$

$$= \frac{-\operatorname{cosec}^2 x^2 \cdot 2x}{\sqrt{\cot x^2}} = \frac{-2x\operatorname{cosec}^2 x^2}{\sqrt{\cot x^2}}$$

Q.No.8 $\cos(\sqrt{x})$

Let $y = \cos(\sqrt{x})$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\cos(\sqrt{x}) = -\sin\sqrt{x}\,\frac{d}{dx}\sqrt{x} = -\sin\sqrt{x}\cdot\frac{1}{2\sqrt{x}}$$

Using $\frac{d}{dx}\cos x = -\sin x$

$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$

Q.No.9 $\because$ we know a function $f(x)$ is said to be differentiable at a point $x=c$ if

$$\lim_{x\to c}\frac{f(x)-f(c)}{x-c} \text{ exists}$$

and we can say $f'(c)$ or value of $f'(x)$ at $x=c$

we have $f(x) = |x-1|,\ x\in R$

and we want to show $f(x)$ is not differentiable at ~~x=1~~ $x=1$

If we substitute $x=1$

$f(x) = |1-1| = 0$

LHD $= Lf'(1) = \lim_{x \to 1^-} \frac{f(x) - f(1)}{x-1}$

$= \lim_{x \to 1^-} \frac{|x-1| - f(1)}{x-1}$

$= \lim_{x \to 1^-} \frac{|x-1| - (0)}{x-1} = \frac{-(x-1) - 0}{x-1} = \frac{-(x-1)}{(x-1)} = -1$

for LHL $x \to 1^- = x < 1$

$= x - 1 < 0 \Rightarrow |x-1| = -(x-1)$

RHD

$\lim_{x \to 1^+} \frac{f(x) - f(1)}{x-1} = \lim_{x \to 1^+} \frac{|x-1| - f(1)}{x-1}$

$= \frac{(x-1) - 0}{(x-1)} = \frac{x-1}{x-1} = 1$

$\because$ LHD $\neq$ RHD

$\therefore$ $f(x)$ is not differentiable at $x = 1$

Q.No.10 $\because$ we have $f(x) = [x]$, $0 < x < 3$

to check differentiability at $x = 1$

if we put $x = 1$ then $f(1) = [1] = 1$

Now LHD $\lim_{x \to 1^-} \frac{f(x) - f(1)}{x-1} = \lim_{x \to 1^-} \frac{[x] - f(1)}{x-1}$

Put $x = 1 - h$, $h \to 0^+$

$\therefore \lim_{h \to 0^+} \frac{[1-h] - 1}{1-h-1} = \frac{[1-h] - 1}{-h}$

as we know that $h \to 0^+$ and $[c - h] = c - 1$ if c will be an integer

$\therefore$ $[1-h] = 1 - 1 = 0$

$\therefore \lim_{x \to 0} \frac{[0] - 1}{-h} = \frac{0-1}{0} = \frac{-1}{0} = \infty$ for $h = 0$

does not exist

$\therefore$ f(x) is not differentiable at $x=1$

Differentiability at $x=2$

$x=2$, $f(2)=[2]=2$

LHD $\lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2} = \lim_{x\to 2^-} \frac{[x]-f(2)}{x-2}$

$x=2-h$, $h\to 0^+$

$\lim_{h\to 0^+} \frac{[2-h]-2}{2-h-2} = \lim_{h\to 0} \frac{1-2}{-h} = \frac{-1}{-h} = \frac{1}{h}$

$\lim_{h\to 0} \frac{1}{h} = \frac{1}{0} = \infty$

does not exist

$\therefore$ f(x) is also not differentiable at $x=2$

Thanks for following

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MATHEMATICS

CLASS-XII-CBSE

DIFFERENTIABILITY EXRCISE 5.3

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Implicit functions

Find $\frac{dy}{dx}$ in the following exercises 1 to 15.

1. $2x+3y=\sin x$

diff. both sides

$$\frac{d}{dx}(2x)+\frac{d}{dx}(3y)=\frac{d}{dx}\sin x$$

$$2+3\frac{dy}{dx}=\cos x$$

$$\boxed{\frac{dy}{dx}=\frac{\cos x-2}{3}}$$ Ans.

2. $2x+3y=\sin y$

Sol: differentiating both sides w.r.t. x

$$\frac{d}{dx}(2x)+\frac{d}{dx}(3y)=\frac{d}{dx}(\sin y)$$

$$2+3\frac{dy}{dx}=\cos y\frac{dy}{dx}$$

$$\cos y\frac{dy}{dx}-3\frac{dy}{dx}=2$$

$$\frac{dy}{dx}(\cos y-3)=2$$

$$\therefore \boxed{\frac{dy}{dx}=\frac{2}{\cos y-3}}$$ Ans

3 $ax + by^2 = \cos y$

diff. both sides

$\frac{d}{dx}(ax) + \frac{d}{dx}(by^2) = \frac{d}{dx}(\cos y)$

$a + 2by\frac{dy}{dx} = -\sin y\frac{dy}{dx}$

$2by\frac{dy}{dx} + \sin y\frac{dy}{dx} = -a$

$\frac{dy}{dx}(2by + \sin y) = -a$

$\frac{dy}{dx} = \frac{-a}{2by + \sin y}$ Ans

4 $xy + y^2 = \tan x + y$

diff. both sides

$\frac{d}{dx}(xy) + \frac{d}{dx}(y^2) = \frac{d}{dx}(\tan x + y)$

$y\frac{d}{dx}x + x\frac{d}{dx}(y) + 2y\frac{dy}{dx} = \frac{d}{dx}\tan x + \frac{d}{dx}y$

$y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$

$x\frac{dy}{dx} + 2y\frac{dy}{dx} - \frac{dy}{dx} = \sec^2 x - y$

$\frac{dy}{dx}(x + 2y - 1) = \sec^2 x - y$

$\therefore \frac{dy}{dx} = \frac{\sec^2 x - y}{x + 2y - 1}$ Ans

5 $x^2 + xy + y^2 = 100$

diff. both sides

$\frac{d}{dx} x^2 + \frac{d}{dx}(xy) + \frac{d}{dx} y^2 = \frac{d}{dx}(100)$

$2x + y\frac{d}{dx}x + x\frac{d}{dx}(y) + 2y\frac{dy}{dx} = 0$

$2x + y(1) + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(x+2y) = -2x - y$

$\frac{dy}{dx} = \frac{-(2x+y)}{x+2y}$ Ans

6 $x^3 + x^2y + xy^2 + y^3 = 81$

diff. both sides

$\frac{d}{dx}(x^3) + \frac{d}{dx}(x^2y) + \frac{d}{dx}(xy^2) + \frac{d}{dx}(y^3) = \frac{d}{dx}(81)$

$3x^2 + y\frac{d}{dx}(x^2) + x^2\frac{dy}{dx} + y^2\frac{d}{dx}(x) + x\frac{d}{dx}(y^2) + 3y^2\frac{dy}{dx} = 0$

$3x^2 + 2xy + x^2\frac{dy}{dx} + y^2 + 2xy\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$

$x^2\frac{dy}{dx} + 2xy\frac{dy}{dx} + 3y^2\frac{dy}{dx} = -3x^2 - 2xy - y^2$

$\frac{dy}{dx}(x^2 + 2xy + 3y^2) = -3x^2 - 2xy - y^2 = -(3x^2 + 2xy + y^2)$

$\frac{dy}{dx} = \frac{-(3x^2 + 2xy + y^2)}{x^2 + 2xy + 3y^2}$ Ans

$\sin 2x + \sin 2y \frac{dy}{dx} = 0$

$\therefore \frac{dy}{dx} = -\frac{\sin 2x}{\sin 2y}$ Ans

$\sin^2 x + \cos^2 y = 1$

diff. both sides

$\frac{d}{dx}(\sin^2 x) + \frac{d}{dx}(\cos^2 y) = \frac{d}{dx}(1)$

$\frac{d}{dx}(\sin x)^2 + \frac{d}{dx}(\cos y)^2 = 0$

$2\sin x \frac{d}{dx}(\sin x) + 2\cos y \frac{d}{dx}(\cos y) = 0$

$2\sin x \cos x + 2\cos y(-\sin y)\frac{dy}{dx} = 0$

$2\sin x \cos x - 2\sin y \cos y \frac{dy}{dx} = 0$

$\sin 2x - \sin 2y \frac{dy}{dx} = 0$

$\sin 2y \frac{dy}{dx} = \sin 2x$

$\therefore \boxed{\frac{dy}{dx} = \frac{\sin 2x}{\sin 2y}}$ Ans

7 $\sin^2 y + \cos xy = \pi$

differentiating both sides

$$\frac{d}{dx}(\sin^2 y) + \frac{d}{dx}\cos xy = \frac{d}{dx}(\pi)$$

$$\frac{d}{dx}(\sin y)^2 + \frac{d}{dx}\cos xy = 0$$

$$2\sin y \frac{d}{dx}\sin y - \sin xy \frac{d}{dx} xy = 0$$

$$2\sin y \cos y \frac{dy}{dx} - \sin xy\left[y\frac{d(x)}{dx} + x\frac{dy}{dx}\right] = 0$$

$$= \sin 2y \frac{dy}{dx} - \sin xy\left[y + x\frac{dy}{dx}\right] = 0 \quad \left[\because 2\sin y\cos y = \sin 2y\right]$$

$$= \sin 2y \frac{dy}{dx} - y\sin xy - x\sin xy\frac{dy}{dx} = 0$$

$$= \sin 2y \frac{dy}{dx} - x\sin xy \frac{dy}{dx} = y\sin xy$$

$$\frac{dy}{dx}(\sin 2y - x\sin xy) = y\sin xy$$

$$\boxed{\frac{dy}{dx} = \frac{y\sin xy}{\sin 2y - x\sin xy}}$$ Ans

8 $\sin^2 x + \sin^2 y = 1$

differentiating both sides

$$\frac{d}{dx}(\sin^2 x) + \frac{d}{dx}(\sin^2 y) = \frac{d}{dx}(1)$$

$$\frac{d}{dx}(\sin x)^2 + \frac{d}{dx}(\sin y)^2 = 0$$

$$2\sin x \frac{d}{dx}\sin x + 2\sin y \frac{d}{dx}\sin y = 0$$

$$2\sin x\cos x + 2\sin y\cos y\frac{dy}{dx} = 0$$

9 $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Now we have inverse trigonometry function
We need such ratio which Could remove inverse
so lets try:

Let $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$\therefore$

$y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$ $\left[\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}\right]$

$y = \sin^{-1}(\sin 2\theta) = 2\theta$

Now diff. both sides w.r.t x

$= 2\tan^{-1}x = y$

$y = 2\tan^{-1}x$

$\frac{dy}{dx} = 2 \cdot \frac{1}{1+x^2} = \frac{2}{1+x^2}$ Ans

10 $y = \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right); \; -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

Let $x = \tan\theta$

$\theta = \tan^{-1}x$

$y = \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right) = \tan^{-1}(\tan 3\theta) = 3\theta$

$\therefore y = 3\theta \Rightarrow 3\tan^{-1}x$

diff. both sides

$$\boxed{\frac{dy}{dx} = 3 \cdot \frac{1}{1+x^2} = \frac{3}{1+x^2}}$$ Ans $\left[\because \frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}\right]$

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$$y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), \quad 0 < x < 1$$

let $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$$\therefore y = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \cos^{-1}(\cos 2\theta) = 2\theta$$

$$y = 2\theta = 2\tan^{-1}x$$

$$y = 2\tan^{-1}x$$

diff. both sides, we get

$$\frac{dy}{dx} = 2 \cdot \frac{1}{1+x^2} = \frac{2}{1+x^2}$$

$$\boxed{\frac{dy}{dx} = \frac{2}{1+x^2}}$$ Ans

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$$y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right), \quad 0 < x < 1$$

let $x = \tan\theta$

$\theta = \tan^{-1}x$

$$y = \sin^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\cos 2\theta)$$

$$= \sin^{-1}\left(\sin\left(\frac{\pi}{2} - 2\theta\right)\right) = \pi/2 - 2\theta.$$

$$\therefore y = \frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2\tan^{-1}x$$

$y = \sin^{-1}(2\sin\theta\sqrt{1-\sin^2\theta})$

$= \sin^{-1}(2\sin\theta\sqrt{\cos^2\theta}) = \sin^{-1}(2\sin\theta\cos\theta)$

$= \sin^{-1}(\sin 2\theta) = 2\theta$

$y = 2\theta$

$y = 2\sin^{-1}x$

diff. both sides

$\frac{dy}{dx} = 2 \cdot \frac{1}{\sqrt{1-x^2}}$

$\therefore \boxed{\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}}$ Ans

15. $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right) =$

Let $x = \cos\theta$

$\theta = \cos^{-1}x$

$y = \sec^{-1}\left(\frac{1}{2\cos^2\theta - 1}\right) = \sec^{-1}\left(\frac{1}{\cos 2\theta}\right)$ $\left[\because \cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1\right.$

$y = \sec^{-1}(\sec 2\theta) = 2\theta$

$y = 2\theta = 2\cos^{-1}x$

diff. both sides

$\frac{dy}{dx} = 2 \cdot \left(\frac{-1}{\sqrt{1-x^2}}\right)$

$\boxed{\frac{dy}{dx} = \frac{-2}{\sqrt{1-x^2}}}$ Ans

Thanks for following

diff both sides

$$\frac{dy}{dx} = -2\left(\frac{1}{1+x^2}\right) = \frac{-2}{1+x^2}$$

$$\boxed{\frac{dy}{dx} = \frac{-2}{1+x^2}}$$ Ans

13 $$y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$$

Let $x = \tan\theta$

$\theta = \tan^{-1}x$

$$\therefore y = \cos^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \cos^{-1}(\sin 2\theta)$$

$$y = \cos^{-1}\left(\cos\left(\frac{\pi}{2}-2\theta\right)\right) = \frac{\pi}{2} - 2\theta$$

$$y = \frac{\pi}{2} - 2\tan^{-1}x$$

diff both sides

$$\frac{dy}{dx} = -2\cancel{\left(\frac{1}{1+\ldots}\right)} = -2\left(\frac{1}{1+x^2}\right) = \frac{-2}{1+x^2}$$

$$\boxed{\frac{dy}{dx} = \frac{-2}{1+x^2}}$$ Ans

14 $$y = \sin^{-1}\left(2x\sqrt{1-x^2}\right), \quad -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$$

Let $x = \sin\theta$

$\theta = \sin^{-1}x$

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DIFFERENTIABILITY EXRCISE 5.4

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EXPONENTIAL & LOGARITHMIC FUNCTIONS

Some Imp logarithmic identities

$$\log_b pq = \log_b p + \log_b q$$

$$\log_b p^2 = 2\log_b p = \log_b p + \log_b p$$

$$\log_b p^n = n \log_b p$$

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

Differentiate the following w.r.t x

1. $\dfrac{e^x}{\sin x}$

diff. [redacted] w.r.t x

$$\frac{d}{dx}\left(\frac{e^x}{\sin x}\right) = \frac{\sin x \frac{d}{dx} e^x - e^x \frac{d}{dx}(\sin x)}{(\sin x)^2}$$

$$= \frac{\sin x \cdot e^x - e^x \cos x}{\sin^2 x} = \frac{e^x(\sin x - \cos x)}{\sin^2 x}$$

2. $e^{\sin^{-1} x}$

$$y = e^{\sin^{-1} x}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\sin^{-1} x}\right) = e^{\sin^{-1} x} \frac{d}{dx} \sin^{-1} x$$

$$= e^{\sin^{-1} x} \cdot \left(\frac{1}{\sqrt{1-x^2}}\right) \text{ Ans}$$

3 e^{x^3}

let $y = e^{x^3}$

diff both sides

$$\frac{dy}{dx} = \frac{d}{dx}(e^{x^3})$$

$$= e^{x^3} \frac{d}{dx} x^3 = e^{x^3} \cdot 3x^2 = \underline{3x^2 \cdot e^{x^3}}$$

$$\left[\because \frac{d}{dx} e^{f(x)} = e^{f(x)} \cdot \frac{d}{dx} f(x) \right.$$

4. $\sin(\tan^{-1} e^{-x})$

let $y = \sin(\tan^{-1} e^{-x})$

$$\frac{dy}{dx} = \frac{d}{dx}(\sin(\tan^{-1} e^{-x}))$$

$$= \cos(\tan^{-1} e^{-x}) \frac{d}{dx} \tan^{-1} e^{-x}$$

$$= \cos(\tan^{-1} e^{-x}) \frac{1}{1+(e^{-x})^2} \frac{d}{dx} e^{-x}$$

$$= \cos(\tan^{-1} e^{-x}) \cdot \frac{1}{1+e^{-x^2}} \cdot e^{-x} \frac{d}{dx}(-x)$$

$$= \cos(\tan^{-1} e^{-x}) \cdot \frac{1}{1+e^{x^2}} \cdot e^{-x}(-1)$$

$$= -\cos(\tan^{-1} e^{-x}) \cdot \frac{1}{1+e^{x^2}} \cdot e^{-x} \quad \underline{\text{Ans}}$$

5. $\log(\cos e^x)$

$y = \log(\cos e^x)$

$$\frac{dy}{dx} = \frac{d}{dx}(\log(\cos e^x)) = \frac{1}{\cos e^x} \frac{d}{dx} \cos e^x$$

$$= \frac{1}{\cos e^x} \cdot (-\sin e^x) \frac{d}{dx} e^x$$

$$= \frac{-1}{\cos e^x} \sin e^x \cdot e^x \frac{d}{dx}(x)$$

$$= -\frac{e^x \cdot \sin e^x}{\cos e^x}(1) \qquad \left[\frac{d}{dx}(x) = 1\right]$$

$$= -e^x \cdot \tan e^x \quad \text{Ans}$$

6 $e^x + e^{x^2} + \cdots + e^{x^5}$

$$y = e^x + e^{x^2} \cdots + e^{x^5}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^x + e^{x^2} + \cdots + e^{x^5}\right)$$

$$= \frac{d}{dx} e^x + \frac{d}{dx} e^{x^2} + \cdots + \frac{d}{dx} e^{x^5}$$

$$= e^x + e^{x^2} \frac{d}{dx} x^2 + e^{x^3} \frac{d}{dx} x^3 + e^{x^4} \frac{d}{dx} x^4 + e^{x^5} \frac{d}{dx} x^5$$

$$= e^x + e^{x^2}(2x) + e^{x^3}(3x^2) + e^{x^4}(4x^3) + e^{x^5}(5)x^4$$

$$= e^x + 2x \cdot e^{x^2} + 3x^2 e^{x^3} + 4x^3 e^{x^4} + 5x^4 e^{x^5} \quad \text{Ans}$$

We have applied $\frac{d}{dx} e^{f(x)} = e^{f(x)} \cdot \frac{d}{dx} f(x)$

7. $\sqrt{e^{\sqrt{x}}}$

Let $y = \sqrt{e^{\sqrt{x}}}$

$$\frac{dy}{dx} = \frac{d}{dx}\sqrt{e^{\sqrt{x}}} = \frac{d}{dx}\left(e^{\sqrt{x}}\right)^{1/2} = \frac{1}{2}\left(e^{\sqrt{x}}\right)^{1/2-1} \frac{d}{dx} e^{\sqrt{x}}$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{e^{\sqrt{x}}}} \cdot e^{\sqrt{x}} \frac{d}{dx}\sqrt{x} = \frac{1}{2} \cdot \frac{1}{\sqrt{e^{\sqrt{x}}}} \cdot e^{\sqrt{x}} \cdot \frac{1}{2} \frac{1}{\sqrt{x}}$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{e^{\sqrt{x}}}} \cdot e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{4} \frac{e^{\sqrt{x}}}{\sqrt{e^{\sqrt{x}}} \cdot \sqrt{x}} \quad \text{Ans}$$

8 $\log(\log x),\ x>1$

$y = \log(\log x)$

$\frac{dy}{dx} = \frac{d}{dx}(\log(\log(x))$

$= \frac{1}{\log x}\frac{d}{dx}\log x = \frac{1}{\log x}\cdot\frac{1}{x}\cdot\frac{d}{dx}(x)$

$= \frac{1}{\log x}\cdot\frac{1}{x}\cdot 1 = \frac{1}{x\log x}$

9 $\frac{\cos x}{\log x}$

$y = \frac{\cos x}{\log x}$

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\cos x}{\log x}\right) = \frac{\log x\frac{d}{dx}\cos x - \cos x\frac{d}{dx}\log x}{(\log x)^2}$

$= \frac{\log x(-\sin x) - \cos x\cdot\left(\frac{1}{x}\right)}{(\log x)^2}$

$= \frac{-\sin x\log x - \cos x\cdot\frac{1}{x}}{(\log x)^2}$

$= -\left[\frac{\sin x\log x + \frac{\cos x}{x}}{(\log x)^2}\right] = -\left[\frac{x\sin x\log x + \cos x}{x(\log x)^2}\right]$

10 $\cos(\log x + e^x) \quad x > 0$

$y = \cos(\log x + e^x)$

$\frac{dy}{dx} = \frac{d}{dx}(\cos(\log x + e^x)) = \frac{d}{dx}(\cos(\log x + e^x))$

$= -\sin(\log x + e^x)\frac{d}{dx}(\log x + e^x)$

$= -\sin(\log x + e^x)\left[\frac{d}{dx}\log x + \frac{d}{dx}e^x\right]$

$= -\sin(\log x + e^x)\left[\frac{1}{x} + e^x\right]$ OR

$= -\frac{1}{x}\sin(\log x + e^x) - e^x\sin(\log x + e^x)$

Thanks for following.

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MATHEMATICS

CLASS-XII-CBSE

DIFFERENTIABILITY EXRCISE 5.5

Prepared By:
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If we are given a function and has its exponent is a function, then the process of differentiating a function after taking the logarithm is called a logarithmic function.

Function will be of the form $(f(x))^{g(x)}$

Note: $\log \frac{a^r b^s c^p}{d^q \cdot e^t}$

$= r\log a + s\log b + p\log c - q\log d - t\log e$

also $\log(u+v) \neq \log u + \log v$

$\log(u-v) \neq \log u - \log v$

1. $\cos x \cos 2x \cos 3x$

Let $y = \cos x \cos 2x \cos 3x$

taking log both sides

$\log y = \log(\cos x \cos 2x \cos 3x)$

$\log y = \log\cos x + \log\cos 2x + \log\cos 3x$

Differentiating both sides wrt x

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{\cos x}\cdot\frac{d(\cos x)}{dx} + \frac{d \log\cos 2x}{dx} + \frac{d \log\cos 3x}{dx}$

$= \frac{1}{\cos x}\cdot(-\sin x) + \frac{1}{\cos 2x}\frac{d}{dx}\cos 2x + \frac{1}{\cos 3x}\frac{d}{dx}\cos 3x$

$= -\frac{\sin x}{\cos x} + \frac{1}{\cos 2x}\cdot(-\sin 2x)\frac{d\,2x}{dx} + \frac{1}{\cos 3x}\cdot(-\sin 3x)\frac{d}{dx}3x$

$= -\frac{\sin x}{\cos x} - \frac{\sin 2x}{\cos 2x}\cdot 2 - \frac{\sin 3x}{\cos 3x}\cdot(3)$

$= -[\tan x + 2\tan 2x + 3\tan 3x)$

$\therefore \frac{dy}{dx} = -y(\tan x + 2\tan 2x + 3\tan 3x)$

$= -\cos x\cos 2x\cos 3x(\tan x + 2\tan 2x + 3\tan 3x)$

2 $\sqrt{\dfrac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$

let $y = \sqrt{\dfrac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}} = \left(\dfrac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}\right)^{\frac{1}{2}}$

taking log both sides

$$\log y = \log\left(\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}\right)^{\frac{1}{2}} = \frac{1}{2}\log\left(\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}\right)$$

$$= \frac{1}{2}\left[\log(x-1)+\log(x-2)-\log(x-3)-\log(x-4)-\log(x-5)\right]$$

differentiating both sides, we get

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\frac{d}{dx}\left(\log(x-1)+\log(x-2)-\log(x-3)-\log(x-4)-\log(x-5)\right)\right]$$

$$= \frac{1}{2}\left[\frac{1}{x-1}\frac{d}{dx}(x-1)+\frac{1}{x-2}\frac{d}{dx}(x-2)-\frac{1}{x-3}\frac{d}{dx}(x-3)-\frac{1}{x-4}\frac{d}{dx}(x-4)-\frac{1}{(x-5)}\frac{d}{dx}(x-5)\right]$$

$$= \frac{1}{2}\left[\frac{1}{x-1}+\frac{1}{x-2}-\frac{1}{x-3}-\frac{1}{x-4}-\frac{1}{x-5}\right]$$

$$\therefore \frac{dy}{dx} = \frac{y}{2}\left[\frac{1}{x-1}+\frac{1}{x+2}-\frac{1}{x-3}-\frac{1}{x-4}-\frac{1}{x-5}\right]$$

$$= \frac{1}{2}\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}\left[\frac{1}{(x-1)}+\frac{1}{(x-2)}-\frac{1}{(x-3)}-\frac{1}{(x-4)}-\frac{1}{(x-5)}\right]$$

3 $y = (\log x)^{\cos x}$

taking log both sides

$\log y = \log(\log x)^{\cos x} = \cos x \log(\log x)$

diff. both sides

$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(\cos x \log(\log x))$

$$\frac{1}{y}\frac{dy}{dx} = \log(\log x)\frac{d}{dx}\cos x + \cos x\frac{d}{dx}\log(\log(x))$$

$$= \log(\log x)(-\sin x) + \cos x\frac{1}{\log x}\frac{d}{dx}\log x$$

$$= -\sin x \cdot \log(\log x) + \cos x \cdot \frac{1}{\log x}\cdot\frac{1}{x}$$

$$= \frac{\cos x}{x\log x} - \sin x \cdot \log(\log x)$$

$$\therefore \frac{dy}{dx} = y\left(\frac{\cos x}{x\log x} - \sin x\log(\log x)\right)$$

$$= (\log x)^{\cos x}\left(\frac{\cos x}{x\log x} - \sin x\log(\log x)\right)$$

Note:→ If we have

$$y = (f(x))^{g(x)} \pm (h(x))^{s(x)}$$

or $\quad y = (f(x))^{g(x)} \pm h(x)$

or $\quad y = (f(x))^{g(x)} \pm k$ where k is a constant

we are not going to start by taking log of both sides function will be taken separately equal to u and v.

then $y = u \pm v$

$$\frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$$

and find separately $\frac{du}{dx}$ and $\frac{dv}{dx}$

4 $y = x^x - 2^{\sin x}$

Let $u = x^x$ and $v = 2^{\sin x}$

taking log both sides

$\log u = \log x^x = x\log x$

Diff. both sides

$\frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(x\log x)$

$= \log x\frac{d}{dx}(x) + x\frac{d}{dx}\log x$

$= \log x\cdot(1) + x\cdot\frac{1}{x}$

$= \log x + 1$

$\therefore \frac{du}{dx} = u(\log x + 1)$

$= x^x(\log x + 1)$

$\frac{dv}{dx} = \frac{d}{dx}2^{\sin x}$

$\frac{1}{v}\frac{dv}{dx} = 2^{\sin x}\log 2\frac{d}{dx}\sin x$

$\left[\because \frac{d}{dx}a^{f(x)} = a^{f(x)}\log a\frac{d}{dx}f(x)\right]$

$\frac{dv}{dx} = \left[2^{\sin x}\log 2\cos x\right]$

$= \left[\cos x\cdot 2^{\sin x}\cdot\log 2\right]$

$\therefore \frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx} = x^x(\log x+1) - \left[\cos x\cdot 2^{\sin x}\cdot\log 2\right]$

$= x^x(\log x+1) - (\cos x\cdot 2^{\sin x}\cdot\log 2)$

5. $(x+3)^2(x+4)^3(x+5)^4$

$y = (x+3)^2(x+4)^3(x+5)^4$

taking log both sides

$\log y = \log\left((x+3)^2(x+4)^3(x+5)^4\right)$

$\log y = \log(x+3)^2 + \log(x+4)^3 + \log(x+5)^4$

$\log y = 2\log(x+3) + 3\log(x+4) + 4\log(x+5)$

Diff. both sides

$\frac{d}{dx}\log y = \frac{d}{dx}\left(2\log(x+3) + 3\log(x+4) + 4\log(x+5)\right)$

$\frac{1}{y}\frac{dy}{dx} = 2\frac{d}{dx}\log(x+3) + 3\frac{d}{dx}(x+4) + 4\frac{d}{dx}(x+5)$

$= 2\frac{1}{x+3}\frac{d}{dx}(x+3) + 3\cdot\frac{1}{(x+4)}\frac{d}{dx}(x+4) + 4\frac{1}{x+5}\frac{d}{dx}(x+5)$

$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x+3}(1) + \frac{3}{(x+4)}(1) + \frac{4}{x+5}(1)$

$$\frac{dy}{dx} = y\left(\frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{(x+5)}\right)$$

$$\frac{dy}{dx} = (x+3)^2(x+4)^3(x+5)^4\left[\frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5}\right)$$

6 $\left(x+\frac{1}{x}\right)^x + x^{\left(x+\frac{1}{x}\right)}$

Let $y = \left(x+\frac{1}{x}\right)^x + x^{\left(x+\frac{1}{x}\right)}$

$u = \left(x+\frac{1}{x}\right)^x$ and $v = x^{\left(x+\frac{1}{x}\right)}$

$y = u + v$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$u = \left(x+\frac{1}{x}\right)^x$

taking log

$$\log u = \log\left(x+\frac{1}{x}\right)^x = x\log\left(x+\frac{1}{x}\right)$$

differentiating both sides wrt x

$$\frac{du}{dx}\log u = \frac{d}{dx}\left(x\log\left(x+\frac{1}{x}\right]\right.$$

$$\frac{1}{u}\frac{du}{dx} = \log\left(x+\frac{1}{x}\right)\frac{d}{dx}(x) + x\frac{d}{dx}\log\left(x+\frac{1}{x}\right)$$

$$= \log\left(x+\frac{1}{x}\right)(1) + x\cdot\frac{1}{\left(x+\frac{1}{x}\right)}\frac{d}{dx}\left(x+\frac{1}{x}\right)$$

$$= \log\left(x+\frac{1}{x}\right) + x^2\frac{1}{x^2+1}\left[\frac{d}{dx}(x) + \frac{d}{dx}\left(\frac{1}{x}\right)\right]$$

$$= \log\left(x+\frac{1}{x}\right) + \frac{x^2}{x^2+1}\left[1+\left(\frac{-1}{x^2}\right)\right]$$

$$= \log\left(x+\frac{1}{x}\right) + \frac{\cancel{x^2}}{x^2+1}\left(\frac{x^2-1}{\cancel{x^2}}\right]$$

$$= u\left[\log\left(x+\frac{1}{x}\right) + \left(\frac{x^2-1}{x^2+1}\right)\right]$$

$$v = x^{(1+\frac{1}{x})}$$

taking log both sides

$$\log v = \log\left[x^{(1+\frac{1}{x})}\right] = \left(1+\frac{1}{x}\right)\log(x) = \left(\frac{x+1}{x}\right)\log x$$

diff. both sides

$$\frac{d}{dx}\log v = \frac{d}{dx}\left[\left(\frac{x+1}{x}\right)\log x\right]$$

$$\frac{1}{v}\frac{dv}{dx} = \log x\frac{d}{dx}\left(\frac{x+1}{x}\right) + \left(\frac{x+1}{x}\right)\frac{d}{dx}(\log x)$$

$$= \log x\left[\frac{x\frac{d}{dx}(x+1) - (x+1)\frac{d}{dx}(x)}{x^2}\right] + \left(\frac{x+1}{x}\right)\left(\frac{1}{x}\right)$$

$$= \log x\left(\frac{(x)(1)-(x+1)(1)}{x^2}\right) + \left(\frac{x+1}{x^2}\right)$$

$$= \log x\left(\frac{x-x-1}{x^2}\right) + \left(\frac{x+1}{x^2}\right)$$

$$= \log x\left(\frac{-1}{x^2}\right) + \left(\frac{x+1}{x^2}\right) = -\frac{\log x}{x^2} + \left(\frac{x+1}{x^2}\right)$$

$$\frac{dv}{dx} = v\left[\frac{x+1}{x^2} - \frac{1\log x}{x^2}\right]$$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$= \left(x+\frac{1}{x}\right)^x\left[\log\left(x+\frac{1}{x}\right) + \left(\frac{x^2-1}{x^2+1}\right)\right] + x^{(x+\frac{1}{x})}\left(\frac{x+1}{x^2} - \frac{1}{x^2}\log x\right)$$

Q.No.7

$y = (\log x)^x + x^{\log x}$

let $u = (\log x)^x$ and $v = x^{\log x}$

$y = u + v$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$u = (\log x)^x$

taking log both side

$\log u = \log(\log x)^x = x \log(\log x)$

differentiating both sides

$\frac{d}{dx}(\log u) = \frac{d}{dx}(x \log(\log x))$

$\frac{1}{u}\frac{du}{dx} = \log(\log x)\frac{d}{dx}(x) + x\frac{d}{dx}(\log(\log x))$

$= \log(\log x)(1) + x \frac{1}{\log x}\frac{d}{dx}\log x$

$= \log(\log x) + x \cdot \frac{1}{\log x} \times \frac{1}{x}$

$= \log(\log x) + \frac{1}{\log x}$

$\therefore \frac{du}{dx} = u\left[\log(\log x) + \frac{1}{\log x}\right] = (\log x)^x\left[\log(\log x) + \frac{1}{\log x}\right]$

$v = x^{\log x}$

taking log both sides

$\log v = \log(x^{\log x}) = \log x(\log x) = (\log x)^2$

diff. both sides

$\frac{d}{dx}(\log v) = \frac{d}{dx}((\log x)^2)$

$\frac{1}{v}\frac{dv}{dx} = 2\log x \frac{d}{dx}\log x = 2\log x \cdot \frac{1}{x}$

$\therefore \quad \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$= (\log x)^x \left[\log(\log x) + \frac{1}{\log x}\right] + 2\log x \cdot \frac{1}{x}$

Q.No. 8 $\quad (\sin x)^x + \sin^{-1}\sqrt{x}$

$y = (\sin x)^x + \sin^{-1}\sqrt{x}$

Let $u = (\sin x)^x$ and $v = \sin^{-1}\sqrt{x}$

$y = u + v$

$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

Now $u = (\sin x)^x$

taking log both sides

$\log u = \log(\sin x)^x$

$\log u = x\log(\sin x)$

diff. both sides

$\frac{d\log u}{dx} = \frac{d}{dx}(x\log(\sin x))$

$\frac{1}{u}\frac{du}{dx} = \log(\sin x)\frac{d}{dx}(x) + x\frac{d}{dx}\log(\sin x)$

$= \log(\sin x)(1) + x \cdot \frac{1}{\sin x}\frac{d}{dx}\sin x$

$= \log(\sin x) + \frac{x}{\sin x} \cdot \cos x$

$v = \sin^{-1}\sqrt{x}$

diff. both sides.

$\frac{dv}{dx} = \frac{d}{dx}\sin^{-1}\sqrt{x}$

$= \frac{1}{\sqrt{1-(\sqrt{x})^2}}\frac{d}{dx}\sqrt{x}$

$= \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}}$

$\frac{dv}{dx} = \frac{1}{2\sqrt{x(1-x)}} = \frac{1}{2\sqrt{x-x^2}}$

$\frac{du}{dx} = \log(\sin x) + \frac{x\cos x}{\sin x} = (\sin x)^x\left[\log(\sin x) + \frac{x\cos x}{\sin x}\right]$

$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} = (\sin x)^x\left[\log(\sin x) + \frac{x\cos x}{\sin x}\right] + \frac{1}{2\sqrt{x-x^2}}$

$= (\sin x)^x\left[\log(\sin x) + x\cdot\cot x\right] + \frac{1}{2\sqrt{x-x^2}}$

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Class ____ Chapter ____
Exercise 5.5/9

QNo.9

$x^{\sin x}+(\sin x)^{\cos x}$

$y = x^{\sin x}+(\sin x)^{\cos x}$

Let

$u = x^{\sin x}$ and $v = (\sin x)^{\cos x}$

$\therefore y = u + v$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

Now

$u = x^{\sin x}$

taking log both sides

$\log u = \log(x^{\sin x})$

$\log u = \sin x \log x$

diff. both sides, we get

$\frac{d}{dx}\log u = \frac{d}{dx}(\sin x \log x)$

$\frac{1}{u}\frac{du}{dx} = \log x \frac{d}{dx}\sin x + \sin x \frac{d}{dx}\log x$

$= \log x \cdot \cos x + \sin x \cdot \frac{1}{x}$

$= \log x \cdot \cos x + \frac{\sin x}{x}$

$\frac{du}{dx} = u\left[\log x \cdot \cos x + \frac{\sin x}{x}\right]$

$= x^{\sin x}\left[\log x \cos x + \frac{\sin x}{x}\right]$

$v = (\sin x)^{\cos x}$

taking log both sides

$\log v = \log((\sin x)^{\cos x}$

$\log v = \cos x \log(\sin x)$

diff. both sides, we get

$\frac{d}{dx}\log v = \frac{d}{dx}(\cos x(\log(\sin x))$

$\frac{1}{v}\frac{dv}{dx} = \log(\sin x)\frac{d}{dx}\cos x + \cos x \frac{d}{dx}\log(\sin x)$

$\frac{1}{v}\frac{dv}{dx} = \log \sin x(-\sin x) + \cos x \cdot \frac{1}{\sin x}\frac{d}{dx}\sin x$

$\frac{1}{v}\frac{dv}{dx} = -\sin x \log \sin x + \cos x \cdot \frac{1}{\sin x} \cdot \cos x$

$\frac{1}{v}\frac{dv}{dx} = -\sin x \log \sin x + \cot x \cos x$

$\frac{dv}{dx} = v\left[\cot x \cos x - \sin x \log \sin x\right]$

$\frac{dv}{dx} = (\sin x)^{\cos x}\left[\cot x \cos x - \sin x \log \sin x\right]$

$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} = x^{\sin x}\left[\log x \cos x + \frac{\sin x}{x}\right] + \sin x^{\cos x}\left[\cot x \cos x - \sin x \log \sin x\right]$

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QNo.10 $x^{x\cos x} + \frac{x^2+1}{x^2-1}$

$y = x^{x\cos x} + \frac{x^2+1}{x^2-1}$

Let $u = x^{x\cos x}$ and $v = \frac{x^2+1}{x^2-1}$

$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

Now $u = x^{x\cos x}$

taking log both sides

$\log u = \log(x^{x\cos x}) = x\cos x \log x$

diff. both sides

$\frac{d}{dx}\log u = \frac{d}{dx}(x \cos x \log x)$

$\frac{1}{u}\frac{du}{dx} = \cos x \log x \frac{d}{dx}(x) + x \cdot \log x \frac{d}{dx}\cos x + x\cos x \frac{d \log x}{dx}$

$= \cos x \log x (1) + x \log x(-\sin x) + x\cos x \cdot \frac{1}{x}$

[using $\frac{d}{dx}(uvw) =$]

$\therefore \frac{du}{dx} = u\left[\cos x \log x - x\sin x \log x + \cos x\right]$

$= x^{x\cos x}\left[\cos x \log x - x \sin x \log x + \cos x\right]$

Now for $v = \frac{x^2+1}{x^2-1}$ [applying Quotient Rule]

$\frac{dv}{dx} = \frac{d}{dx}\left(\frac{x^2+1}{x^2-1}\right) = \frac{(x^2-1)\frac{d}{dx}(x^2+1) - (x^2+1)\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$

$= \frac{(x^2-1)(2x) - (x^2+1)(2x)}{(x^2-1)^2} = \frac{\cancel{2x^3} - 2x - \cancel{2x^3} - 2x}{(x^2-1)^2}$

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$= \frac{-4x}{(x^2-1)^2}$

$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$= x^{x\cos x}\left[\cos x \log x - x\sin x \log x + \cos x\right] - \frac{4}{(x^2-1)^2}$

Q.No.11 $(x\cos x)^x + (x\sin x)^{1/x}$

$y = (x\cos x)^x + (x\sin x)^{1/x}$

Let $u = (x\cos x)^x$ and $v = (x\sin x)^{1/x}$

$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

Now

$u = (x\cos x)^x$

taking log both sides

$\log u = \log(x\cos x)^x = x\log(x\cos x)$

Diff. both sides

$\frac{d}{dx}(\log u) = \frac{d}{dx}(x\log(x\cos x))$

$\frac{1}{u}\frac{du}{dx} = \log(x\cos x)\frac{d}{dx}(x) + x\frac{d}{dx}(\log(x\cos x))$

$= \log(x\cos x)(1) + x \cdot \frac{1}{x\cos x}\frac{d}{dx}(x\cos x)$

$= \log(x\cos x) + \frac{1}{\cos x}\left[\cos x\frac{d}{dx}(x) + x\frac{d}{dx}\cos x\right]$

$= \log(x\cos x) + \frac{1}{\cos x}\left[\cos x(1) + x(-\sin x)\right]$

$= \log(x\cos x) + \frac{1}{\cos x}\left[(\cos x - x\sin x)\right]$

$\therefore \frac{du}{dx} = u\left[\log(x\cos x) + \left[\frac{\cos x \times 1}{\cos x} - \frac{x\sin x}{\cos x}\right]\right]$

$= (x\cos x)^x\left[\log(x\cos x) + 1 - x\tan x\right]$

$v = (x \sin x)^{1/x}$

taking log both sides

$\log v = \log(x \sin x)^{1/x}$

$\log v = \frac{1}{x} \log(x \sin x)$

diff. both sides

$\frac{d}{dx} \log v = \frac{d}{dx}\left(\frac{1}{x}(\log(x \sin x))\right)$

$\frac{1}{v}\frac{dv}{dx} = \log(x \sin x)\frac{d}{dx}\left(\frac{1}{x}\right) + \frac{1}{x}\frac{d}{dx}\log(x \sin x)$

$= \log(x \sin x)\left(-\frac{1}{x^2}\right) + \frac{1}{x} \cdot \frac{1}{x \sin x}\frac{d}{dx} x \cdot \sin x$

$= \frac{-1}{x^2}\log(x \sin x) + \frac{1}{x} \cdot \frac{1}{x \sin x}\left[\sin x \frac{d(x)}{dx} + x \frac{d}{dx} \sin x\right]$

$= \frac{-1}{x^2}\log(x \sin x) + \frac{1}{x^2 \sin x}\left[\sin x(1) + x \cdot \cos x\right]$

$\frac{dv}{dx} = v\left[-\frac{1}{x^2}\log(x \sin x) + \frac{1}{x^2 \sin x}\left[\sin x + x \cos x\right]\right]$

$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$= (x \cos x)^x\left[\log(x \cos x - x \tan x + 1\right] +$

$(x \sin x)^{1/x}\left[-\frac{1}{x^2}\log(x \sin x) + \frac{1}{x^2} + \frac{1}{x}\cot x\right]$

Q.No. 12

$$x^y + y^x = 1$$

let $u = x^y$

$v = y^x$

$u + v = 1$

diff. both sides

$$\frac{du}{dx} + \frac{dv}{dx} = 0$$

Now $u = x^y$

Taking log both sides

$\log u = \log x^y$

$\log u = y \log x$

Diff. both sides

$$\frac{d}{dx} \log u = \frac{d}{dx}(y \log x)$$

$$\frac{1}{u}\frac{du}{dx} = \log x . \frac{dy}{dx} + y \frac{d}{dx} \log x = \frac{dy}{dx} . \log x + y . \frac{1}{x}$$

$$\frac{du}{dx} = u\left[\frac{dy}{dx} . \log x + \frac{y}{x}\right]$$

$$= x^y\left[\frac{dy}{dx} . \log x + \frac{y}{x}\right] = x^y \frac{dy}{dx} \log x + x^{y-1} . y$$

Now $v = y^x$

taking log both sides

$\log v = \log y^x = x \log y$

diff. both sides

$$\frac{d}{dx}(\log v) = \frac{d}{dx}(x \log y) = \log y \frac{d}{dx}(x) + x \frac{d}{dx} \log y$$

$$\frac{1}{v}\frac{dv}{dx} = \log y (1) + x . \frac{1}{y} = \log y + \frac{x}{y}\frac{dy}{dx}$$

$$\frac{dv}{dx} = v\left[\log y + \frac{x}{y}\right] = y^x\left[\log y + \frac{x}{y}\frac{dy}{dx}\right]$$

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$$= y^x \log y + y^{x-1} \cdot x \frac{dy}{dx}$$

Now $\frac{du}{dx} + \frac{dv}{dx} = 0$

$$x^y \frac{dy}{dx} \log x + x^{y-1} \cdot y + y^x \log y + y^{x-1} \cdot x \frac{dy}{dx} = 0$$

$$x^y \frac{dy}{dx} \log x + y^{x-1} \cdot x \frac{dy}{dx} = -x^{y-1} y - y^x \log y$$

$$\frac{dy}{dx}\left[x^y \log x + y^{x-1} \cdot x \frac{dy}{dx}\right] = -\left[x^{y-1} y + y^x \log y\right]$$

$$\frac{dy}{dx} = \frac{-\left[x^{y-1} y + y^x \log y\right]}{x^y \log x + y^{x-1} \cdot x}$$ Ans

Q No. 13 $y^x = x^y$

taking log both sides

$$\log y^x = \log x^y$$

$$x \log y = y \log x$$

diff. both sides

$$\frac{d}{dx}(x \log y) = \frac{d}{dx}(y \log x)$$

$$\log y \frac{d}{dx}(x) + x \frac{d}{dx} \log y = \log x \frac{d}{dx} y + y \frac{d}{dx} \log x$$

$$\log y (1) + x \frac{1}{y} \frac{dy}{dx} = \log x \frac{dy}{dx} + y \cdot \frac{1}{x}$$

$$\log y + \frac{x}{y} \frac{dy}{dx} = \log x \cdot \frac{dy}{dx} + \frac{y}{x}$$

$$\frac{x}{y}\frac{dy}{dx} - \log x\frac{dy}{dx} = \frac{y}{x} - \frac{x}{y} \quad \frac{y}{x} - \log y$$

$$\frac{dy}{dx}\left[\frac{x}{y} - \log x\right] = \frac{y - x\log y}{x}$$

$$\frac{dy}{dx}\left[\frac{x - y\log x}{y}\right] = \frac{y - x\log y}{x}$$

$$\therefore \frac{dy}{dx} = \frac{\frac{y - x\log y}{x}}{\frac{x - y\log x}{y}} = \frac{y - x\log y \times y}{x(x - y\log x)}$$

$$\frac{dy}{dx} = \frac{y^2 - xy\log y}{x^2 - xy\log x} \quad \text{Ans}$$

Q.No. 14

$$(\cos x)^y = (\cos y)^x$$

Taking log both sides

$$\log(\cos x)^y = \log(\cos y)^x$$

$$y\log(\cos x) = x\log(\cos y)$$

diff. both sides

$$\frac{d}{dx}(y\log(\cos x)) = \frac{d}{dx}(x\log(\cos y))$$

$$\log(\cos x)\frac{d}{dx}(y) + y\frac{d}{dx}\log(\cos x) = \log(\cos y)\frac{d}{dx}(x) + x\frac{d}{dx}\log(\cos y)$$

$$\log\cos x \cdot \frac{dy}{dx} + y \cdot \frac{1}{\cos x} \cdot \frac{d}{dx}\cos x = \log\cos y\,(1) + x \cdot \frac{1}{\cos y}\frac{d}{dx}\cos y$$

$$\log\cos x\frac{dy}{dx} + \frac{y}{\cos x} \cdot (-\sin x) = \log\cos y + x \cdot \frac{1}{\cos y}(-\sin y)\frac{dy}{dx}$$

$$\log\cos x\frac{dy}{dx} - y\tan x = \log\cos y - x\tan y\frac{dy}{dx}$$

$$\log\cos x\frac{dy}{dx} + x\tan y\frac{dy}{dx} = \log\cos y + y\tan x$$

$$\frac{dy}{dx}(\log\cos x + x\tan y) = \log\cos y + y\tan x$$

$$\boxed{\frac{dy}{dx} = \frac{\log\cos y + y\tan x}{\log\cos x + x\tan y}} \text{ Ans}$$

QNo.15

$$xy = e^{x-y}$$

taking log both sides

$$\log(xy) = \log(e^{x-y})$$

$$\log x + \log y = (x-y)\log e \qquad [\log e = 1]$$

$$\log x + \log y = x - y$$

diff. both sides

$$\frac{d}{dx}(\log x + \log y) = \frac{d}{dx}(x-y)$$

$$\frac{d}{dx}\log x + \frac{d}{dx}\log y = \frac{d}{dx}(x) - \frac{d}{dx}(y)$$

$$\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 1 - \frac{dy}{dx}$$

$$\frac{1}{y}\frac{dy}{dx} + \frac{dy}{dx} = 1 - \frac{1}{x} = \frac{x-1}{x}$$

$$\frac{dy}{dx}\left(\frac{1}{y}+1\right) = \frac{x-1}{x}$$

$$\frac{dy}{dx}\left(\frac{1+y}{y}\right) = \frac{x-1}{x}$$

$$\therefore \frac{dy}{dx} = \frac{(x-1)\times y}{(x)(1+y)} = \frac{y}{x}\cdot\frac{x-1}{(y+1)}$$

$$\boxed{\frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}} \text{ Ans}$$

Q No. 16 $f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$ find derivative and hence $f'(1)$

We have

$$f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$$

Taking log both sides

$$\log(f(x)) = \log(1+x)(1+x^2)(1+x^4)(1+x^8)$$

$$\log(f(x)) = \log(1+x) + \log(1+x^2) + \log(1+x^4) + \log(1+x^8)$$

diff. both sides

$$\frac{1}{f(x)}\frac{d}{dx}(f(x)) = \frac{d}{dx}\log(1+x) + \frac{d}{dx}\log(1+x^2) + \frac{d}{dx}\log(1+x^4) + \frac{d}{dx}\log(1+x^8)$$

$$= \frac{1}{1+x}\frac{d}{dx}(1+x) + \frac{1}{1+x^2}\frac{d}{dx}(1+x^2) + \frac{1}{1+x^4}\frac{d}{dx}(1+x^4) + \frac{1}{1+x^8}\frac{d}{dx}(1+x^8)$$

$$= \frac{1}{1+x}\cdot 1 + \frac{1}{1+x^2}\cdot 2x + \frac{1}{1+x^4}\cdot 4x^3 + \frac{1}{1+x^8}\cdot 8x^7$$

$$\therefore \frac{d}{dx}f(x) = f(x)\left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8}\right]$$

$$= (1+x)(1+x^2)(1+x^4)(1+x^8)\left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8}\right]$$

Now putting $x=1$

$$\therefore f'(1) = (1+1)(1+(1)^2)(1+(1)^3)(1+(1)^4)\left[\frac{1}{1+1} + \frac{2(1)}{1+(1)^2} + \frac{4(1)^3}{1+(1)^4} + \frac{8(1)^7}{1+(1)^8}\right]$$

$$= (2)(2)(2)(2)\left[\frac{1}{2} + \frac{2}{2} + \frac{4}{2} + \frac{8}{2}\right] = 16\left[\frac{1}{2} + 1 + 2 + 4\right]$$

$$= 16\left[\frac{1+2+4+8}{2}\right] = 8 \times 15 = 120$$

$$\therefore \boxed{f'(1) = 120}$$ Ans

Q No. 17 $y=(x^2-5x+8)(x^3+7x+9)$

Using product formula

$y=(x^2-5x+8)(x^3+7x+9)$

diff. both sides

$\frac{dy}{dx}=\frac{d}{dx}(x^2-5x+8)(x^3+7x+9)$

$=(x^3+7x+9)\frac{d}{dx}(x^2-5x+8)+(x^2-5x+8)\frac{d}{dx}(x^3+7x+9)$

$=(x^3+7x+9)(2x-5)+(x^2-5x+8)(3x^2+7)$

$=2x^4+14x^2+18x-5x^3-35x-45+3x^4-15x^3+24x^2$
$+7x^2-35x+56$

$=5x^4-20x^3+45x^2-52x+11$

(ii) by expanding

$y=(x^2-5x+8)(x^3+7x+9)$

$=x^5-5x^4+8x^3+7x^3-35x^2+56x+9x^2-45x+72$

$y=x^5-5x^4+15x^3-26x^2+11x+72$

diff. both sides

$\frac{dy}{dx}=\frac{d}{dx}(x^5-5x^4+15x^3-26x^2+11x+72)$

$=5x^4-15x^3+45x^2-52x+11$ Ans

iii) To find $\frac{dy}{dx}$ by logarithmic function.

$y=(x^2-5x+8)(x^3+7x+9)$

taking log both sides

$\log y=\log[(x^2-5x+8)(x^3+7x+9)]$

$=\log(x^2-5x+8)+\log(x^3+7x+9)$

Class ________ Chapter ________

Exercise ________ 5.5/19

$\frac{d}{dx}\log y$

$$\frac{d}{dx}\log y = \frac{d}{dx}\left[\log(x^2-5x+8)+\log(x^3+7x+9)\right]$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}\log(x^2-5x+8)+\frac{d}{dx}\log(x^3+7x+9$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2-5x+8}\frac{d}{dx}(x^2-5x+8)+\frac{1}{x^3+7x+9}\frac{d}{dx}(x^3+7x+9)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2-5x+8}(2x-5)+\frac{1}{x^3+7x+9}\cdot(3x^2+7)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{(2x-5)(x^3+7x+9)+(3x^2+7)(x^2-5x+8)}{(x^2-5x+8)(x^3+7x+9)}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2x^4+14x^2+18x-5x^3-35x-45+3x^4-15x^3+24x^2+7x^2-35x+56}{(x^2-5x+8)(x^3+7x+9)}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{5x^4-20x^3+45x^2-52x+11}{(x^2-5x+8)(x^3+7x+9)}$$

$$\therefore \frac{dy}{dx} = y\left[\frac{5x^4-20x^3+45x^2-52x+11}{(x^2-5x+8)(x^3+7x+9)}\right]$$

$$= \cancel{(x^2-5x+8)(x^3+7x+8)}\left[\frac{5x^4-20x^3+45x^2-52x+11}{\cancel{(x^2-5x+8)(x^3+7x+9)}}\right]$$

$$= 5x^4-20x^3+45x^2-52x+11$$

We can say that the value of $\frac{dy}{dx}$ is same using three different method

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MATHEMATICS

CLASS-XII-CBSE

DIFFERENTIABILITY EXRCISE 5.6

Prepared By:
PARDEEP KUMAR

DNG TUTORIAL
Crack the Concepts, Not just the Exams

If x and y are connected parametrically by the equations given in exercises 1 to 10 without eliminating the parameter find $\frac{dy}{dx}$

1. It involves finding the derivatives of a function expressed in parametric form, where both x and y are defined in terms of a third variable (the parameter often t)
The idea behind this is to use chain rule to relate the derivative of x and y with respect to the parameter.

e.g if $\quad x = \cos t \qquad y = \sin t$

$$\frac{dx}{dt} = -\sin t \qquad \frac{dy}{dt} = \cos t$$

$$\therefore \quad \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t}{-\sin t}$$

$$\frac{dy}{dx} = -\cot t$$

So if $\quad x = x(t)$ and $y = y(t)$, then

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \quad \text{provided that } \frac{dx}{dt} \neq 0$$

Q No. 1 $\quad x = 2at^2$ and $y = at^4$

diff. both sides w.r.t t

$$\frac{dx}{dt} = 4at \qquad \frac{dy}{dt} = 4at^3$$

we know $\quad \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4at^3}{4at} = t^2$

2 $x = a\cos\theta$ and $y = b\cos\theta$

diff. both sides w.r.t θ

$\frac{dx}{d\theta} = \frac{d}{d\theta}(a\cos\theta)$ $\frac{d}{d\theta}(y) = \frac{d}{d\theta}(b\cos\theta)$

$= a\frac{d}{d\theta}\cos\theta$ $\frac{dy}{d\theta} = b\frac{d}{d\theta}\cos\theta$

$= a(-\sin\theta)$ $\frac{dy}{d\theta} = b(-\sin\theta) = -b\sin\theta$

$= -a\sin\theta$

$\because$ we know $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-b\sin\theta}{-a\sin\theta} = \frac{b}{a}$ Ans

Q.No.3 $x = \sin t$ and $y = \cos 2t$

diff. both sides w.r.t t

$\frac{dx}{dt} = \frac{d}{dt}(\sin t)$ $\frac{dy}{dt} = \frac{d}{dt}(\cos 2t)$

$= \cos t$ $= -\sin 2t\frac{d}{dt}(2t)$

$= -2.\sin 2t$

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2\sin 2t}{\cos t} =$ ~~Ans~~

$\frac{dy}{dx} = -\frac{2\sin 2t}{\cos t}$ Ans

or

$\frac{dy}{dx} = \frac{-2\cdot 2\sin t\cos t}{\cos t} = -4\sin t$ Ans

Q No.4 $x = 4t$ $\quad y = \frac{4}{t}$

diff. both sides w.r.t t

$\frac{dx}{dt} = \frac{d}{dt}(4t)$ $\quad \frac{dy}{dt} = \frac{d}{dt}\left(\frac{4}{t}\right) = \frac{d}{dt}(4t^{-1})$

$= 4 \times 1 = 4$ $\quad = 4\frac{d}{dt}(t^{-1})$

$= -4(t^{-2}) = \frac{-4}{t^2}$

$\because \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-4/t^2}{4} = \frac{-1}{t^2}$

Q No.5 $x = \cos\theta - \cos 2\theta$ $\quad y = \sin\theta - \sin 2\theta$

diff both sides w.r.t θ

$\frac{dx}{d\theta} = \frac{d}{d\theta}(\cos\theta - \cos 2\theta)$

$= \frac{d}{d\theta}(\cos\theta) - \frac{d}{d\theta}(\cos 2\theta)$

$= -\sin\theta - (-\sin 2\theta)\frac{d}{d\theta}(2\theta)$

$= -\sin\theta + \sin 2\theta \cdot 2$

$= 2\cdot\sin 2\theta - \sin\theta \rightarrow$ i)

$= 2\cdot 2\sin\theta\cos\theta - \sin\theta$

$= \sin\theta(4\cos\theta - 1) \rightarrow$ ii)

$\frac{dy}{d\theta} = \frac{d}{d\theta}(\sin\theta - \sin 2\theta)$

$= \frac{d}{d\theta}(\sin\theta) - \frac{d}{d\theta}(\sin 2\theta)$

$= \cos\theta - \cos 2\theta \frac{d}{d\theta} 2\theta$

$= \cos\theta - 2\cos 2\theta$

$= \cos\theta - 2\cos 2\theta \rightarrow$ iii)

If we use (i) and (iii)

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cos\theta - 2\cos 2\theta}{2\sin 2\theta - \sin\theta}$ Ans

If we use ii) and (iii)

$\frac{dy}{dx} = \frac{\cos\theta - 2\cos 2\theta}{\sin\theta(4\cos\theta - 1)}$ Ans

Q.No.6 $x = a(\theta - \sin\theta)$ $y = a(1+\cos\theta)$

diff. both sides w.r.t. θ.

$$\frac{dx}{d\theta} = \frac{d}{d\theta}(a(\theta - \sin\theta))$$
$$= a\frac{d}{d\theta}(\theta - \sin\theta)$$
$$= a\left[\frac{d}{d\theta}(\theta) - \frac{d}{d\theta}(\sin\theta)\right]$$
$$= a\left[1 - \cos\theta\right]$$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}a(1+\cos\theta)$$
$$= a\left[\frac{d}{d\theta}(1) + \frac{d}{d\theta}(\cos\theta)\right]$$
$$= a\left[-\sin\theta\right]$$
$$= -a\sin\theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-a\sin\theta}{a(1-\cos\theta)} = \frac{-\sin\theta}{(1-\cos\theta)}$$
$$= -\frac{2\sin\theta/2\cos\theta/2}{2\sin^2\theta/2} = -\frac{\cos\theta/2}{\sin\theta/2}$$
$$= -\cot\theta/2$$

Note $\because$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$$
$$\cos\theta = 1 - 2\sin^2\theta/2$$
$$\therefore 2\sin^2\frac{\theta}{2} = 1 - \cos\theta$$
$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

Q.No.7 $x = \dfrac{\sin^3 t}{\sqrt{\cos 2t}}$

diff. both sides w.r.t. t

$$\frac{dx}{dt} = \frac{d}{dx}\left(\frac{\sin^3 t}{\sqrt{\cos 2t}}\right) = \frac{d}{dx}\frac{(\sin t)^3}{\sqrt{\cos 2t}}$$

$x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$ $\quad y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$

$x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$

diff. both sides wrt t

$\frac{dx}{dt} = \frac{d}{dt}\left(\frac{\sin^3 t}{\sqrt{\cos 2t}}\right) = \frac{d}{dt}\frac{(\sin t)^3}{\sqrt{\cos 2t}}$

$= \frac{\sqrt{\cos 2t}\,\frac{d}{dt}(\sin t)^3 - \sin^3 t\,\frac{d}{dt}\sqrt{\cos 2t}}{(\sqrt{\cos 2t})^2}$

$= \frac{\sqrt{\cos 2t}\; 3\sin^2 t\,\frac{d}{dt}\sin t - \sin^3 t \cdot \frac{1}{2\sqrt{\cos 2t}}\,\frac{d}{dt}\cos 2t}{(\sqrt{\cos 2t})^2}$

$= \frac{3\sin^2 t\cos t\cdot\sqrt{\cos 2t} - \frac{\sin^3 t}{2\sqrt{\cos 2t}}\cdot(-\sin 2t)\frac{d}{dt}(2t)}{\cos 2t}$

$= \frac{3\sin^2 t\cos t\cdot\sqrt{\cos 2t} + \frac{\sin^3 t}{\cancel{2}\sqrt{\cos 2t}}\cdot\sin 2t\cdot\cancel{2}}{\cos 2t}$ (LCM)

$= \frac{3\sin^2 t\cos t\cdot\cos 2t + \sin^3 t\cdot 2\sin t\cos t}{\sqrt{\cos 2t}\cdot\cos 2t}$ $\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$

$= \frac{\sin^2 t\cos t\,(3\,\cancel{\sin t}\cos 2t + 2\sin^2 t)}{(\cos 2t)^{3/2}}$

$\therefore \frac{dx}{dt} = \frac{\sin^2 t\cos t\,(3\cos 2t + 2\sin^2 t)}{(\cos 2t)^{3/2}}$

$$y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$$

diff. both sides w.r.t t

$$\frac{dy}{dt} = \frac{d}{dt}\left[\frac{\cos^3 t}{\sqrt{\cos 2t}}\right] = \frac{d}{dt}\frac{(\cos t)^3}{\sqrt{\cos 2t}}$$

$$= \frac{\sqrt{\cos 2t}\,\frac{d}{dt}(\cos t)^3 - (\cos t)^3 \frac{d}{dt}\sqrt{\cos 2t}}{(\sqrt{\cos 2t})^2}$$

$$= \frac{\sqrt{\cos 2t}\cdot 3(\cos^2 t)\frac{d}{dt}\cos t - \cos^3 t\,\frac{1}{2\sqrt{\cos 2t}}\cdot\frac{d}{dx}\cos 2t}{(\sqrt{\cos 2t})^2}$$

$$= \frac{3\cos^2 t(-\sin t)\sqrt{\cos 2t} - \frac{\cos^3 t}{2\sqrt{\cos 2t}}\cdot(-\sin 2t)\frac{d}{dt}(2t)}{(\sqrt{\cos 2t})^2}$$

$$= \frac{-3\sin t\cos^2 t\sqrt{\cos 2t} + \frac{\cos^3 t}{2\sqrt{\cos 2t}}\cdot \sin 2t\cdot 2}{(\sqrt{\cos 2t})^2}$$

$$= \frac{-3\sin t\cos^2 t\sqrt{\cos 2t} + \frac{\cos^3 t}{\sqrt{\cos 2t}}\cdot 2\sin t\cos t}{(\sqrt{\cos 2t})^2}$$

$$= \frac{-3\sin t\cos^2 t\cdot\cos 2t + 2\sin t\cos^4 t}{\sqrt{\cos 2t}\cdot(\sqrt{\cos 2t})^2}$$

$$= \frac{\sin t\cos^2 t(-3\cos 2t + 2\cos^2 t)}{(\sqrt{\cos 2t})^{3/2}}$$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$= \frac{\dfrac{\sin t\cos^2 t\,(2\cos^2 t - 3\cos 2t)}{(\cos 2t)^{3/2}}}{\dfrac{\sin^2 t\cos t\,(3\cos 2t + 2\sin^2 t)}{(\cos 2t)^{3/2}}}$$

$$= \frac{\cos t}{\sin t}\left(\frac{2\cos^2 t - 3\cos 2t}{3\cos 2t + 2\sin^2 t}\right)$$

$$\frac{dy}{dx} = \cot t\cdot\left(\frac{2\cos^2 t - 3\cos 2t}{3\cos 2t + 2\sin^2 t}\right)$$

$$= \cot t\left(\frac{2\cos^2 t - 3(2\cos^2 t - 1)}{3(1-2\sin^2 t) + 2\sin^2 t}\right)$$

$$= \cot t\left[\frac{2\cos^2 t - 6\cos^2 t + 3}{3 - 6\sin^2 t + 2\sin^2 t}\right]$$

$$= \cot t\left[\frac{3 - 4\cos^2 t}{3 - 4\sin^2 t}\right] = \frac{\cos t}{\sin t}\left[\frac{3 - 4\cos^2 t}{3 - 4\sin^2 t}\right]$$

$$= \frac{3\cos t - 4\cos^3 t}{3\sin t - 4\sin^3 t} = \frac{-(4\cos^3 t - 3\cos t)}{(3\sin t - 4\sin^3 t)}$$

$$= -\left(\frac{\cos 3t}{\sin 3t}\right) = \boxed{-\cot 3t}$$

$$\therefore \boxed{\frac{dy}{dx} = -\cot 3t}$$ Ans.

Q No.8 $x = a(\cos t + \log\tan\frac{t}{2})$; $y = a\sin t$

let $x = a(\cos t + \log\tan\frac{t}{2})$

diff. both sides w.r.t. t

$\frac{d}{dt}(x) = \frac{d}{dt}(a(\cos t + \log\tan\frac{t}{2}))$

$\frac{dx}{dt} = a\frac{d}{dt}(\cos t + \log\tan\frac{t}{2})$

$= a\left[\frac{d}{dt}(\cos t) + \frac{d}{dt}\log\tan\frac{t}{2}\right.$

$= a\left[-\sin t + \frac{1}{\tan\frac{t}{2}}\frac{d}{dt}(\tan\frac{t}{2})\right.$

$= a\left[-\sin t + \frac{1}{\tan\frac{t}{2}}\cdot\sec^2\frac{t}{2}\frac{d}{dt}\frac{t}{2}\right.$

$= a\left[-\sin t + \frac{1}{\tan\frac{t}{2}}\cdot\sec^2\frac{t}{2}\left(\frac{1}{2}\right)\right]$

$= a\left[-\sin t + \frac{\cos t/2}{\sin t/2}\cdot\frac{1}{\cos^2\frac{t}{2}}\cdot\frac{1}{2}\right]$

$= a\left[-\sin t + \frac{1}{2\sin\frac{t}{2}\cdot\cos t/2}\right]$

$= a\left[-\sin t + \frac{1}{\sin 2(\frac{t}{2})}\right] = a\left[-\sin t + \frac{1}{\sin t}\right]$

$= a\left[\frac{1}{\sin t} - \sin t\right] = a\left[\frac{1-\sin^2 t}{\sin t}\right] = \frac{a\cos^2 t}{\sin t}$

$= a\frac{\cos^2 t}{\sin t}$

$y = a\sin t$

diff. both sides wrt to t

$\frac{dy}{dt} = \frac{d}{dt}(a\sin t)$

$= a\frac{d}{dt}(\sin t) = a\cos t$

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cancel{a}\cos t}{\cancel{a}\cos^2 t} \times \sin t = \frac{\sin t}{\cos t}$

$= \tan t$

$\therefore \boxed{\frac{dy}{dx} = \tan t}$ Ans

Q No. 9 $x = a\sec\theta$ $y = b\tan\theta$

diff. both sides wrt θ

$\frac{dx}{d\theta} = \frac{d}{d\theta}(a\sec\theta) = a\frac{d}{d\theta}\sec\theta = a\sec\theta\tan\theta$

$y = b\tan\theta$

diff. both sides wrt θ

$\frac{dy}{d\theta} = \frac{d}{d\theta}(b\tan\theta)$

$= b\frac{d}{d\theta}\tan\theta$

$= b\sec^2\theta$

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\sec\theta\tan\theta}{b\sec^2\theta} = \frac{a}{b}\frac{\sin\theta}{\cos\theta} \times \cos\theta$

$= \frac{a}{b}\sin\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{b\sec^2\theta}{a\sec\theta\tan\theta} = \frac{b}{a}\frac{\sec\theta}{\sin\theta} \times \cos\theta$$

$$= \frac{b}{a}\frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta} = \frac{b}{a} \times \frac{1}{\sin\theta} = \frac{b}{a}\cancel{\sin\theta}$$

$$= \frac{b}{a}\operatorname{cosec}\theta.$$

Q No. 10 $x = a(\cos\theta + \theta\sin\theta)$; $y = a(\sin\theta - \theta\cos\theta)$

Sol: $x = a(\cos\theta + \theta\sin\theta)$

diff. both sides wrt θ

$$\frac{dx}{d\theta} = \frac{d}{d\theta}(a(\cos\theta + \theta\sin\theta)$$

$$= a\frac{d}{d\theta}(\cos\theta + \theta\sin\theta)$$

$$= a\left[\frac{d}{d\theta}\cos\theta + \frac{d}{d\theta}\theta\sin\theta\right]$$

$$= a\left[-\sin\theta + d\left[\sin\theta\frac{d}{d\theta}(\theta) + \theta\frac{d}{d\theta}\sin\theta\right]\right]$$

$$= a\left[-\sin\theta + \left[\sin\theta(1) + \theta\cdot(\cos\theta)\right]\right]$$

$$= a\left[-\cancel{\sin\theta} + \cancel{\sin\theta} + \theta\cdot\cos\theta\right]$$

$$= a\cdot\theta\cdot\cos\theta$$

Now

$$y = a(\sin\theta - \theta\cos\theta)$$

diff both sides wrt θ

$$\frac{dy}{d\theta} = \frac{d}{d\theta}(a(\sin\theta - \theta\cos\theta)$$

$$\frac{dy}{d\theta} = a\left[\frac{d}{d\theta}\sin\theta - \frac{d}{d\theta}\theta.\cos\theta\right]$$

$$= a\left[\cos\theta - \left[\cos\theta\frac{d}{d\theta}(\theta) + \theta\frac{d}{d\theta}\cos\theta\right]\right]$$

$$= a\left[\cos\theta - (1)\cos\theta - \theta(-\sin\theta)\right]$$

$$= a\left[\cancel{\cos\theta} - \cancel{\cos\theta} + \theta\sin\theta\right]$$

$$= a(\theta\sin\theta)$$

$$\frac{dy}{dx} = \frac{dy|d\theta}{dx|d\theta} = \frac{a\cdot\theta\sin\theta}{a\theta\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\therefore \boxed{\frac{dy}{dx} = \tan\theta} \quad \text{Ans}$$

Q.No.11 If $x = \sqrt{a^{\sin^{-1}t}}$ and $y = \sqrt{a^{\cos^{-1}t}}$

show that $\frac{dy}{dx} = -\frac{y}{x}$

Sol: $x = \sqrt{a^{\sin^{-1}t}}$

diff. both sides wrt t

$$\frac{dx}{dt} = \frac{d}{dx}\left(\sqrt{a^{\sin^{-1}t}}\right)$$

$$= \frac{1}{2\sqrt{a^{\sin^{-1}t}}}\frac{d}{dx}a^{\sin^{-1}t} = \frac{1}{2\sqrt{a^{\sin^{-1}t}}}\cdot a^{\sin^{-1}t}\log a\frac{d}{dx}\sin^{-1}t$$

$$= \frac{1}{2\sqrt{a^{\sin^{-1}t}}}\cdot a^{\sin^{-1}t}\cdot\frac{1}{\sqrt{1-t^2}}\cdot\log a$$

$$= \frac{1}{2(a^{\sin^{-1}t})^{\frac{1}{2}}}\cdot a^{\sin^{-1}t}\cdot\frac{1}{\sqrt{1-t^2}}\cdot\log a$$

$$= \frac{1}{2}\left(a^{\sin^{-1}t}\right)^{\frac{1}{2}} \cdot \frac{1}{\sqrt{1-t^2}} \cdot \log a$$

$$= \frac{1}{2}\left(a^{\sin^{-1}t}\right)^{\frac{1}{2}} \cdot \log a \cdot \frac{1}{\sqrt{1-t^2}}$$

Now $y = \sqrt{a^{\cos^{-1}t}}$

diff both sides w.r.t t

$$\frac{dy}{dt} = \frac{d}{dt}\left(\sqrt{a^{\cos^{-1}t}}\right)$$

$$= \frac{a^{\cos^{-1}t}}{2\sqrt{a^{\cos^{-1}t}}} \log a \cdot \frac{d}{dt} \cos^{-1}t$$

$$= \cancel{a^{\cos}}\, \frac{1}{2}\left(a^{\cos^{-1}t}\right)^{\frac{1}{2}} \cdot \log a \cdot \left(\frac{-1}{\sqrt{1-t^2}}\right)$$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cancel{\frac{1}{2}}\left(a^{\cos^{-1}t}\right)^{\frac{1}{2}} \cdot \cancel{\log(a)}\left(-\cancel{\frac{1}{\sqrt{1-t^2}}}\right)}{\cancel{\frac{1}{2}}\left(a^{\sin^{-1}t}\right)^{\frac{1}{2}} \cdot \cancel{\log a} \cdot \cancel{\frac{1}{\sqrt{1-t^2}}}}$$

$$= \frac{-\sqrt{a^{\cos^{-1}t}}}{\sqrt{a^{\sin^{-1}t}}} = \frac{-y}{x}$$

$$\therefore \boxed{\frac{dy}{dx} = \frac{-y}{x}}$$ Ans

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MATHEMATICS

CLASS-XII-CBSE

DIFFERENTIABILITY EXRCISE 5.7

Prepared By:
PARDEEP KUMAR

DNG TUTORIAL
Crack the Concepts, Not just the Exams

Q.No.1 x^2+3x+2

Sol: let $y = x^2+3x+2$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(x^2+3x+2) = 2x+3$$

again diff. both sides

$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}(2x+3) = 2$$

$$\therefore \boxed{\frac{d^2y}{dx^2} = 2}$$ Ans.

Q.No.2 x^{20}

Sol: $\text{d}\ y = x^{20}$

$$\therefore \frac{dy}{dx} = 20x^{19}$$

diff. again both sides

$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}(20x^{19}) = 380x^{18}$$

$$\therefore \boxed{\frac{d^2y}{dx^2} = 380x^{18}}$$ Ans

Q.No.3 $x \cdot \cos x$

$y = x \cdot \cos x$

diff. both sides

$$\frac{dy}{dx} = \frac{d}{dx}(x \cdot \cos x) = \cos x \frac{d}{dx}(x) + x\left(\frac{d}{dx} \cdot \cos x\right)$$

$$= \cos x + x(-\sin x) = \cos x - x\sin x$$

diff. again both sides

$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}(\cos x - x\sin x)$$

$$= \frac{d^2}{dx^2}(\cos x) - \frac{d^2}{dx^2}(x\sin x)$$

$$= -\sin x - \left[\sin x \frac{d^2}{dx^2}(x) + x\frac{d^2}{dx^2}(\sin x)\right]$$

$$= -\sin x - [\sin x + x\cos x]$$

$$= -\sin x - \sin x + x\cos x$$

$$= -2\sin x - x\cos x$$

$$\therefore \frac{d^2y}{dx^2} = -(2\sin x + x\cos x) \quad \text{Ans}$$

Q No. 4 $\log x$

$y = \log x$

diff. both sides

$$\frac{dy}{dx} = \frac{d}{dx}(\log x) = \frac{1}{x}$$

again diff. both sides

$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$$

$$\therefore \boxed{\frac{d^2y}{dx^2} = -\frac{1}{x^2}} \quad \text{Ans}$$

Q.No.5 $x^3 \log x$

let $y = x^3 \log x$

diff. both sides

$$\frac{dy}{dx} = \frac{d}{dx}(x^3 \log x)$$

$$= \log x \frac{d}{dx} x^3 + x^3 \frac{d}{dx} \log x$$

$$= \log x \cdot 3x^2 + x^3 \cdot \frac{1}{x} = 3x^2 \log x + x^2.$$

Now again diff. both sides

$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}(3x^2 \log x + x^2)$$

$$= \frac{d^2}{dx^2}(3x^2 \log x + \frac{d^2}{dx^2}(x^2)$$

$$= \left[\log x \frac{d^2}{dx^2}(3x^2 + 3x^2 \frac{d^2}{dx^2} \log x\right] + 2x$$

$$= \left[6x \cdot \log x + 3x^2 \cdot \frac{1}{x}\right] + 2x$$

$$= 6x \cdot \log x + 3x + 2x$$

$$= 6x \cdot \log x + 5x$$

$$\therefore \boxed{\frac{d^2y}{dx^2} = 6x \cdot \log x + 5x}$$ Ans

Q No.6 $e^x \sin 5x$

let $y = e^x \sin 5x$

diff. both sides

$$\frac{dy}{dx} = \frac{d}{dx}(e^x \sin 5x)$$

$$= \sin 5x \frac{d}{dx} e^x + e^x \frac{d}{dx}(\sin 5x)$$

$$= e^x \sin 5x + e^x \cos 5x \frac{d}{dx}(5x)$$

$$= e^x \sin 5x + e^x \cos 5x \cdot 5$$

$$= e^x \sin 5x + 5e^x \cos 5x$$

diff. again both sides

$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}(e^x \sin 5x + 5e^x \cos 5x)$$

$$= \frac{d^2}{dx^2}(e^x \sin 5x) + \frac{d^2}{dx^2}(5e^x \cos 5x)$$

$$= \sin 5x \frac{d^2}{dx^2}(e^x) + e^x \frac{d^2}{dx^2}(\sin 5x) + 5\left[\cos 5x \frac{d^2}{dx^2} e^x + e^x \frac{d^2}{dx^2} \cos 5x\right.$$

$$= \sin 5x\, e^x + e^x \cdot \cos 5x \frac{d}{dx} 5x + 5\left[e^x \cos 5x + e^x(-\sin 5x)\frac{d^2}{dx^2}(5x)\right.$$

$$= e^x \sin 5x + e^x \cos 5x(5) + 5\left[e^x \cos 5x - 5e^x \sin 5x\right)$$

$$= e^x \sin 5x + 5e^x \cos 5x + 5e^x \cos 5x - 25e^x \sin 5x$$

$$= 10e^x \cos 5x - 24e^x \sin 5x$$

$$\therefore \boxed{\frac{d^2y}{dx^2} = 2e^x(5\cos 5x - 12\sin 5x)}$$ Ans

Q No.7 $e^{6x}\cos 3x$

let $y = e^{6x}\cos 3x$

diff. both sides, we have

$$\frac{dy}{dx} = \frac{d}{dx}(e^{6x}\cos 3x) = \cos 3x \frac{d}{dx}e^{6x} + e^{6x}\frac{d}{dx}\cos 3x$$

$$= e^{6x}\frac{d}{dx}(6x)\cdot\cos 3x + e^{6x}(-\sin 3x)\frac{d}{dx}(3x)$$

$$= e^{6x}\cdot 6(1)\cdot\cos 3x - e^{6x}\sin 3x(3)$$

$$= 6e^{6x}\cos 3x - 3e^{6x}\sin 3x$$

again diff. both sides, we have

$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}(6e^{6x}\cos 3x - 3e^{6x}\sin 3x)$$

$$= \cos 3x\frac{d}{dx}(6e^{6x}) + 6e^{6x}\frac{d}{dx}\cos 3x - 3\left[\frac{d}{dx}e^{6x}\cdot\sin 3x + e^{6x}\frac{d}{dx}\sin 3x\right]$$

$$= 6e^{6x}\cos 3x\frac{d}{dx}6x + 6e^{6x}(-\sin 3x)\frac{d}{dx}(3x) - 3\left[e^{6x}\frac{d}{dx}6x\cdot\sin 3x + e^{6x}\frac{d}{dx}(\sin 3x)\frac{d}{dx}(3x)\right]$$

$$= 6e^{6x}\cos 3x \times 6 + 6e^{6x}(-\sin 3x)(3) - 3\left[6e^{6x}\sin 3x + e^{6x}(\cos 3x)(3)\right]$$

$$= 36e^{6x}\cos 3x - 18e^{6x}\sin 3x - 18e^{6x}\sin 3x - 9e^{6x}\cos 3x$$

$$= 27e^{6x}\cos 3x - 36e^{6x}\sin 3x$$

$$= 9e^{6x}(3\cos 3x - 4\sin 3x)$$ Ans

Q No. 8 $\tan^{-1}x$

let $y = \tan^{-1}x$

diff. both sides

$$\frac{dy}{dx} = \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$$

Now again diff. both sides

$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}\left(\frac{1}{1+x^2}\right)$$

$$= \frac{(1+x^2)\frac{d^2}{dx^2}(1) - 1\frac{d^2}{dx^2}(1+x^2)}{(1+x^2)^2}$$

$$= \frac{(1+x^2)(0) - (1)\left[\frac{d^2}{dx^2}(1) + \frac{d^2}{dx^2}(x^2)\right]}{(1+x^2)^2}$$

$$= \frac{-1\left[0+2x^2\right]}{(1+x^2)^2} = \frac{-2x^2}{(1+x^2)^2}$$

$\therefore$ $\boxed{\frac{d^2y}{dx^2} = \frac{-2x^2}{(1+x^2)^2}}$ Ans.

Q No. 9 $\log(\log x)$

$y = \log(\log x)$

diff. both sides

$$\frac{dy}{dx} = \frac{d}{dx}(\log(\log x)) = \frac{1}{\log x}\frac{d}{dx}(\log x) = \frac{1}{\log x}\cdot\frac{1}{x} = \frac{1}{x}\cdot\frac{1}{\log x}$$

$$= \frac{1}{x\cdot\log x}$$

Now again diff. both sides

$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}\left(\frac{1}{x\log x}\right)$$

$$= \frac{x\cdot\log x \frac{d}{dx}(1) - (1)\frac{d}{dx}(x\cdot\log x)}{(x\log x)^2}$$

$$= \frac{x\log x\cdot(0) - 1\left[x\frac{d}{dx}\log x + \log x\frac{d}{dx}(x)\right]}{(x\log x)^2}$$

$$= -1\left[\frac{x\times\frac{1}{x} + \log x(1)}{(x\log x)^2}\right]$$

$$= -1\left[\frac{1+\log x}{(x\log x)^2}\right]$$

$$\therefore \frac{d^2y}{dx^2} = -\frac{(1+\log x)}{(x\log x)^2}$$

QNo. 10 $\sin(\log x)$

let $y = \sin(\log x)$

diff. both sides, we get

$$\frac{dy}{dx} = \frac{d}{dx}(\sin(\log x)) = \cos(\log x)\frac{d}{dx}\log x$$

$$= \cos(\log x)\cdot\frac{1}{x} = \frac{\cos(\log x)}{x}$$

Now diff. both sides again

$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}\left(\cos(\log x)\cdot\frac{1}{x}\right)$$

$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}\left(\frac{\cos(\log x)}{x}\right)$$

$$= \frac{x\frac{d^2}{dx^2}\cos(\log x) - \cos(\log x)\frac{d}{dx^2}(x)}{x^2}$$

$$= \frac{x\left(-\sin(\log x)\frac{d^2}{dx^2}.\log x - \cos(\log x)(1)\right.}{x^2}$$

$$= \frac{(x)(-\sin(\log x)\cdot\frac{1}{x} - \cos(\log x)}{x^2}$$

$$= \frac{-\sin(\log x) - \cos(\log x)}{x^2}$$

$$\therefore \frac{d^2y}{dx^2} = -\left(\frac{\sin(\log x) + \cos(\log x)}{x^2}\right)$$

Q No. 11 $y = 5\cos x - 3\sin x$ show that $\frac{d^2y}{dx^2} + y = 0$

diff. both sides

$$\frac{dy}{dx} = \frac{d}{dx}(5\cos x - 3\sin x)$$

$$= \frac{d}{dx}(5\cos x) - \frac{d}{dx}(3\sin x)$$

$$= -5\sin x - 3\cos x = -\left[5\sin x + 3\cos x\right]$$

diff. again both sides

$$\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}\left[-(5\sin x + 3\cos x)\right]$$

$$= -\left[\frac{d^2}{dx^2}(5\sin x) + \frac{d^2}{dx^2}(3\cos x)\right]$$

$$= -\left[5\cos x - 3\sin x\right]$$

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Now

$$\frac{d^2y}{dx^2}+y=-\left[5\cos x-3\sin x\right]+5\cos x-3\sin x$$

$$=-5\cos x+3\sin x+5\cos x-3\sin x$$

$$=0$$

$$\therefore \boxed{\frac{d^2y}{dx^2}+y=0} \quad \text{Ans}$$

Q No. 12 If $y=\cos^{-1}x$ find $\frac{d^2y}{dx^2}$ in terms y alone

$\because \quad y=\cos^{-1}x \Rightarrow x=\cos y$

diff. both sides

$$\frac{dy}{dx}=\frac{d}{dx}(\cos^{-1}x)=\frac{-1}{\sqrt{1-x^2}}=\frac{-1}{\sqrt{1-\cos^2 y}}=-\frac{1}{\sin y}$$

$$\frac{dy}{dx}=-\operatorname{cosec} y$$

diff. again both sides

$$\frac{d^2y}{dx^2}=\frac{d^2}{dx^2}(-\operatorname{cosec} y)=-\frac{d^2}{dx^2}(\operatorname{cosec} y)$$

$$=-\operatorname{cosec} y\cot y\frac{dy}{dx}$$

$$=-\operatorname{cosec} y\cot y(-\operatorname{cosec} y)$$

$$\therefore \boxed{\frac{d^2y}{dx^2}=\operatorname{cosec}^2 y\cot y} \quad \text{Ans}$$

Q.No.13 If $y = 3\cos(\log x) + 4\sin(\log x)$
then find $x^2y_2 + xy_1 + y = 0$

$\because$ $y = 3\cos(\log x) + 4\sin(\log x)$

diff. both sides, we get

$\frac{dy}{dx} = \frac{d}{dx}(3\cos(\log x) + 4\sin(\log x))$

$= \frac{d}{dx}(3\cos(\log x) + \frac{d}{dx} 4\sin(\log x)$

$= 3(-\sin(\log x) \cdot \frac{d}{dx}\log x + 4\cos(\log x)\frac{d}{dx}\log x$

$= -3\sin(\log x) \cdot \frac{1}{x} + 4\cos(\log x) \cdot \frac{1}{x}$

Now if we find

$xy_1 = x\left[\frac{-3\sin(\log x) + 4\cos(\log x)}{x}\right]$

$= 4\cos(\log x) - 3\sin(\log x) \rightarrow (i)$

Now we need x^2y_2

diff. both sides of (i)

$\therefore$

$\frac{d^2}{dx}$

$\frac{d^2}{dx^2}(xy_1) = \frac{d^2}{dx^2}(4\cos(\log x) - 3\sin(\log x))$

$y_1\frac{d^2}{dx^2}x + x\frac{d^2}{dx^2}(y_1) = \frac{d^2}{dx^2}(4\cos(\log x) - \frac{d^2}{dx^2}(3\sin(\log x))$

$xy_2 + y_1 \quad = -4\sin(\log x) \cdot \frac{1}{x} - 3\cos(\log x) \cdot \frac{1}{x}$

$xy_2 + y_1 \quad = \frac{-4\sin(\log x) \cdot - 3\cos(\log x)}{x}$

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$x^2y_2 + xy_1 = -4\sin(\log x) - 3\cos(\log x)$

$x^2y_2 + xy_1 = -(4\sin(\log x) + 3\cos(\log x))$

$x^2y_2 + xy_1 = -y$

$\therefore\ x^2y_2 + xy_1 + y = 0$

Q.No.14 If $y = Ae^{mx} + Be^{nx}$, show that

$$\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$$

$\because$ we have

$$y = Ae^{mx} + Be^{nx}$$

diff. both sides

$$\frac{dy}{dx} = \frac{d}{dx}(Ae^{mx} + Be^{nx}) = \frac{d}{dx}(Ae^{mx}) + \frac{d}{dx}Be^{nx}$$

$$= Ae^{mx}\frac{d}{dx}mx + Be^{nx}\frac{d}{dx}nx$$

$$= Ae^{mx}\cdot m + Be^{nx}\cdot n = Ae^{mx}\cdot m + Be^{nx}\cdot n$$

$$= A$$

Now $\frac{d^2y}{dx^2} = \frac{d^2}{dx^2}(Ae^{mx}\cdot m + Be^{nx}\cdot n)$

$$= \frac{d^2}{dx^2}(Am\cdot e^{mx}) + \frac{d^2}{dx^2}(Bn\cdot e^{nx}$$

$$= Am\frac{d^2}{dx^2}e^{mx} + Bn\cdot\frac{d^2}{dx^2}e^{nx}$$

$$= Am\cdot e^{mx}\cdot m + Bn\cdot e^{nx}\cdot n$$

$$= Am^2\cdot e^{mx} + Bn^2e^{nx}$$

Now

$$\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny$$

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$$= Am^2\cdot e^{mx} + Bn^2e^{nx} - (m+n)(Ame^{mx} + Bne^{nx}) + mny$$

$$= Am^2e^{mx} + Bn^2e^{nx} - Am^2e^{mx} - Bnme^{nx} - Amne^{mx} - Bn^2e^{nx} + mny$$

$= -Bnme^{nx} - Amne^{mx} + mny$

$= -mn(Be^{nx} + Ae^{mx}) + mny$

$= -mn(y) + mny$

$= -\cancel{mny} + \cancel{mny} = 0$

hence proved